

DESIGNING AND CONDUCTING SURVE RESEARCH

A COMPREHENSIVE GUIDE

LOUIS M. REA RICHARD A. PARKER

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DESIGNING AND CONDUCTING SURVEY RESEARCH

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DESIGNING AND Conducting Survey Research

A Comprehensive Guide

FOURTH EDITION

Louis M. Rea Richard A. Parker



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CONTENTS

Figures, Tables, Exhibits, and Worksheets vii Preface xi

The Authors xv

PART ONE: DEVELOPING AND ADMINISTERING QUESTIONNAIRES 1

- 1 An Overview of the Sample Survey Process 3
- 2 Designing Effective Questionnaires: Basic Guidelines 36
- 3 Developing Survey Questions 59
- 4 Using Focus Groups in the Survey Research Process 80

PART TWO: ENSURING SCIENTIFIC ACCURACY 97

- 5 Descriptive Statistics: Measures of Central Tendency and Dispersion 99
- 6 The Theoretical Basis of Sampling 135
- 7 Confidence Intervals and Basic Hypothesis Testing 146

Contents

- 8 Determining the Sample Size 163
- 9 Selecting and Obtaining a Representative Sample 177

PART THREE: PRESENTING AND ANALYZING SURVEY RESULTS 201

- 10 Analyzing Cross-Tabulated Data 203
- 11 Testing the Difference between Means 235
- 12 Regression and Correlation 261
- 13 Preparing an Effective Final Report 292

Resource A: Table of Areas of a Standard Normal Distribution 305 Resource B: Glossary 311 Bibliography 321 Index 325

FIGURES, TABLES, EXHIBITS, AND WORKSHEETS

Figures

- 5.1 The Normal Curve 123
- 5.2 Distribution with Positive Skew 125
- 5.3 Distribution with Negative Skew 125
- 5.4 Histogram 129
- 6.1 Area under the Normal Curve 137
- 6.2 Proportionate Areas under the Normal Curve for Military Weights 138
- 6.3 Standardized Proportionate Areas under the Normal Curve: An Example 139
- 6.4 Distribution of Sample Means 142
- 6.5 Hypothetical Normal Distributions for Sample Means Compared to Raw Data 143
- 6.6 Properties of Normal Curves Applied to Hypothetical Distribution of Sample Mean Weights 144
- Dual-Direction Research Question (Two-Tail Test at 95 Percent Confidence Level) 155
- 7.2 Single-Direction Research Question (One-Tail Test at 95 Percent Confidence Level) 156

- 8.1 Determining Sample Size 175
- 11.1 ANOVA Table for Public Works Employees 249
- 12.1 Rehabilitation Subsidy/Property Value Increase Analysis 262
- 12.2 Scatter Plot and Regression Line 263
- 12.3 Violent Crimes and Population 265
- 12.4 Prior Convictions and Sentence Length 266
- 12.5 ANOVA Table for Regression Line of Subsidy and Value Increases 269
- 13.1 Favor or Oppose Building Seawater Desalination Facilities 300
- 13.2 Rating of Police Response to Emergency and Nonemergency Calls 300
- 13.3 Mean Satisfaction Ratings for Various Features of Bus Service 301
- 13.4 Frequency of Sprinkler Adjustment by Households with an Automatically Controlled Sprinkler System 302

Tables

- 2.1 Ways in Which City Government Can Serve Community Needs 57
- 5.1 Political Leaning 100
- 5.2 Rating of Local Library Facilities 102
- 5.3 Number of Children per Household in Mt. Pilot, South Dakota 103
- 5.4 Consultants' Hourly Fees Paid by Cities in Pennsylvania 104
- 5.5 Travel Time to Work in Jersey City, New Jersey 107
- 5.6 Police Nonemergency Response Times 110
- 5.7 Willingness to Spend for Street and Highway Beautification 127
- 5.8 Impression of Effect on Montgomery City Government on Business 131
- 6.1 Distribution of 100 Hypothetical Sample Mean Weights 142
- 8.1 Minimum Sample Sizes for Variables Expressed as Proportions 169
- 8.2 Minimum Sample Sizes for Selected Small Populations 171
- 9.1 Proportionate Sample Representation for a Hypothetical Ethnic Distribution 186
- 9.2 Population of Davis Planning Areas 188
- 9.3 Outdoor Recreation Facilities Used Most Often by Davis Residents, by Planning Area 190
- 10.1 Number of Years as a Customer of the Richmond Water District 205

Figures, Tables, Exhibits, and Worksheets

- 10.2 Number of Years as a Customer of the Richmond Water District, by Area of Residence 205
- 10.3 Number of Years as a Customer of the Richmond Water District, by Area of Residence: Male Respondents 207
- 10.4 Opinion Concerning Capital Punishment by Political Party 212
- 10.5 Value of Physical Therapy, by Age of Respondent 218
- 10.6 Political Orientation of Teachers in the Santa Rosa School District, by Gender 221
- 10.7 Educational Level, by Socioeconomic Status 228
- 10.8 Most Frequently Used Mode of Transportation to Work, by Subarea 232
- 11.1 Promotional Examination Score, by Department Affiliation 236
- Statistically Significant Differences for Various Potential Uses of Recycled Water 303

Exhibits

- 2.1 Mail-Out Introduction 40
- 2.2 Telephone Introduction 41
- 2.3 In-Person Introduction 41
- 2.4 Series of Questions Demonstrating Sensitizing of Respondents 45
- 2.5 Filter or Screening Questions 48
- 2.6 Screening Used to Disqualify Respondents 48
- 4.1 Sample Focus Group Confirmation E-Mail 89
- 4.2 Structured Format for a Focus Group Discussion 91
- 5.1 Salaries of Employees of a Small, Private Business 126
- 7.1 Values of Calculated Z Scores to Determine Significance (according to Research Hypothesis Direction) 157
- 7.2 Critical Values of the *t* Distribution 159
- 9.1 Sample Sizes and Weights for the City of Davis Planning Areas 189
- 9.2 Sample Sizes and Weights for Stratified Sample for Table 9.1 191
- 10.1 Critical Values of the Chi-Square Distribution 215
- 10.2 Interpretation of Calculated Cramér's V, Phi, and Lambda Measures of Association 219
- 10.3 Interpretation of Calculated Gamma 229
- 11.1 Table of *F*-Statistics, P = .05 251
- 11.2 Table of *F*-Statistics, P = .01 252
- 11.3 Studentized Range (q) Distribution 254

- 12.1 Choosing the Appropriate Statistical Test or Measure 290
- 13.1 Example of a Cover Page 293

Worksheets

- 5.1 Determination of Median 104
- 5.2 Calculation of Mean Number of Children, Mt. Pilot, South Dakota 107
- 5.3 Calculation of Mean Travel Time to Work in Jersey City, New Jersey 108
- 5.4 Calculation of Standard Deviation for Individual Case Data 112
- 5.5 Calculation of Standard Deviation for a Frequency Distribution with Single-Value Categories 113
- 5.6 Calculation of Standard Deviations for Table 5.6 114
- 9.1 Calculation of Weighted Citywide Total for Outdoor Recreation Facilities Used Most Often 191
- 10.1 Chi-Square Matrix (n = 500) 213
- 11.1 Public Works Employees' Earnings 247
- 12.1 Calculation of Regression Line 264
- 12.2 Residual Sum of Squares 268

PREFACE

The sample survey research industry can expect to continue its rapid expansion in the years ahead. As we move through the twenty-first century, myriad technological and analytical innovations have firmly entrenched the probability sample as an indispensable part of life. The growing population and associated socioeconomic complexities, the strengthening of capitalism as a worldwide economic system, and the concurrent forces of democracy surfacing with strength around the globe guarantee the continued significance of sample survey research as a means of gathering data and understanding the interests, concerns, and behavior of people everywhere.

There is a shortage of well-focused, easily understood, yet theoretically and methodologically sound treatments of the sample survey process. Existing texts are generally highly technical and can be appreciated and used only by experts, or they are overly descriptive and not conducive to the successful implementation of a sample survey research project. Furthermore, sample survey research is frequently treated as a relatively small component of broader books that focus on quantitative methods; this treatment is often insufficiently detailed to serve practitioners in their professional capacities.

At the root of sample survey research is the discipline of statistics. Statistics is an advanced field of study, and traditionally people have had some difficulty mastering it and understanding its wide-ranging applicability to practical problems. One of the most difficult areas of statistics to comprehend is sampling theory. Yet it is through sample surveys that students and researchers most often gather the data they need to carry out their research agendas. Many statistics textbooks do not adequately convey the relationship between sampling theory and its application to the conduct of a survey research project. They tend to inadequately explain the linkage between the theory and its ultimate manifestation in practice.

As teachers and research consultants, we are particularly cognizant of the need to find an appropriate balance between statistical theory and its application. Accordingly, the purpose of this book is to enable readers to conduct a sample survey research project from the initial conception of the research focus to the preparation of the final report, including basic statistical analysis of the data. Eight years of working with the third edition prompted us to supplement, clarify, and refine certain aspects of the book in order to serve our audience better.

Audience

Designing and Conducting Survey Research is intended to serve two distinct audiences. One major audience consists of working professionals who wish to conduct a survey research project or have a framework for planning such a project. Sociologists, political scientists, psychologists, public administrators, city planners, and other social scientists who are faced with gathering data through a survey research project will find this book a useful reference for specific technical and procedural aspects of the survey research project. We also anticipate that professionals in government, private enterprise, and research agencies will make use of this book as a reference guide when the need to conduct, commission, or review sample surveys arises.

An equally important audience is undergraduate and graduate students who want to understand survey research as part of their education in quantitative and research methodology.

Overview of the Contents

Part 1 covers a major component of the survey research process: construction of the survey instrument. Chapter 1 presents an overview of the sample survey research process, including the advantages and disadvantages of the

Preface

major types of survey research: mail-out, telephone, web-based, intercept, and in-person interview surveys. The procedures for the administration of these surveys are also discussed. This chapter includes updated material on web panels, advances in telephone interviewing such as computer-assisted technology and the role of cell phones, ethical considerations in survey research, and the increasing usefulness of Internet surveys.

Chapters 2 and 3 pursue in detail the process of questionnaire development. Drawing on specific survey research projects we have conducted, we demonstrate the key components and principles associated with questionnaire design. Chapter 2 provides an overview of the process, including thorough discussions of question type and sequence. Chapter 3 delves into the specific guidelines for constructing individual questions and deals with such topics as phraseology, format, and, in particular, the avoidance of biasinducing questions.

Chapter 4 discusses the role of focus groups in preparing survey questionnaires and analyzing survey findings. The focus group process is presented in four stages: planning, recruiting, conducting, and analyzing. This chapter recognizes the rapidly growing importance of focus group research and draws on our own extensive experience in this area of research.

Part 2 addresses the more technical aspects of the survey research process, from its theoretical underpinnings to scientific sample selection procedures. Chapter 5 establishes the groundwork for the ensuing chapters through a treatment of descriptive statistics, including measures of central tendency, measures of dispersion, and the normal distribution. Chapter 6 explains how generalizations about an entire population can be made from just one sample consisting of a relatively small subset of that total population. Chapter 7 presents practical applications of the theory explained in the preceding chapter in terms of confidence intervals and the testing of simple hypotheses based on a sample. Chapter 8 thoroughly discusses the important considerations associated with determining an appropriate sample size for the conduct of sample surveys. It presents and explains equations for determining sample sizes and includes tables that demonstrate how the required sample size can be referenced quickly.

Chapters 6, 7, and 8 are particularly valuable contributions to the survey research literature. Their relevance is derived from a clear and focused presentation of complex material that historically has represented a difficult obstacle for students and practitioners to overcome.

Chapter 9 introduces the concepts of general population, working population, and sampling frame. Methods of probability and nonprobability sampling are explored. Cluster sampling, stratified sampling, and weighting procedures are discussed in concert with relevant examples. This chapter also introduces a new section on nonresponse bias.

Part 3 explores the analysis of survey findings, including basic statistical techniques. It presents the statistical techniques most commonly used in testing the statistical significance of survey data. It addresses the question, "Are the apparent relationships in the survey data, as displayed in tables, genuine, or are they the result of chance occurrences?" Chapter 10, referencing our personal research experience, presents a discussion of the appropriate use of cross-tabulated tables and introduces the chi-square test of significance along with Cramér's *V*, gamma, and lambda.

Chapter 11 discusses independent and paired samples *t* tests, difference of proportions, and analysis of variance, and chapter 12 addresses regression and correlation.

Chapter 13 provides rules and guidelines for the preparation of final research reports. It focuses on the graphical depictions of data and the appropriate use and organization of tables, while discussing the merging of text and statistical information into an integrated work that effectively communicates a study's findings to its intended audience.

This book is accompanied by a website supplement (www.www.wiley .com/college/rea) that provides chapter summaries and manual exercises that coordinate with the chapters in the book. Throughout the book, where appropriate, we have included detailed instructions for using the Statistical Package for the Social Sciences (SPSS) computer software package to perform the calculations and other statistical techniques that are demonstrated in the book. These instructions are geared to two online data sets (Transportation.sav and Water.sav). Chapters 5, 7, 9, 10, 11, and 12 are supplemented accordingly. The introduction of computer applications is the most important addition to this fourth edition of the book. The website supplement also includes computer exercises that begin with chapter 5.

Years of teaching statistics and survey research as well as extensive experience in private consulting motivated us to prepare a book that could serve both practitioners and academics. It is this combination of classroom and field experience that gives this work its unique approach and instructional value. We trust that readers will find this new edition to be useful and highly conducive to their research endeavors.

San Diego, California July 2014 Louis M. Rea Richard A. Parker

THE AUTHORS

Louis M. Rea is an emeritus professor of city planning in the School of Public Affairs at San Diego State University where he served as school director for nearly twenty-five years. He teaches graduate courses in statistical analysis, transportation planning, survey research, program design and evaluation, and urban/fiscal problems, as well as undergraduate courses in introductory public administration and policy and evaluation in Criminal Justice.

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Rea has had extensive experience as a researcher and consultant in the San Diego area since 1975 and has conducted surveys and focus group studies in numerous consulting and research assignments for municipal jurisdictions and private businesses throughout California. He has also prepared environmental impact reports and market analyses for various commercial and recreational developments. Among other projects, he has conducted program evaluations, analyzed the feasibility of assessment districts and direct benefit financing, conducted research in the areas of transportation and water management, and prepared demographic and economic profiles and projections for a multitude of public and private agencies. Rea has published a variety of articles in such journals as Urban Affairs Quarterly, Transportation Quarterly, and the American Review of Public Administration. He has participated in panel discussions and delivered numerous papers at professional conferences throughout the United States.

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Parker specializes in sample survey research for various governmental agencies and market research for retail, commercial, residential, and recreational development. He has placed particular emphasis on market research conducted in the fields of transportation and water, with myriad rider/on-board surveys, intercept surveys, telephone surveys, and focus group projects performed for several transportation and water agencies. He has testified as an expert witness in court on several occasions concerning sampling and survey research.

Parker has further established his reputation with regard to fiscal impact studies, urban development, and environmental impact and socioeconomic and demographic analyses. He has published a variety of articles, monographs, and books on these subjects and received the 2008 Bernays Mark of Merit for Special Purpose Publications. He has participated in various panel discussions, delivered expert testimony to legislatures, and appeared on several radio and television programs.

Together, Parker and Rea prepared the first fiscal impact analysis of the provision of public services to undocumented immigrants in the state of

The Authors

California. The analysis included extensive survey research, focus group discussions, and the use of advanced statistical techniques.

The authors have also conducted extensive on-board bus and rail surveys for some of the largest transportation authorities in the United States. These projects have required the use of advanced sampling techniques and the application of survey research principles that inform and benefit this edition of the book.

DESIGNING AND CONDUCTING SURVEY RESEARCH



PART ONE

DEVELOPING AND ADMINISTERING QUESTIONNAIRES



CHAPTER ONE

AN OVERVIEW OF THE SAMPLE SURVEY PROCESS

Curveys have become a widely used and acknowledged research tool in Omost of the developed countries of the world. Through reports presented by newspapers, magazines, television, and radio, the concept of considering information derived from a relatively small number of people to be an accurate representation of a significantly larger number of people has become a familiar one. Surveys have broad appeal, particularly in democratic cultures, because they are perceived as a reflection of the attitudes, preferences, and opinions of the very people from whom the society's policymakers derive their mandate. Politicians rely heavily on surveys and public opinion polls for popular guidance in mapping out campaign strategies and carrying out their professional responsibilities. Commercial enterprises use survey findings to formulate market strategies for the potential widespread use, distribution, and performance of new and existing products. Television and radio programs are evaluated and scheduled largely in accordance with the results of consumer surveys. Government programs designed to provide assistance to various communities often rely on the results of surveys to determine program effectiveness. Private social organizations obtain information from their members through the use of survey techniques. Libraries, restaurants, financial institutions, recreational facilities, and churches, among many others, make use of polls to solicit information from their constituents and clientele concerning desired services.

As a research technique in the social sciences and professional disciplines, survey research has derived considerable credibility from its widespread acceptance and use in academic institutions. Many universities have established survey research institutes where the techniques of survey research are taught and surveys can be conducted within the confines of propriety and scientific rigor. Students are often encouraged to use survey research for gathering primary data, thereby satisfying the requirement of conducting original research. Professors publish countless articles and books based on the results of funded and unfunded survey research projects.

Despite the broad-based societal acceptance of survey research, there remains a lingering doubt, especially within the general population at large, concerning the reliability of information derived from relatively few respondents purporting to represent the whole. They frequently ask, for instance, "How can fifteen hundred respondents to a survey be said to represent millions of people?" or, "Why should two thousand television viewers dictate to program directors on a national scale what Americans choose to watch?" The answers to these and other such questions lie in the systematic application of the technique of scientific sample survey research.

Survey research involves soliciting self-reported verbal information from people about themselves. The ultimate goal of sample survey research is to allow researchers to generalize about a large population by studying only a small portion of that population. Accurate generalization derives from applying the set of orderly procedures that comprise scientific sample survey research. These procedures specify what information is to be obtained, how it will be collected, from whom it will be solicited, and how it will be analyzed.

If the researcher needs personal, self-reported information that is not available elsewhere and if generalization of findings to a larger population is desired, sample survey research is the most appropriate method as long as enough general information is known or can conveniently be obtained about the subject matter under investigation to formulate specific questions and as long as the population that is needed to be sampled is accessible and willing to provide self-reported information. The theoretical underpinnings of scientific sample survey research, its procedural applications, and analysis of the data it generates constitute the substance of this book.

Gathering Information through Research

Researchers must be aware that survey research is only one among several methods associated with the process of data collection. The three main

techniques used to collect primary data (data collected firsthand, directly from the subjects under study) are survey research, direct measurement, and observation, all of which in one way or another can make use of sampling. Secondary research is a fourth means of data collection. It consists of compiling and analyzing data that have already been collected and exist in usable form. These alternative techniques, when they are not appropriate in and of themselves, can often be used as complements to the survey research process. A brief description of these alternative techniques follows:

- Secondary research: Certain data may already exist that can serve to satisfy the research requirements of a particular study. Any study should investigate existing sources of information as a first step in the research process to take advantage of information that has already been collected and may shed light on the study. Sources of secondary information include libraries, government agencies, and private foundations, among others.
- *Direct measurement:* This technique involves testing subjects or otherwise directly counting or measuring data. Testing cholesterol levels, monitoring airport noise levels, measuring the height of a building to make certain it complies with local ordinances, and counting ballots in a local election are all examples of direct measurement.
- *Observation:* A primary characteristic of observation is that it involves the direct study of behavior by simply watching the subjects of the study without intruding on them and recording certain critical natural responses to their environment. For example, a government official can obtain important information about the issues discussed in a speech by observing the audience's reactions to that speech.

However, there is no better method of research than the sample survey process for determining, with a known level of accuracy, detailed and personal information about large populations. Opinions, which are the keys to public policy, are obtainable with defined and determinable reliability only through the survey research process. By combining surveys with scientific sampling, the researcher is using the only method of gaining this information to a known level of accuracy. The survey process is particularly suited to collecting data that can inform the researcher about research questions such as the following:

• How do Americans feel about proposed changes in social security regulations?

- What is the average income of people twenty-five years of age and older whose highest level of completed education is high school?
- What factors influence people's choice of banks?
- What are the reactions among employees of a local factory concerning a newly proposed union policy?
- How do members of the New York State Bar Association feel about capital punishment?
- What do various state legislators think about a proposed mandatory balanced-budget amendment?
- What proportion of drivers observe seat belt laws?
- To what extent has the Latino community in Texas experienced job discrimination?

The particular use for which a survey is conducted determines the informational requirements of that survey. Surveys typically collect three types of information: descriptive, behavioral, and attitudinal.

Surveys frequently include questions designed to elicit descriptive information or facts about the respondent. Such important data as the respondent's income, age, education, ethnicity, household size, and family composition are integral to most sample survey studies. These socioeconomic characteristics provide important information that enables the researcher to better understand the larger population represented by the sample.

In many survey research projects, the researcher is interested in the respondent's behavior. Patterns of transportation use, recreation, entertainment, and personal behavior are often the desired information in sample survey studies. For example, such information as frequency of public transit ridership or use of various types of recreational and entertainment facilities is typical of behaviorally oriented information that can be obtained from sample surveys.

In addition to descriptive and behavioral information, many surveys solicit, as their primary focus, the respondent's attitudes and opinions about a variety of conditions and circumstances. The hallmark of this type of sample survey is the public opinion poll, which seeks opinions and preferences regarding issues of social and political relevance. The primary objective of such studies is to be predictive and future oriented.

Very rarely does a study include only one of the informational categories we have noted. Scientific investigation requires that relationships be identified in terms of descriptive, behavioral, and attitudinal data so that we may fully understand the differential complexities of the population from which a sample has been drawn. For instance, in a political public opinion poll, it is much more desirable to know not only the breakdown of votes for each candidate but also such factors as the voter's political party, age, gender, past voting patterns, and opinion on a variety of key issues. Such a survey requires the researcher to derive information from each of the above categories in one sample survey.

Advantages of Sample Survey Research

Generalizations based on a mere fraction of the total population (a sample) did not gain acceptance until the beginning of the twentieth century, when a researcher for a liquor distillery in England named W. S. Gossett was faced with the problem of testing the quality of his company's product. Testing the plant's output involved tasting, and therefore consuming, the product. Testing the entire output of the plant, or even as few as one in ten bottles, was clearly not economically feasible. Gossett, writing under the pseudonym "Student," developed a theoretical basis for making generalizations about the quality of the plant's product by sampling only a small portion of that output.

The foremost advantage of the sample survey technique, as indicated by Gossett's experience, is the ability to generalize about an entire population by drawing inferences based on data drawn from a small portion of that population. The cost and time requirements of conducting a sample survey are significantly less than those involved with canvassing the entire population. When implemented properly, the sample survey is a reasonably accurate method of collecting data. It offers an opportunity to reveal the characteristics of institutions and communities by studying individuals and other components of those communities that represent these entities in a relatively unbiased and scientifically rigorous manner.

Surveys can be implemented in a timely fashion. That is, the survey project can be organized so that the actual data gathering is performed in a relatively short period of time. Besides the convenience afforded by this approach, there is also the advantage of obtaining a snapshot of the population. Other techniques may involve a longer-term study, during which opinions or facts may change from the beginning of the study to the end.

Well-structured sample surveys generate standardized data that are extremely amenable to quantification and consequent computerization and statistical analysis. This quality has been enhanced through rapid advances in computer technology as well as through the development and refinement of complex analytical statistical software packages and techniques. For purposes of comparisons among individuals, institutions, or communities, surveys offer a further advantage: replicability. A questionnaire that has been used in one city or community can be reimplemented in another community or administered once again in the same community at a later date in order to assess differences attributable to location or time.

Sampling started gaining general acceptance beginning in 1935 when George Gallup established the American Institute of Public Opinion in order to conduct weekly polls on national political and consumer issues for private and public sector clients. In as much as Gallup was operating a business for profit and since he was committed to delivering weekly polls, he was necessarily highly sensitive to cost and time factors. Gallup developed a method of sampling fifteen hundred to three thousand respondentsquite a small number compared to other surveys at that time. His method established sample quotas based on age, sex, and geographical region. In the 1936 presidential election between Franklin D. Roosevelt and Alfred Landon, Gallup forecast a Roosevelt victory, while one of the most respected polls at that time, the Literary Digest poll of 2.5 million subscribers, forecast a Landon landslide. The final results are well known: a Roosevelt victory with 61 percent of the vote. The scientifically implemented small sample thereafter became established as the survey method of choice. Advancements in the understanding of sample survey methodology that were developed in World War II and refined thereafter now provide even greater accuracy than Gallup had in 1936, with still smaller sample sizes.

Types of Sample Survey Research

Survey information can be collected by means of any of five general methods of implementation: mail-out, web-based, telephone, in-person interviews, and intercept. This section addresses the advantages and disadvantages of these types of surveys and discusses the procedures for administering the surveys.

Mail-Out Surveys

The mail-out format for collecting survey data involves the dissemination of printed questionnaires through the mail (commonly the postal service) to a sample of predesignated potential respondents. Respondents are asked to complete the questionnaire on their own and return it by mail to the researcher (postage paid by the researcher). An Overview of the Sample Survey Process

Advantages and Disadvantages. The advantages of the mail-out technique can be stated as follows:

- *Possible cost savings*: Other techniques require trained interviewers, and the recruitment, training, and employment of interviewers can be quite costly. Access to respondents by mail can, under some circumstances, be less expensive than telephone surveys and certainly less expensive than in-person interviews.
- *Convenience*. The questionnaire can be completed at the respondent's convenience.
- *Ample time*: The respondent has virtually no time constraints. There is enough time to elaborate on answers and consult personal records if necessary to complete certain questions.
- *Authoritative impressions*: The researcher can prepare the mail-out questionnaire form so that it has significant legitimacy and credibility.
- *Anonymity:* Because there is no personal contact with an interviewer, the respondent may feel that the responses given are more anonymous than is the case with other formats.
- *Reduced interviewer-induced bias:* The mail-out questionnaire exposes all respondents to precisely the same wording on questions. Thus, it is not subject to interviewer-induced bias in terms of voice inflection, misreading of the questions, or other clerical or administrative errors.
- *Complexity:* Mail-out questions can be longer and more complex than telephone questions.
- *Visual aids:* Mail-out questionnaires can make use of photographs and maps that would be impossible to use in a telephone survey.

Mail-out questionnaires, however, have the following disadvantages:

- *Comparatively long time period:* Many follow-ups and substitutions of sample respondents are required in order to achieve the appropriate sample size and adequate random distribution necessary for purposes of generalization. The mail-out therefore generally requires a few weeks for questionnaires to be returned.
- *Self-selection:* Mail-outs typically achieve a lower response rate than telephone surveys. Low response rates can imply some bias in the sample. For instance, poorly educated respondents or those with reading or language deficiencies tend to exclude themselves from this form of survey more often than from surveys administered by an interviewer.

- *Lack of interviewer involvement:* The fact that no interviewer is present means that unclear questions cannot be explained, there is no certainty that the questions will be answered in the order written (which may be important), and spontaneously volunteered reactions and information are not likely to be recorded by the respondent and cannot be probed by an interviewer as would be the case with other methods.
- *Incomplete open-ended questions:* It is more likely that questions requiring an original written response in lieu of fixed answers will be avoided.

Administration of Mail-Out Surveys. Certain guidelines should be followed in administering a mail-out questionnaire. First, the questionnaire should be designed in the form of a booklet in order to ensure a professional appearance and to make it more usable by the respondent. Any resemblance to an advertising brochure should be strictly avoided. The aesthetic appearance of the questionnaire is important in terms of generating satisfactory response rates. There should be adequate spacing between questions, and questions should not begin on one page and end on another. Instructions to the respondent should be clear and easily distinguished from the survey questions themselves. Graphics, such as maps and illustrative photographs, should be carefully integrated into the design of the questionnaire.

The cover letter should be prepared in accordance with the principles discussed in chapter 2 and should become the first page of the booklet. The last page of the booklet should be reserved for three purposes only: to express appreciation to the respondents for their participation, provide a return mailing address and prepaid postage through a business reply permit, and provide instructions for returning the completed questionnaire. An alternative is to provide postage-paid, preaddressed return envelopes, but the cost of this approach is somewhat higher.

Questionnaires should be stamped with an identification number or bar code for purposes of monitoring the follow-up process. This number or code must be explained to the respondent in the cover letter, accompanied by assurances of privacy and confidentiality.

The questionnaire booklet is mailed by first-class postage to the respondent in an envelope. The envelope is addressed with the name and address of the respondent individually imprinted (in the case of small, more personalized surveys) or with a mailing label (most commonly used in largescale, high-volume surveys). A target date should be designated for the return of the questionnaire; this target date is generally recommended to be approximately two weeks from the initial mailing date. Two weeks after the initial mailing, a follow-up postcard reminder should be sent to potential respondents who have not yet replied, as determined by their prestamped identification number. The reminder should be friendly in tone and indicate that if the completed questionnaire and the reminder postcard have crossed in the mail, the respondent should disregard the reminder; it should also again express appreciation for the respondent's cooperation.

Four weeks from the initial mailing, a second follow-up can be mailed to all survey recipients who have not yet responded. This follow-up should include a new cover letter that does not specify a target due date but instead stresses the importance of responding. Another copy of the questionnaire should accompany the letter in case the original questionnaire has been misplaced or discarded.

It can be reasonably expected that this procedure will yield a response rate that can approach 50 percent for the general public and a somewhat higher rate for specialized populations. The researcher should wait two weeks after the second follow-up before closing the mailing process. A response rate of 50 percent can be considered satisfactory for purposes of analysis and reporting of findings as long as the researcher is satisfied in the representativeness of the respondents (see chapter 9). If the researcher wishes to increase the response rate and has adequate resources and time to do so, the following additional procedures are suggested:

- In lieu of using mailing labels, envelopes and the cover letter can be individually imprinted with the potential respondent's name and address.
- The cover letters should be individually signed in blue ink to avoid the impression that they were impersonally mass-produced.
- The follow-up mailings should include eye-catching but tasteful illustrations and graphics.
- Six weeks after the initial mailing, nonrespondents can be given a reminder telephone call.
- A third follow-up mailing, again with a new cover letter and copy of the questionnaire, can be sent to all nonrespondents eight weeks after the first mailing. This third follow-up should be delivered by certified mail.

These additional procedures are designed to achieve a response rate higher than 70 percent for the general population and as high as 90 percent for certain specialized groups.

Web-Based Surveys

The web-based survey is an alternative to the traditional mail-out technique; individuals are contacted by e-mail and asked to participate in a survey that is designed to be completed and submitted through the Internet.

Advantages and Disadvantages. The advantages of the web-based survey method are as follows:

- *Convenience:* This technique represents a convenient and efficient way of reaching potential respondents. They are able to receive the question-naire and complete it in the privacy of their home or office. This advantage is becoming particularly significant as the availability of computers becomes increasingly widespread.
- *Rapid data collection:* Information, especially information that must be timely (e.g., a political public opinion poll related to an upcoming election), can be collected and processed within days.
- *Cost-effectiveness:* This technique is more cost-effective than the traditional mail-out survey because there is no need for postage or paper supplies. It is also more cost-effective than the telephone and in-person surveys because it is not at all labor intensive.
- *Ample time:* The respondent is not pressed for time in responding to the web-based survey and has the opportunity to consult records in answering the questions. There is time to consider response choices and respond to open-ended questions in the form of text.
- *Ease of follow-up:* Potential respondents can be reminded to respond to the survey through follow-up e-mail messages.
- *Confidentiality and security:* Personal or sensitive information supplied by the respondents can be protected on a secure server through the efforts of the research team.
- *Specialized populations:* The survey is particularly useful in reaching specialized or well-identified populations whose e-mail addresses are readily available. For example, we have successfully used this technique to conduct surveys of satisfaction among employees and stakeholders of large public organizations.
- *Complexity and visual aids:* As with mail-out surveys, web-based surveys can use visual images and more complex questions.

Web-based surveys also have certain disadvantages:

• *Limited respondent bases:* A major disadvantage of this technique is that it is limited to populations that have access to e-mail and a computer.

12

Furthermore, the technique assumes a certain minimal level of computer literacy that is necessary for the completion and submission of the questionnaire. Such literacy is improving rapidly within the general population.

- *Self-selection:* As in the traditional mail-out, there is a self-selection bias that leads to lower response rates. Those who do not use e-mail or are not comfortable with web-based technology exclude themselves from the sample. Also, individuals with reading or language issues tend not to respond to web-based surveys. Some researchers send the survey by e-mail in multiple languages in an effort to obviate this problem.
- *Lack of interviewer involvement:* Since there is no interviewer involvement in the web-based survey, unclear questions cannot be explained, and respondents may not follow instructions. These problems can seriously compromise the scientific reliability of the survey even though telephone contacts are provided to the respondents in the event that they need help.

Administration of Web-Based Surveys. The web-based survey requires expertise to prepare a survey instrument for online administration. If the research team members have the necessary expertise and the necessary equipment, including a secure server, they may set up the survey for online response themselves. Alternatively, the researcher can use a service that can be purchased, such as Survey Monkey, where the instructions for setting up the survey structure, sending it to the intended recipients, and receiving responses are explicit and relatively straightforward. If such a service is used, it is possible to convert the data file to the Statistical Package for the Social Sciences (SPSS) for a more robust analysis. The following administrative tasks are required to implement a web-based survey successfully:

• The researcher must have the capability of providing a survey Internet link to the intended respondents. For example, respondents may be employees in a company who are asked to complete an online job satisfaction survey. The researcher will be given access to the e-mail addresses of these respondents in order to send them the survey link. The researcher should be able to send e-mail messages to everyone simultaneously as well as to other selected individuals as desired and necessary. In other types of research efforts, the researcher may be interested in a survey with a broader base of potential respondents—perhaps extending to an entire community. In this case, the researcher can advertise in local newspapers and through community organizations that the survey is available by visiting a specific website and clicking on the appropriate survey link.

- The researcher should strive to prepare an online questionnaire that is as user friendly as possible. The online program should guide the respondent efficiently through the questions to the final submission of the completed survey.
- The initial e-mail message or the introduction to the online survey itself should specify a deadline for the return of the survey form—about ten days. If the researcher knows the e-mail addresses of those who received the survey link, approximately five days after sending the initial e-mail, the researcher may consider sending a reminder e-mail to those who have not yet responded.

It is important that the questionnaire be submitted to a secure server so that the privacy of the respondents is maintained.

Web Panels. Web panels are large reservoirs or banks of potential survey respondents who are recruited to participate in various online surveys. Panel or web companies generally recruit potential respondents to join the web panel through advertisements on the Internet and in local newspapers and newsletters. Web companies encourage panel participation through a series of incentives, including an opportunity to earn money, a chance to have a voice in new products and services, and an avenue to enjoy a variety of interesting surveys and thereby become introduced to various subject areas.

Potential panel recruits are directed to the company's website, where they are asked to provide varying amounts of personal and demographic information. This information is used to screen respondents for participation in surveys requiring a specific demographic profile. For example, a researcher may wish to purchase from the web panel administrator the e-mail addresses of panel members who are women between the ages of twenty and forty and who live in the New England states. Large web panels consist of 2 million to 3 million people. Like all other Internet surveys, those using a web panel have two essential disadvantages: (1) participant self-selection negates random selection and (2) individuals who do not have access to the Internet cannot join a web panel. The absence of such individuals can create a systematic bias in the sample population.¹

The Telephone Survey

The telephone survey collects information through the use of telephone interviews between a trained interviewer and selected respondents.
An Overview of the Sample Survey Process

Advantages and Disadvantages. The advantages of the telephone survey interviewing process can be stated as follows:

- *Rapid data collection:* Information, especially information that must be timely (e.g., a political public opinion poll related to an upcoming election), can be collected and processed within days. It is possible to complete a telephone survey in the time it would take simply to plan a mail-out or in-person survey.
- *Possible cost savings:* The cost of implementing a telephone survey is considerably less than that of in-person interviews; under certain circumstances, it can even be less than that of a mail-out survey.
- *Anonymity:* A telephone survey is more anonymous than an in-person interview. Hence, the interviewer can conduct in-depth questioning in a less-threatening environment than exists in face-to-face situations.
- Assurance that instructions are followed: As with the in-person interview, the telephone interviewer can make certain that the questions are answered in precisely the order intended so that the integrity of the questionnaire sequence is maintained.

Telephone surveys also have certain disadvantages:

- *Less control:* The interviewer has less control over the interview situation in a telephone survey than in an in-person interview. The respondent can easily end the interview at any time by hanging up the telephone.
- *Less credibility:* The interviewer will have greater difficulty establishing credibility and trust with a respondent over the telephone than would be the case in person or by mail.
- *Lack of visual materials:* Unlike the mail-out survey and the in-person interview, the telephone survey does not permit the use of visual aids, such as maps, pictures, or charts, as components of the questions.
- *Less complexity*: Related to the lack of visual materials is the fact that telephone questions must be much less complex than most other survey forms.

Administration of Telephone Survey. The telephone survey is less complex to implement than the mail-out. The most important aspect of this survey technique is the use of personal interviewers; the proper selection and training of these interviewers is critical to the success of the research project. Selection of Telephone Interviewers. Individuals may be recruited from a number of sources to serve as telephone interviewers. The single best source of interviewers, when available, is a local university. Students, especially upper-division undergraduate students and graduate students, are motivated to become involved in the interviewing process for two basic reasons. First, there is frequently some substantive interest in the research project and its potential findings. Second, students often seek ways to augment their income to help fund their education while at the same time gaining relevant experience and therefore may be willing to work for wages that are relatively modest in relation to their skill level. If the researcher does not already have an affiliation with a university, professors in appropriate disciplines should be contacted and arrangements made to recruit potential interviewers. University bulletin boards and newsletters can also be used. If universities are not easily available, temporary agencies can be a source of employees, and when universities and temporary agencies are not readily accessible or when additional assistance is required, newspaper "help wanted" ads may be of some additional help. Newspapers that can be considered for placement of such ads include not only the major metropolitan dailies but also neighborhood weekly newspapers. Another source of recruitment is contact with local organizations such as social service delivery groups, civic organizations, and church groups, which are frequently able to publicize recruitment needs among their memberships.

The content of the recruiting advertisement should enable potential applicants to determine if they are interested in the job and meet its requirements. Thus, the job notice should include such information as work hours, pay rate, location of the work site (home or central telephone facility), and whether fluency in a language other than English is necessary. The job notice should also indicate times and dates for group meetings, which are designed to dispense additional information, answer questions, and receive interviewer job applications; these applications should contain questions about work history, education, professional references, and availability to perform the required tasks. Group sessions are an efficient way to avoid unscheduled and frequent individual recruitment sessions, which can be time consuming for the researcher.

Having reviewed the job applications, the researcher should narrow the list of applicants by screening out those who clearly do not meet the basic requirements. After a brief personal interview, the remaining applicants are asked to administer a practice questionnaire as a final screening device. This process will enable the researcher to determine an applicant's ability to read at the appropriate level, follow directions, and relate to other people. Final selection should be based on the written application, the personal interviews, the practice questionnaire, and any potential biasing characteristics that the interviewer feels the applicant may possess. A poor performance during the practice questionnaire should not necessarily eliminate the applicant from consideration; interviewer training after selection may help to mitigate some of the problems that are seen during the practice session.

Training of Telephone Interviewers. Interviewer training consists of a twopronged process. The researcher first provides the interviewer with general training regarding the fundamental techniques of the interviewing process and then instructs the interviewer in proper administration of the specific survey questionnaire. Several procedures can be used to assist in the training process. To begin, an overview of the questionnaire should be provided that is specific to the study, with the various types of questions identified and all interviewer instructions pointed out, especially those pertaining to filtering and screening. It is advisable to pay particular attention to questions that permit more than one response and to make certain that "Other" categories and open-ended questions are recorded with precision. The researcher should also discuss the answer code format and explain the purpose of the variable fields.

Interviewers should be provided with a general understanding of the scope and substantive purpose of the research project. The organization sponsoring the survey should also be indicated. It is also important to make interviewers aware of the role they play within the survey process as a whole; that is, the interviewers should become aware of the sample size, the sample selection process employed, and how their role relates to the entire survey process, including data entry, data analysis, and the preparation of the final report. Trained telephone interviewers become very valuable to the researcher, and the services of these employees will be in demand for many subsequent surveys. These interviewers become a bank of part-time or even full-time personnel to call on as the need arises.

The interviewer must be careful to minimize the amount of bias introduced into the interviewing process. The introductory greeting, as discussed in chapter 2, should be delivered with sincerity. Questions should be read verbatim with appropriate pacing and in a pleasant conversational tone. The interviewer should be satisfied that the respondent understands the question and must be careful to record responses accurately.

The interviewer should not express any opinions or make extraneous comments in reaction to statements made by the respondent. Despite these

efforts to minimize bias, there is always the potential for the respondent's answers to be affected to some extent by her or his reaction to one or more characteristics of the interviewer, such as ethnic or regional accents, gender, or age. The researcher should be cognizant of these potential problems and plan the conduct of the research study accordingly. For example, during the survey pretest, if it is found that a large number of respondents are having difficulty understanding an interviewer with a particularly heavy regional accent or it appears that their responses are affected in some manner by the accent, the researcher may find it necessary to reassign that interviewer to another aspect of the study.

In the not-too-distant past, telephone surveys were conducted by traditional paper-and-pencil procedures. That is, interviewers would ask the question and mark the respondent's answer on an interview form. During the past twenty years, computer technology has displaced these traditional techniques. Questions on the survey are programmed to appear on the computer screen designed to be read by the interviewer. The respondent's answer is recorded by marking the button that corresponds to the appropriate numerical code and having that response entered directly into the computer. One of the most popular programs for computer-based telephone interviewing is the Computer-Assisted Telephone Interviewing (CATI) program. The primary advantages of computer technology for telephone interviewing are as follows:

- This technology enables the interviewer to follow complex question skip patterns. For example, if the respondent provides the answer "yes" to question 3 and the survey directs such responses to question 6, the computer would automatically place question 6 on the computer screen.
- These programs minimize interviewer error because they remove a great deal of interviewer discretion in the conduct of the interview. Such technology saves interview time, which has important cost implications.
- Computer-assisted technology allows the recorded responses to be stored and transferred into an SPSS data file for immediate analysis.
- These programs have the ability to rotate response categories in order to minimize the potential bias associated with presenting responses in the same patterned order.
- Other features of the program include tracking nonresponses and programming automatic callbacks.

Computer-assisted telephone interviewing has become common practice and the state of the art in telephone interviewing. As such, it is important that telephone research makes use of this technology in order to remain competitive and to achieve the most accurate survey results.

Conducting Telephone Surveys. Interviewing should be conducted in the early evening (6:00 p.m. to 9:00 p.m. local time) and on weekends (noon to 9:00 p.m.). Evenings provide the interviewer greater opportunity to reach working adult household members, whereas daytime calling during the week reaches only adults who are not working outside the home. After 9:00 p.m., the interviewer should stop placing calls to avoid disturbing those who may have retired for the night. Similarly, on weekends, calls prior to noon may interfere with needed extra hours of sleep or time spent at religious services. The overriding principle is to reach as many adult household members as possible at a convenient time.

If the interviewer encounters a busy signal, the call should be tried again in thirty minutes; if the line is still busy, the call should be placed again the next day. If the first call on the next day is once again met with a busy signal, the interviewer should again wait thirty minutes and try one more time. When there are repeated busy signals, the interviewer is required to contact the telephone company to ascertain the working status of the number. If the telephone company indicates that the line is operating, the interviewer may try calling on another day at a time totally different from the previous attempts. If the line is still busy, the interviewer should classify the number as "nonresponse" to avoid spending an inordinate amount of time in pursuit of one potential respondent. When, instead of a busy signal, the first call elicits no answer, the call should be repeated the next day. If there is still no answer after four or five such attempts, the telephone number can be treated as a nonresponse.

When the sample comprises households rather than specific individuals at a given phone number, the interviewer must speak to an adult member of the selected household unless the survey is specifically geared to minors. The interviewer should try to speak to a representative mix of men and women and sometimes may have to specifically request to speak to an "adult male" or "adult female" in order to maintain representativeness by gender.

When the interviewer has exhausted the sample list of telephone numbers, he or she should tell the researcher how many nonresponses have been encountered. The researcher will provide the interviewer with a list of replacement telephone numbers selected in accordance with the appropriate sample selection method (see chapter 9). The interviewer then proceeds to make these calls as described, returning to the researcher once again reporting all nonresponses from the list. This process continues until the interviewer has completed the number of interviews assigned.

Types of Telephone Sampling. The type of telephone sampling method to be used in a telephone survey depends on the requirements of the survey and the source of telephone numbers. The three major types of telephone sampling techniques are random digit dialing, cell phone–only dialing, and dialing from a fixed list of telephone numbers.

Random digit dialing (RDD) has become the most prevalent method of conducting a telephone survey since its inception in the mid-1960s. RDD telephone samples typically use a random sample of telephone numbers generated from the entire universe of telephone numbers that could be assigned within a specified geographic area. These random telephone numbers are associated with traditional landline telephones, the basic telephone service that was used almost exclusively by American households through the late 1980s. Researchers purchase lists of random landline telephone numbers, according to zip code, from independent sample suppliers.

Cell phone–only dialing: The growing popularity of cell phones in the United States threatens the representativeness of telephone surveys when RDD of landline numbers is conducted exclusively. The reason is that the prevalence of cell phones has been increasing at a steady pace since the early 2000s. More and more households are eliminating their landline phone entirely in favor of cell phone use exclusively. Overall, according to a Centers for Disease Control (CDC) report, two in every five American homes (39 percent) had only wireless phones as of the first half of 2013. About 38 percent of US adults (or 90 million) and 45 percent of US children (33 million) lived in wireless-only households. The wireless-only share has been rising steadily ever since the CDC began asking people about their phone status in 2011 when only 26.6 percent of US households were wireless only.²

Wireless-only households are especially predominant among the poor and the young. According to the CDC, nearly two-thirds (66 percent) of adults ages twenty-five to twenty-nine lived in households with only wireless phones, as did three in five (59.9 percent) thirty- to thirty-four-yearolds and a majority (54 percent) of adults ages eighteen to twenty-four. A majority of adults living in poverty (54.7 percent) lived in a wireless-only household versus 48 percent of what the CDC calls the "near-poor" and 35 percent of nonpoor adults; wireless-only households also predominate among Hispanics, renters, and adults living with roommates.

Based on these data, it is suggested that telephone surveys that use RDD of landline telephones also include a random sample of cell phone–only

users. This represents a dual-frame telephone sampling design and will mitigate against the geographic and demographic biases that are likely to result if cell phone–only users are excluded from the survey. Random cell phone numbers can also be purchased from sample suppliers that maintain current lists.

The growing body of research suggests that the quality of data obtained from cell phone calls is not different from the data quality obtained from landline calls. Nonetheless, there are reasons to suspect that data obtained from cell phones may be less reliable.

When individuals receive cell phone calls, they could be located at a variety of different public places. Although these cell phone users appear to be perfectly willing to carry on the telephone conversation, they may feel somewhat constrained in answering certain questions of a sensitive and private nature. This reluctance, either conscious or unconscious, could threaten the integrity of the responses and the validity of the entire survey.

The location of a cell phone user (generally outside of the home) may be associated with a number of distractions that could cause the respondent to be less focused on the telephone interview at hand. The data provided may be less accurate and less thoughtful than if the respondent were located in a more serene location. The respondent may be walking or in a restaurant or coffee shop where others in the vicinity are engaged in a somewhat loud conversation.

At times, cell phones receive poor transmission signals, jeopardizing the audio quality of the call. This can potentially cause problems for the respondent to clearly understand the question and for the interviewer to correctly record the appropriate response.

As a result of potentially poor sound quality and the possibility of distractions, cell phone calls may last longer than landline calls. If the interview begins to run inordinately long, the user may very well break off the interview before the survey is completed. By the same token, the interviewer, in sensing that the interview is running long, may rush through the interview. In both cases, data quality is jeopardized. If these circumstances come to pass, the cost of the project is likely to increase since more cell phone calls will have to be made to make up for the incomplete data and ultimately complete the survey project.

Cell phone samples are sold according to area code and zip codes much as landline samples are. However, cell phone owners frequently retain their cell phone area code even when they move to another part of the state or country where a different area code is used. These individuals may well be included in a sample of numbers associated with a zip code that is relevant to the project but must be excluded from the survey when contacted. This screening process can be time consuming and adds to the cost of the survey project.

Fixed List of Telephone Numbers: In some survey projects, the client provides the researcher with a fixed list of telephone numbers from which the random sample is to be drawn. These are simply the contact numbers provided by the potential respondents and can be either landline or cell phone numbers. The fixed lists often represent a customer base where the contact information is frequently updated. For example, a water district may be interested in conducting a satisfaction survey among its customers or a bus company may wish to survey customers who are enrolled in their special service for the disabled.

There are a number of additional rules of interviewing that the researcher should insist on having followed. These rules include the following:

- An interviewer should never interview more than one adult in the same household.
- A friend or relative of the research team should not be interviewed. If a friend or relative is part of the sample list, the researcher should be notified so that the person in question can be reassigned to another interviewer.
- The interviews should be conducted in as much privacy as possible to avoid distraction. It is likely that a professional facility will ensure privacy and quiet.
- The interviewer should not delegate assigned interviews to anyone else unless for reasons of language.
- Interviews should never be falsified.

In-Person Interviews

In-person, or face-to-face, surveys are structured to permit an interviewer to solicit information directly from a respondent in personal interviews.

Advantages and Disadvantages. The advantages of the in-person interview survey technique are as follows:

- *Flexibility:* The interviewer can probe for more detail, explain unclear questions, and use visual aids such as maps or photographs.
- *Greater complexity:* Interviewers can administer highly complex questionnaires and provide detailed instructions and lengthy lists of alternative

responses that many respondents would find confusing and intimidating if the questionnaire were administered by any other means.

- *Ability to contact hard-to-reach populations:* Certain groups, for instance, the homeless or criminal offenders, are difficult or impossible to reach by any method other than personal interviews.
- Assurance that instructions are followed: The interviewer can make certain that the questions are answered in precisely the order intended so that the integrity of the questionnaire sequence is maintained.

In-person interviews also have certain disadvantages:

- *High cost:* Administering in-person interviews can be very costly in terms of time per interview, travel time, interviewer training, and field supervision.
- *Interviewer-induced bias:* Although the interviewer obviously serves many useful functions in this process, he or she can also be a source of bias. For example, the interviewer may inadvertently react in some way to a response rather than remaining neutral. This action could affect future responses by the interviewee and, hence, the validity of the entire questionnaire. By the same token, the respondent may alter his or her responses to gain perceived approval from the interviewer.
- *Respondents' reluctance to cooperate:* If respondents must allow interviewers into their homes to participate in a face-to-face survey, they may tend to be somewhat less inclined to participate than in a telephone survey. Many telephone calls and return visits may be necessary in order to complete an interview.
- *Greater stress:* The in-person interview format is clearly the most intense and stressful for both the respondent and the interviewer. It tends to be a longer and more complex interviewing process, and it is the only one in which a stranger is present in the respondent's environment. Such situations can cause increased stress and fatigue, which may have unfavorable effects on the quality of the responses.
- *Less anonymity:* The advantages of the anonymity perceived by the respondent in mail-out and telephone surveys are greatly reduced in the face-to-face format.

Administration of In-Person Interviews. The administration of in-person, or face-to-face, interviews creates even greater challenges for the researcher than do other methods. As with the telephone interview, the selection and training of interviewers is critical to the successful solicitation of data.

Selection of In-Person Interviewers. The process of selecting in-person interviewers should be precisely the same as that used for selecting telephone interviewers, with a certain emphasis on physical characteristics that is not as important in the telephone survey process. Because in-person interviewing involves face-to-face interaction between the respondent and the interviewer, the respondent's willingness to participate is highly dependent on the comfort level the respondent perceives. Physical characteristics such as attire, cleanliness, neatness, manners, and overall grooming loom considerably larger in the in-person format than in the telephone survey, and they set the tone for the seriousness of the research study. Consequently, these characteristics must be emphasized in the selection process.

A secondary component of the interviewer's physical characteristics can bear strongly on the in-person interview. A series of studies throughout the years has established that people have been socialized to react differently to another person depending on his or her sex, age, ethnicity, and social status. These studies indicate that an interviewer with roughly the same characteristics as the respondent will tend to obtain more reliable information, especially if this information pertains to issues that the respondent perceives are sensitive in nature. In the interest of obtaining as much reliable information as possible, the researcher must incorporate these considerations into the interviewer selection process.

Training of In-Person Interviewers. The principles of interviewer training that have been stated with regard to the training of telephone interviewers apply also to in-person interviewers. A few additional considerations exist, a result of the differences in format between the two methods. Such considerations include maintaining a neat personal appearance and developing a facility for displaying visual material to the respondent.

Prearranging the In-Person Interview. It is important to remember that inperson interviews must be prearranged in order to protect the privacy and safety of both the respondent and the interviewer, in contrast to telephone calling, which is performed spontaneously. In addition to refraining from making verbal reactions to the respondent, the interviewer should avoid any facial expressions or other gestures that may bias or otherwise disturb the respondent. It is recommended that all potential respondents be sent a letter not dissimilar from the one that introduces a mail-out questionnaire, including a description of the nature of the study and a statement concerning the importance of the recipient's participation. The letter should further state that a telephone call will soon follow in which the interviewer will seek to arrange an appointment for a personal interview at a place convenient to the respondent—often the respondent's home or place of work. Approximately one week after delivery of the letter, interviewers should begin placing the telephone calls. The guidelines for conducting these calls should follow the same format in terms of time of day and follow-up calling procedures as telephone interview calls.

Intercept Surveys

The intercept survey is a variation of the in-person survey whereby information can be obtained from respondents as they pass by a populated public area such as a retail mall, a workplace, a transit station, or an airport, among others. The interviewer actually "intercepts" individuals and asks them to participate in the survey. If the intercept survey is particularly long, some respondents may not have time to complete the survey. To avoid this problem, the researcher may wish to build a mail-back option into the survey. This can be readily accomplished by using a business reply permit, which can be purchased through the US Postal Service. When this permit is printed on the survey form, the respondent simply drops the completed survey in a mailbox and it will be returned to the researcher (postage paid by sponsoring agency).

Advantages and Disadvantages. The advantages of the intercept survey can be stated as follows:

- *Complexity:* The availability of an interviewer provides the opportunity to explain unclear questions and use visual aids in the conduct of the interview. The use of maps and graphs is particularly important for transportation-related surveys.
- *Interviewer involvement:* The interviewer can ensure that questions are not skipped and that all questions that the respondent wishes to answer are completed.
- *Informs larger questionnaires:* The intercept survey is useful in informing the preparation of questionnaires for larger telephone, web-based, mailout, and in-person interview surveys and can also be helpful in structuring the discussion guide for focus groups.
- *Observed data:* The interviewer can observe certain personal characteristics of the respondent (such as gender, ethnicity, age, or physical

disability), thereby avoiding the need to ask the respondent. This advantage serves to maintain the brevity of the survey and minimizes verbalizing potentially sensitive questions.

• *Cost-effective:* The intercept survey is more cost-effective than the traditional telephone, mail-out, and in-person surveys.

The disadvantages of the intercept survey are as follows:

- *Interviewer errors:* Interviewers may fail to follow a preestablished random procedure for selecting potential respondents. This can occur because interviewers may sense that certain potential respondents are threatening or unfriendly.
- *Limited information:* Since the intercept survey is of necessity quite short, the amount of information that can reasonably be obtained is limited. The researcher faces the challenge of asking the most important and relevant questions.
- *Lack of anonymity:* The anonymity of the respondent that is perceived in the telephone and mail-out surveys is greatly compromised in the intercept survey.
- *Interviewer bias:* The interviewer can inadvertently serve as a source of bias through hand gestures, body language, facial expressions, and extraneous comments. Similarly, the respondent may respond to the interviewer in a less objective fashion in an effort to seek the interviewer's approval.

Administration of the Intercept Survey. The appropriate administration of an intercept survey requires adherence to certain rules and guidelines:

- A questionnaire should be prepared that takes about three minutes to administer. The survey should be pretested to ensure its effectiveness and brevity.
- The researcher should select a suitable location for the intercept survey. This location should be an area populated by potential respondents who are the target for the survey. The researcher should make sure that all legal requirements for the use of the location have been met.
- Ideally, a small incentive or prize should be offered to the respondent for completing the survey (e.g., a pen or candy).
- Response rates are higher when the interviewer represents a public agency such as a city or county government since these affiliations tend to provide credibility to the research effort.

- Interviewers should wear a badge or some other form of observable identification that identifies them as part of the research project underway. They should carry legal identification in the event that authorities challenge their presence at the designated location.
- The researcher should establish time periods on weekdays as well as weekends as appropriate and necessary for the study to ensure that a reasonable cross section of the target population has been accessed.
- Interviewers who reflect the demographic characteristics of the population to be surveyed should be recruited and trained by the researchers. This training should include interviewing techniques, obtaining the attention and eventual participation of respondents, accurate recording of responses, and the importance of conviviality and confidentiality.
- The researcher must inform interviewers of the procedures to follow if they encounter problems during the administration of the survey instrument, procedures for contacting supervisors, and the importance of cleanliness and proper attire.
- The researcher should provide interviewers careful instructions concerning where to pick up blank surveys and where and when to return the completed ones.
- Survey forms should be prepared in the language or languages of the potential respondents, and interviewers must be selected who are bilingual in the appropriate languages. For intercept surveys in most states, survey forms must be prepared in both English and Spanish at a minimum. In several states, various Asian and African languages may also be necessary.
- The researcher should establish procedures for monitoring interviewers in the field as a form of quality control.

On-Board Surveys. The on-board bus and rail survey is a form of intercept that can be an effective and useful method for collecting information for transportation planning. It enables researchers to gain information about travel patterns that aid in the planning of bus and rail routes. Also, it elicits opinions about customer satisfaction with bus and rail service so that service improvements that meet the needs of bus and rail riders can be considered in the planning process. The on-board survey involves placing an interviewer on buses or rail cars to interview riders in person or provide survey forms for them to complete and return on-board or by business reply mail, as discussed. Many of the instructions provided for the administration of the more general intercept survey can be readily adapted for the on-board surveys.

Stages of the Survey Research Process

To conduct any of the major types of surveys in a rigorous and unbiased fashion, it is important to adhere to specific procedures and apply them in a systematic manner. Although the stages are presented here as distinct steps, there is actually a great deal of overlap over the course of the survey research process. An overview of the process is presented here, with each stage explained in the following chapters. The following list displays these stages, which are explained more fully below it:

Stage 1: Identifying the focus of the study and method of research
Stage 2: Determining the research schedule and budget
Stage 3: Establishing an information base
Stage 4: Determining the sampling frame
Stage 5: Determining the sample size and sample selection procedures
Stage 6: Designing the survey instrument
Stage 7: Pretesting the survey instrument
Stage 8: Selecting and training interviewers
Stage 9: Implementing the survey
Stage 10: Coding the completed questionnaires and computerizing the data
Stage 11: Analyzing the data and preparing the final report

Stage 1: Identifying the Focus of the Study and Method of Research

During the initial stage, the researcher must be satisfied that sample survey research is the most appropriate method of collecting the necessary information for the study under consideration among the other potential data-gathering techniques of secondary research, direct measurement, and observation. The following factors make sample survey research appropriate:

- Adequate secondary data are not available.
- There is a desire to generalize findings from a small subpopulation to a larger population.
- The target respondent population is accessible.
- The data to be obtained are of a personal, self-reported nature.

Once survey research has been determined to be the most appropriate research method, the researcher has two fundamental tasks to consider: (1) the goals and objectives of the study should be elaborated and refined,

(2) the researcher should identify the format for collecting the data (mailout, telephone, web-based, intercept, or in-person interview). The second decision will be greatly influenced by the complexity of the data to be obtained, the accessibility of the sample population, the budget available for the study, and the time constraints that have been imposed for completing the project.

Stage 2: Determining the Research Schedule and Budget

Once the parameters and objectives of the study have been identified, the researcher must establish a timetable for completing the survey research project. The timetable should be flexible enough to accommodate unforeseen delays and yet be capable of satisfying the needs of the research sponsor. In conjunction with this timetable, a detailed budget should be prepared. Insofar as budgetary and time considerations permeate and constrain each step of the survey research process, it is critical that this stage be carefully implemented.

Stage 3: Establishing an Information Base

Prior to the development of a survey instrument (questionnaire), it is necessary to gather information about the subject matter under investigation from interested parties and key individuals. Such individuals might be brought together in an informal group setting where relevant issues and problems can be freely discussed and debated. The goals and objectives of the research can be clearly defined, and the practical relevance of the proposed survey can be explained. For example, a research organization may have the objective of studying the travel behavior and travel preferences of economically disadvantaged residents in a major city in the United States. At the outset, it would be important to hold a focus group meeting where representatives of social service organizations such as the county welfare agency, economically disadvantaged residents, and the researchers involved in conducting the study gather to exchange ideas and concerns. The hope is that an open and frank discussion will reveal the type of survey information that would be helpful in outlining key issues and identifying relevant sectors of the population to be targeted in such a study. A detailed treatment of the focus group technique is presented in chapter 4.

In some research endeavors, the subject matter is found to be new or vague, and as a result of this lack of general knowledge, it is not immediately feasible to devise a series of specific questions to be used in a formal survey process. In such situations, it may be necessary to conduct, as a preliminary technique, some form of semistructured direct observation of the population using professional observers who are trained to record information about the subject population in a systematic way. Such semistructured research techniques have been successfully used in anthropological and sociological studies of geographical, economic, and behaviorally distinct subcultures. This base level of information may then be used to devise a questionnaire for the formal survey process. Without such preliminary information, the survey questions could prove to be peripheral or tangential to the goals of the research study. A thorough reconnaissance of information at this point is critical in terms of producing a focused and well-directed study. This chapter has already provided some background discussion regarding information collection. A more detailed treatment of this topic is in chapter 2.

Stage 4: Determining the Sampling Frame

The population that is identified for formal interviewing derives from applying the sampling frame for the survey research project. The researchers must be confident that the sample possesses the knowledge and information required to fulfill the requirements of the research project. That is, the sample represents the population from which they are supposed to be selected (called the *general population*).

After the general population, or *universe*, is defined in a conceptual sense, a list of identifiable and contactable members of this general population must be obtained. From this list, a sample of respondents will be drawn. This list is called the *working population*. The sequence of steps that moves the researcher from the general population to the sample is known as the *sampling frame*. For example, in a survey project concerning residential preferences and relocation tendencies, the general population may be defined as one that has demonstrated some mobility within a given metropolitan area. One way of operationalizing this concept of mobility is to obtain a list of residents whenever a new gas or electric hookup is requested, so new hookups within a given period of time could easily identify a mobile population. Concepts related to identifying an appropriate population are discussed in chapter 8.

Stage 5: Determining the Sample Size and Sample Selection Procedures

The researcher must attempt to select a sample that is an approximate microcosm of the working population. Given equally representative

samples, larger samples generally yield a higher degree of accuracy than smaller ones. The researcher must weigh the desired degree of accuracy against the increased time and cost that a larger sample size entails. Once the overall sample size is determined, several alternative procedures must be considered for selecting a sample. Foremost among these are simple random sampling, systematic random sampling, stratified random sampling, and cluster (multistage) sampling. The theoretical basis of sampling is discussed in chapters 6 and 7, the criteria for determining sample size are described in chapter 8, and various sampling procedures are given in chapter 9.

Stage 6: Designing the Survey Instrument

The development of the survey instrument or questionnaire is a crucial component of the survey research process. At this stage, the researcher must devise a series of unbiased, well-structured questions that will systematically obtain the information identified in stage 1. Developing the questionnaire can be an extremely detailed and time-consuming process. Decisions must be made concerning the wording of questions and the format depending on whether the survey is an in-person interview, intercept, mail-out, web-based, or telephone survey. The number of fixed-answer and open-ended questions must be determined, and the element of time with respect to questionnaire length should be considered. The longer the questionnaire is, the greater are the variable costs associated with its implementation, such as interviewing time, computerization of data, and production and distribution costs. Furthermore, longer questionnaires tend to lead to lower response rates. The questionnaire must be easily understood and internally consistent and must lend itself to appropriate and meaningful data analysis. Questionnaire design is fully discussed in chapters 2 and 3.

Stage 7: Pretesting the Survey Instrument

After a draft questionnaire has been prepared and the researcher believes that the questions will obtain the information necessary to achieve the goals of the study, it is important to pretest the instrument under actual survey conditions. During the course of the pretest, poorly worded questions will be identified and the overall quality of the survey instrument refined. Based on the experience of the pretest, the questionnaire will be fine-tuned for use in the actual survey process. The pretest is discussed in chapter 2.

Stage 8: Selecting and Training Interviewers

Telephone and in-person surveys require trained interviewers. These interviewers can be selected from the student ranks, they can be trained professionals, or they can be part-time nonstudent interviewers. Researchers select interviewers according to the nature of the study and the characteristics of the sample respondents.

Prospective interviewers should be thoroughly trained by the researchers in the use of the questionnaire. It has been found that when interviewers have facility with the survey instrument, they are better able to generate and sustain respondents' interest in the survey. In order to gain this facility with the survey instrument, interviewers should receive specific instructions on conducting their interviews and should also be given guidelines for handling uncooperative respondents. They should practice survey implementation many times before actually engaging in the field research. Interviewer selection and training is described previously in this chapter.

Stage 9: Implementing the Survey

The implementation of the survey instrument is a critical phase of the research process. Care must be taken that the established random sampling procedure is adhered to and that the timetable is strictly maintained. Ensuring the privacy and minimizing the inconvenience of potential respondents should be major concerns. In addition, the researcher must adhere to a number of ethical standards in the conduct of the survey research process. Some ethical guidelines can be found at the end of this chapter. (The administrative procedures in the conduct of survey research have already been discussed previously in this chapter.)

Stage 10: Coding the Completed Questionnaires and Computerizing the Data

The final questionnaire must be formatted in such a way that responses can be entered directly into the computer for data processing. (We discuss coding and formatting issues in chapter 2.) Once the questionnaires have been returned, the very important process of cleaning up the forms begins. This is especially important for mail-out surveys, where no interviewer was present to make certain that instructions were followed. The cleanup entails making certain that the appropriate number of entries have been marked for each question, ensuring that there are no extraneous responses, and

33

making sure that enough questions have been answered to validate the questionnaire. All open-ended answers must be categorized and coded on the form itself for ready computer entry. A variety of statistical software packages are available for this purpose including the SPSS.

Stage 11: Analyzing the Data and Preparing the Final Report

The recorded data input must be summarized, placed in tabular or graphical form, and prepared for statistical analyses that will shed light on the research issues at hand, using statistical significance tests, measures of central tendency, determinations of variability, and correlations among variables. These formal statistics and data summaries form the basis of the report that will be the culmination of the survey research process. Chapters 10 through 12 elaborate on the essential statistical concepts involved in the analysis of survey data. Chapter 13 provides guidelines for the preparation of the final report.

Monitoring and Supervision of the Interview Process

For larger projects, a supervisor, hired by the researcher, should be expected to work at least twenty hours per week, especially in the early stages of the interview process. With smaller projects, the researcher may also be able to serve as the supervisor, thereby eliminating the need to employ additional staff.

Telephone interviewing and the scheduling of personal interviews are best conducted from a centralized facility. This tends to produce higher response rates compared with interviews conducted or arranged privately from interviewers' homes or offices. It also affords the supervisor ample opportunities to directly monitor telephone conversations by listening to them. When such direct monitoring takes place, the respondent must be informed.

When telephone interviews and scheduling are conducted from private locations, the supervisor should randomly select at least 10 percent of the proposed sample and call these households to verify that contact has in fact taken place and to ascertain the respondents' degree of satisfaction with the conversation.

The supervisor should review the interviewers' work, be available for questions, and have frequent contact with the interviewers in the form of regular telephone or personal conferences. The supervisor should be prepared to reassign cases among interviewers if this is necessitated by such factors as language difficulties or varying completion rates. Production objectives should be established in terms of the number of interviews to be completed in a given amount of time. It is the supervisor's responsibility to constantly monitor interviewer performance in terms of these objectives.

Ethical Considerations in Survey Research

The American Association of Public Opinion Research (AAPOR) has approved a code of ethics (latest revision in May 2010). This organization has established ethical guidelines for survey and opinion researchers and stated its goals as follows: "Our goals are to support sound and ethical practice in the conduct of survey and public opinion research and in the use of such research for policy and decision-making in the public and private sectors, as well as to improve public understanding of survey and public opinion research methods and the proper use of those research results."

The key principles from this code of ethics can be summarized as follows:

- Researchers should avoid practices or methods that may harm, endanger, humiliate, or seriously mislead survey respondents or prospective respondents.
- Regarding private clients, all proprietary information obtained about the client and about the conduct and findings of the research undertaken for the client must be held in confidence by survey researchers.
- Public sector clients should be informed about the AAPOR standards for disclosure. Survey researchers should make efforts to encourage public clients to disclose essential information about the research that is being conducted: a description of the sampling frame, sampling design, sample sizes and associated margins of error, respondent selection process, and any weighting or estimating procedures that may have been used.
- Researchers must make it clear to potential respondents that participation in surveys and other forms of public opinion research is voluntary, except for the decennial US Census and other government surveys as specified by law.
- Researchers should not misrepresent their research or conduct other activities, such as sales, fundraising, or political campaigning, under the guise of conducting survey and public opinion research.

An Overview of the Sample Survey Process

- Unless the respondent explicitly waives confidentiality, researchers will hold as privileged and confidential all information that could be used, alone or in combination with other reasonably available information, to identify a respondent with his or her responses.
- Researchers must not knowingly engage in fabrication or falsification of survey data or survey results.

Conclusion

This chapter has introduced sample survey research as a useful technique for gathering information. The fundamental advantage of this kind of research is the ability to generalize about an entire population by drawing inferences based on data derived from a small portion of that population. Sample survey research can be applied to any facet of descriptive data, behavioral patterns, and attitudinal information about societal preferences and opinions. The chapter presents the stages of the survey research process and it emphasizes the types of surveys (mail-out, web-based, telephone, in-person, and intercept) as well as the procedures for their administration.

Notes

- 1. For a thorough review of web panels, consult American Association of Public Opinion Research, *AAPOR Report on Online Panels* (March 2010).
- 2. http://www.pewresearch.org/fact-tank/2013/12/23/for-most-wireless-only-households-look-south-and-west/



CHAPTER TWO

DESIGNING EFFECTIVE QUESTIONNAIRES

Basic Guidelines

At the heart of survey research is the questionnaire development process. The key considerations in this process, including the placement of questions within the survey instrument and their format in terms of the method of implementation (telephone, mail-out, web-based, intercept, or in-person interviews), form the basis of this chapter. The discussion of these issues takes place within the context of sample questions and exhibits derived from actual questionnaires and survey instruments that we have implemented in our professional sample survey projects. This chapter discusses certain general, macro-questionnaire development considerations; chapter 3 delves more deeply into specific questions—or a microlevel examination of questionnaire development.

Be aware that no questionnaire can be regarded as ideal for soliciting all the information deemed necessary for a study. Most questionnaires have inherent advantages as well as inherent flaws. The researcher must use experience and professional judgment in constructing a series of questions that maximizes the advantages and minimizes the potential drawbacks. The guidelines detailed in this chapter recognize that there are a large number of considerations that the researcher must address in the process of questionnaire development. Sound questionnaire construction is a highly developed art form within the practice of scientific inquiry. In the initial stages of the survey research process, it is important to determine the relevant issues that bear on the purpose of the research. Because social science research spans so many disciplines, it is impossible for any researcher to be fully knowledgeable in all the fields of study that might call on survey research services and skills. In addressing the complex multidisciplinary nature of survey research in the social sciences, the researcher can respond in two ways.

First, the principal investigator often seeks to construct a team of experts who jointly plan and implement the research study. This team represents both technical expertise and substantive knowledge of the political, socioeconomic, and cultural environment associated with the project. Second, with or without such a research team in place and as a prelude to the development of survey questions, the investigators must gather preliminary information about issues of importance from interested parties and key individuals. These issues will derive in whole or in part from the three types of information elaborated on in chapter 1: descriptive, behavioral, and preferential. This preliminary information is best generated in a group setting where issues and problems of relevance to the study can be debated, discussed, and refined openly and constructively. Foremost among these preliminary information-gathering techniques is the focus group, a semistructured discussion among individuals deemed to have some knowledge of or interest in the issues associated with the research study. Group participants are brought together in roundtable discussions run by a group leader or moderator. The discussion that ensues should contribute significantly to an understanding of the key substantive issues necessary for the development of the questionnaire. A full discussion of the use of focus groups in the survey research process is presented in chapter 4.

At the conclusion of this preliminary information-gathering stage, the key issues that have emerged must be outlined and specified. This list of issues should be submitted to members of the discussion groups for clarification, confirmation, and perhaps further explanation. After this review, the researchers can prepare a draft questionnaire or survey instrument. If the research study has been commissioned by public agencies or private clients, as is frequently the case, the draft questionnaire should be reviewed by these parties for content and to ensure that the questions are consistent with the objectives of the study.

Once the researcher is satisfied with the draft questionnaire, the next step is to conduct a pretest—a small-scale implementation of the draft questionnaire that assesses such critical factors as the following:

- *Questionnaire clarity:* Will respondents understand the questions? The researchers may find that certain ambiguities exist that confuse respondents. Are the response choices sufficiently clear to elicit the desired information?
- *Questionnaire comprehensiveness:* Are the questions and response choices sufficiently comprehensive to cover a reasonably complete range of alternatives? The researchers may find that certain questions are irrelevant, incomplete, or redundant and that the stated questions do not generate all of the important information required for the study.
- *Questionnaire acceptability:* Such potential problems as excessive questionnaire length or questions that are perceived to invade the privacy of the respondents, as well as those that may abridge ethical or moral standards, must be identified and addressed by the researchers.

The sample size for the pretest is generally in the range of twenty to forty respondents; however, for very large sample surveys, it is not uncommon for a pretest to contain a larger sample. The researcher is not really interested in statistical accuracy at this point; rather, interest centers on feedback concerning the overall quality of the questionnaire's construction. Accordingly, the researcher will select respondents from among the working population but need not be concerned about selecting them through a random sampling procedure (chapter 9) or in accordance with sample size requirements (chapter 8). Because statistical inferences are not the primary intent of the pretest, the researcher can be particularly sensitive to cost and time considerations—hence, the relatively small number of respondents. For example, a study that attempts to obtain information about teenagers might conduct a pretest using one or two high school classes. The members of the classes would very likely be individuals in the appropriate age category, and the classes could be surveyed quickly, conveniently, and efficiently. Clearly not all teenagers are high school students; therefore, the high school classes would not necessarily represent the exact characteristics of the respondents in the final study. However, this degree of precision in the selection of pretest respondents is not required. The only requirement is that the pretest respondents bear a reasonable resemblance to the study's actual general population.

Following the pretest, the researchers must revise the questions as needed. They may want to perform a further pretest if these revisions are extensive. Otherwise the final questionnaire can be drafted and prepared for implementation in an actual study.

Introducing the Study

A questionnaire is a conversation, and, like most other conversations, it builds on itself, beginning with an introduction. It is important to inform potential respondents about the purpose of the study in order to convey its importance and alleviate any concerns that potential respondents are likely to have. From the researcher's point of view, there is a need to convince potential respondents that their participation is useful to both the survey's sponsor or client and the respondents themselves. Any concerns that respondents may have regarding time and inconvenience, confidentiality, and safety should be allayed. The respondent must be assured that all answers are valuable—that there are no "correct" or "incorrect" responses.

An introductory statement should contain certain components. First, the organization or agency conducting the study should be mentioned (unless that information itself would be biasing). The introduction should state the relationship between the sponsoring institution and the potential respondent. A great deal of credibility can be gained for the study if the sponsor is a governmental body that in some way represents the respondent. An introduction that contains a reference similar to the following can be quite successful in establishing credibility: "The City of Chicago is conducting a survey of residents in order to assess community opinions about services provided by your local police department."

A general statement establishing the objectives and goals of the study and the significance of the results to the respondents themselves should follow the client reference. Potential respondents are more likely to participate when they perceive that the study's findings will have a direct impact on their well-being—for example: "It is the purpose of this study to identify those needs that the residents of the city feel should be addressed in order to maintain a peaceful and secure community."

The basis of sample selection should be made clear so that the respondent understands that there are no hidden agendas or undisclosed motivations behind the questionnaire. The characteristics the respondent possesses that led to his or her inclusion in the sample should be clearly delineated—for example: "Chicago is particularly interested in the opinions of new residents, and as such you have been selected at random from a list of new residents of the city."

The respondent must be assured that his or her participation is valued and that answers are neither correct nor incorrect. He or she must be assured that participation is strictly protected in terms of confidentiality—for example: "You should know that there are no right or wrong answers and that your responses will be treated confidentially. Survey results will in no way be traceable to individual respondents."

Because of the more personal nature of telephone, intercept, and inperson interviews, the interviewer should, as a matter of courtesy, identify himself or herself by name and obtain permission to proceed with the survey questions.

A telephone, intercept, or in-person interview preamble might also include some estimate of the time required in order to complete the questionnaire. In the case of a mail-out questionnaire, the respondent should be able to judge this by direct observation of the instrument received in the mail.

A mail-out questionnaire should also include brief return mail instructions, such as, "Please drop your postage-paid, preaddressed response in the mail by June 15."

Exhibit 2.1 is an example of a mail-out or web-based introduction that addresses the issues discussed above. Verify that the preamble contains the essential information. Exhibit 2.2 is an example of a telephone interview

EXHIBIT 2.1. MAIL-OUT INTRODUCTION.

Dear Omaha Resident [applicable respondent characteristic]:

We need your help [*participative value*]! The City of Omaha [*organization identi-fication/credibility*] is conducting a survey of all households in the city [*basis of sample selection*]. The information you provide will be useful in helping your City Council provide services and programs to meet the needs and wishes of the residents [*goals and objectives of study*].

Please take the time to complete the enclosed questionnaire. There are no correct or incorrect responses, only your much-needed opinions [*responses neither right nor wrong*]. This form contains an identification number that will be used for followup purposes only. All responses will be treated confidentially and will in no way be traceable to individual respondents [*confidentiality*] once the survey process has been concluded. Please drop your postage-paid, preaddressed envelope in the mail by June 24 [*return mail instructions*].

Thank you for your assistance. We care what you think [participative value].

Sincerely,

Joan W. Jensen Mayor [credibility]

EXHIBIT 2.2. TELEPHONE INTRODUCTION.

Good evening (afternoon/morning). My name is Joseph Sparks [*interviewer's name*]. The City of Lincoln [*organization identification/credibility*] is currently conducting a survey of Lincoln residents [*applicable respondent characteristic*] concerning the future development of library facilities for the city [*goals/objectives of study*].

Your household was selected at random [*basis of study selection*] to provide information and opinions regarding library facilities in the city of Lincoln.

We would greatly appreciate a small amount of your time [*time*] and your input on this important issue [*participative value*]. There are no correct or incorrect responses, so please feel free to express your opinions [*responses neither right nor wrong*]. Your responses will be treated confidentially and will in no way be traceable to you [*confidentiality*].

May I ask you a few short questions [time/permission]?

EXHIBIT 2.3. IN-PERSON INTRODUCTION.

Hello, my name is Emily Norton [*interviewer name*]. The City of Lincoln [*organization identification/credibility*] is conducting a survey of its residents [*applicable respondent characteristic*] concerning the city's future development of library facilities [*goals/ objectives of study*].

Your household was randomly selected [*basis of sample selection*] to provide information and opinions about library facilities.

Would you be willing to answer a few short questions [*time/permission*] on this important issue [*participative value*]? Please feel free to express your opinions, because there are no correct or incorrect responses [*responses neither right nor wrong*].

The questionnaire form we complete today will not be marked in any way that would identify you [*confidentiality*].

introduction. Once again, cross-check the highlighted issues against the example.

Because of the personal, physical presence of the researcher in face-toface interviewing, exhibit 2.2 can be revised into a somewhat less formal, more conversational format in this type of questionnaire administration. Exhibit 2.3 reflects these changes.

Sequence of Questions

The order in which questions are presented can affect the overall study quite significantly. A poorly organized questionnaire can confuse

respondents, bias their responses, and jeopardize the quality of the entire research effort. The following series of guidelines for sequencing questions has been created to enable researchers to develop a well-ordered survey instrument.

Introductory Questions

The first questions should be related to the subject matter stated in the preamble but should be relatively easy to answer. Introductory questions should elicit a straightforward and uncomplicated opinion or derive basic factual—but not overly sensitive—information—again, as would a conversation. The main purpose of the early questions is to stimulate interest in continuing with the questionnaire without offending, threatening, confusing, or boring the respondent.

For a study designed to determine customer satisfaction among residents of a water district in Southern California, the authors developed a questionnaire that began with the following questions:

- Q1. How long have you been a customer of the Applewood Water District?
 - 1. _____ Less than 1 year
 - 2. _____1 year and under 3 years
 - 3. _____ 3 years and under 5 years
 - 4. _____5 years and under 10 years
 - 5. _____ 10 years and under 15 years
 - 6. _____ 15 years and under 20 years
 - 7. _____ 20 years or more
- Q2. In what area of the Applewood Water District do you live?
 - 1. _____ Canyon District
 - 2. _____ Foothill District
 - 3. _____ Beach District
- Q3. On a scale of 1 to 5, where 1 = very good and 5 = very poor, how would you rate your overall satisfaction with the Applewood Water District as your water service provider?
 - 1. _____ Very good
 - 2. _____ Good
 - 3. _____ Neither good nor poor
 - 4. _____ Poor
 - 5. _____ Very poor

42

The first two questions are of a basic, factual nature. The third question, although eliciting an opinion, is uncomplicated; however, it is germane to the key focus and sufficiently stimulating to secure the respondent's continued interest.

Sensitive Questions

Certain questions deal with sensitive issues, such as religious affiliation, ethnicity, sexual practices, income, and opinions regarding highly controversial ethical and moral dilemmas. It is recommended that these questions be placed late in the questionnaire, for two fundamental reasons.

First, if respondents react negatively to such questions and decide to terminate their participation in the questionnaire, the information obtained on all previous questions may still be usable in the overall survey results, because enough information may have been obtained to warrant acceptance of the interview as a completed survey with only a few questions remaining unanswered. Second, if rapport has been established between the interviewer and the respondent during the course of the survey process, there is an increased likelihood that the respondent will answer sensitive questions that come late in the questionnaire.

Related Questions

Questionnaires generally have a certain frame of reference, as indicated by their goals and objectives. Within this overall context, there are several categories of questions. For instance, the questionnaire soliciting opinions from water district customers contained questions relating to communication, water rates, conservation, outdoor watering, desalination, water recycling, social media, and demographic data.

Proper questionnaire design dictates that related questions be placed together within the questionnaire so that the respondent can focus and concentrate on specific issues without distraction. In order to facilitate this, it is sometimes appropriate to separate categories of questions by providing a distinct heading that characterizes each section. In terms of communication issues regarding the Applewood Water District website, the following question sequence can be considered to be an acceptable one.

- Q1. Have you ever visited the Applewood Water District website?
 - 1. _____Yes
 - 2. _____ Have access to Internet but have not visited website \rightarrow GO TO Q3

3. _____ Do not have access to the Internet—— \rightarrow GO TO Q4

- 9. _____ Do not know \rightarrow GO TO Q4
- Q2. [IF Q1 = 1] On a scale of 1 to 5, with 1 being very good and 5 being very poor, how would you rate the user friendliness of the website? Would you say:
 - 1. _____ Very good
 - 2. _____ Good
 - 3. _____ Neither good nor poor
 - 4. _____ Poor
 - 5. _____ Very poor
- Q3. [IF Q1 = 2] Do you think that you would use the website more if a chat function were available that allows you to ask questions directly to a customer service representative?
 - 1. _____Yes
 - 2. _____ No
 - 9. _____ Do not know

If these same questions were to be commingled with questions from other categories, the resulting questionnaire would be much less likely to produce clear, well-formulated responses. You should be able to verify this by examining this less acceptable question order:

- Q1. Do you feel that water costs too much, too little, or priced about right?
 - 1. _____ Water costs too much
 - 2. _____ The cost of water is just about right
 - 3. _____ Water costs too little
- Q2. Does your landscaping include a lawn?
 - 1. _____Yes
 - 2. _____ No
- Q3. Have you ever visited the Applewood Water District website?
 - 1. _____Yes
 - 2. _____ Have access to Internet, but have not visited website
 - 3. _____ Do not have access to the Internet

While it is generally desirable to arrange questions pertaining to a particular subject in the same section of the questionnaire, it is also important to be cognizant of creating a patterned series of responses. Consecutive questions that tend to evoke reflexive responses, given without adequate thought, should be minimized. **Designing Effective Questionnaires**

Note that the sequence of questions in exhibit 2.4, which is part of a commercial business survey, could well produce an automatic, unidirectional set of responses unless the respondent is sensitized to the subtle but important differences among the questions. This process of sensitizing will tend to minimize the risk of reflexive responses and is accomplished in this example by italicizing the essential distinctions.

EXHIBIT 2.4. SERIES OF QUESTIONS DEMONSTRATING SENSITIZING OF RESPONDENTS.

1. What types of additional businesses, if any, do you feel are needed in the City of Poway to help serve *your business needs?* (Please check no more than three types of businesses.)

Types of Businesses

- _____ Food/market
- _____ Food/specialty store (bakery, deli, etc.)
- _____ Restaurant/dinner house
- _____ Restaurant/other (specify)
- _____ Retail/department store
- _____ Retail/specialty store
- _____ Professional
- _____ Services/supplies/equipment
- _____ Light industry
- _____ Other (specify) ______
- _____ Other (specify) ______
 - _____ Other (specify) _____
- 2. What types of additional businesses, if any, do you feel are needed in the City of Poway to help serve the needs of *your employees?* (Please check no more than three types of businesses.)
 - _____ Food/market
 - _____ Food/specialty store (bakery, deli, etc.)
 - _____ Restaurant/dinner house
 - _____ Restaurant/other (specify)
 - _____ Retail/department store
 - _____ Retail/specialty store
 - _____ Professional
 - _____ Services/supplies/equipment

Other (specify)
Other (specify)
Other (specify)

3. What types of additional businesses, if any, do you feel are needed in the City of Poway to help serve the needs of *your customers?* (Please check no more than three types of businesses.)

Types of Businesses

 _ Food/market
 _ Food/specialty store (bakery, deli, etc.)
 _ Restaurant/dinner house
 _ Restaurant/other (specify)
 _ Retail/department store
 _ Retail/specialty store
 _ Professional
 _ Services/supplies/equipment
 _ Other (specify)
 Other (specify)
 Other (specify)

Alternative approaches to minimizing this risk of patterned responses may include the more frequent use of open-ended questions (without fixed alternative responses), questions that change the order of the fixed responses from question to question, or questions that vary substantially in terms of wording or length. The potential disadvantages of such tactics are that the respondent's thought focus may be disrupted or the respondent might become confused, thereby defeating the purpose of grouping these questions in the first place. Because several considerations must be balanced in the grouping of questions, the pretest becomes of paramount importance to identify the potential for inadvertently eliciting response patterns and to minimize any such impact on the study.

Logical Sequence

There is frequently a clear, logical order to a particular series of questions contained within the survey instrument. For instance, an appropriate time sequence should be followed. If questions are to be posed concerning an individual's travel behavior by bus, they should be structured in such a way that the respondent is asked to answer them in a sequential or temporal order—for instance, from the most recent to the least recent bus over a specified period of time:

Please list all buses you will use to complete your one-way trip today:

- 1. First bus: Route # ____
- 2. Transfer to second bus: Route # _____
- 3. Transfer to third bus: Route # _____

Filter or Screening Questions

Other portions of the questionnaire might involve establishing the respondent's qualifications to answer subsequent questions. Through what are called filter or screening questions, as shown in exhibit 2.5, the researcher can determine whether succeeding questions apply to the particular respondent. The first question requires that some respondents be screened out of certain subsequent questions. Only those who have read the district's newsletter or bill inserts are asked how they would rate the usefulness of these materials. After the appropriate respondents are asked about the newsletter or bill inserts, respondents who read these materials as well as those who do not are asked about how they pay their water bill most months.

Further screening occurs among those who do not use the Internet to pay their bill. These respondents are asked how the district can make bill paying online more appealing.

Under some circumstances, filter questions may be used to disqualify certain respondents from participating in the survey process at all. Exhibit 2.6 draws from a telephone questionnaire that was used in a survey of registered voters. It was the intent of the survey to query not all registered voters but only those who were likely to vote. For purposes of the survey, those who were most likely to vote were considered to be those who had voted for the mayor or US senator in the previous major election. The survey screened out entirely those who did not satisfy the appropriate preconditions by providing explicit instructions for the interviewer concerning disqualification.

Reliability Checks

When a question is important or is particularly sensitive or controversial, the degree of truthfulness or thoughtfulness of the response may be in

EXHIBIT 2.5. FILTER OR SCREENING QUESTIONS.

- Q1. Do you read the newsletter or bill inserts that come in the mail with your monthly water bill . . .
 - 1. _____ Every time,
 - 2. _____ Most times,
 - 3. _____ Sometimes, or
 - 4. _____ Never \rightarrow GO TO Q3
- Q2. [IF Q1 = 1 or 2 or 3] On a scale of 1 to 5, with 1 being very good and 5 being very poor, how would you rate the usefulness of these materials? Would you say . . .
 - 1. _____ Very good
 - 2. _____ Good
 - 3. _____ Neither good nor poor
 - 4. _____ Poor
 - 5. _____ Very poor
- Q3. How do you pay your water bill most months?
 - 1. _____ Send check by mail
 - 2. _____ Automatic bank deduction
 - 3. _____ Credit card over the telephone
 - 4. _____ In person at the Applewood Water District office
 - 5. _____ In person at payment center
 - 6. _____ Online (Internet) [GO TO Q5]
- Q4. [IF Q3 ≠ 6] What can the district do to make paying online a more appealing option for you? _____

EXHIBIT 2.6. SCREENING USED TO DISQUALIFY RESPONDENTS.

- 1. Are you registered to vote in the City of Sonoma?
 - 1. Yes _____ (CONTINUE)
 - 2. No _____ (DISQUALIFY)
 - 3. Not sure _____ (DISQUALIFY)
 - 4. Refused _____ (DISQUALIFY)
- 2. Did you vote in the 2012 elections for mayor or US senator?
 - 1. Yes _____ (ASK QUESTION C)
 - 2. No _____ (DISQUALIFY)
 - 3. Not sure _____ (DISQUALIFY)

doubt. In such situations, it may be appropriate to include in the questionnaire a check of the respondent's consistency of response by asking virtually the same question in a somewhat different manner and at a different place within the survey instrument.

In a survey research project seeking to identify the demand for market rate housing in downtown San Diego, the following question was asked of respondents:

Please indicate the likelihood of your choosing to live in downtown San Diego.

- 1. ____ Very possible
- 2. ____ Somewhat possible
- 3. ____ Not very likely
- 4. ____ Highly unlikely

The researchers suspected that there might be a casual or less careful response pattern to this question, in which respondents might indicate their willingness to live downtown without giving the matter adequate thought. Therefore, later in the questionnaire, this question was posed:

When you consider the possibility of living in downtown San Diego, do you feel

- 1. ____ Excited
- 2. ____ Interested
- 3. ____ Indifferent
- 4. ____ Uncomfortable
- 5. ____ Frightened
- 6. ____ Other; please specify: _____

In this study, in order for a respondent to be considered a "possible downtown resident," he or she had to choose the first or second response to both questions. Because any other combination might indicate a tentative or inconsistent willingness to consider downtown as a possible place to live, respondents with such answers were not considered strong candidates for downtown living. Without the benefit of this reliability check, respondents who were less likely to live downtown might well have been wrongly included with those who were more inclined to do so.

Question Format: Open-Ended or Closed-Ended

Most questions in a questionnaire have *closed-ended* response choices or categories. Such questions provide a fixed list of alternative responses and ask the respondent to select one or more of them as indicative of the best possible answer. In contrast, *open-ended* questions have no preexisting response categories and permit the respondent a great deal of latitude in responding to them.

Advantages of Closed-Ended Questions

There are several advantages to closed-ended questions. One is that the set of alternative answers is uniform and therefore facilitates comparisons among respondents. For purposes of data entry, this uniformity permits the direct transfer of data from the questionnaire to the computer without intermediate stages. The respondent's answers can be directed by a fixed list of alternatives, which limits extraneous and irrelevant responses. Here is an example of a closed-ended question:¹

What is the highest level of formal education that you have achieved?

- 1. ____ Some high school or less
- 2. ____ High school graduate
- 3. ____ Some college
- 4. ____ Four-year-college graduate
- 5. ____ Postgraduate degree

If, instead, the question were open-ended, as shown below, the responses might not be quite so specific:

How much education do you have?

Another advantage is that the fixed list of response possibilities tends to make the question clearer to the respondent. A respondent who may otherwise be uncertain about the question can be enlightened as to its intent by the answer categories. Furthermore, such categories may in fact remind the respondent of alternatives that otherwise would not have been considered or would have been forgotten.

Sensitive issues are frequently better addressed by asking questions with a preestablished, implicitly "acceptable" range of alternative answers rather
than by asking someone to respond with specificity to an issue that might be considered particularly personal. For example, for medical purposes, an abortion clinic might require information about a client's history in terms of previous abortions. The questions, "Have you ever had an abortion? If so, how many have you had?" will tend to intimidate certain respondents who have had prior abortions and perceive that abortion carries a degree of social stigma. Their responses therefore might be biased toward minimizing the actual number. Recognizing that this tendency exists and always will in regard to socially sensitive issues, the researcher would improve response accuracy by constructing the question as follows:

How many abortions have you had?

 1. _____ None

 2. _____ One

 3. _____ Two

 4. _____ Three

 5. _____ Four

 6. Five or more

Phrasing sensitive questions in this way, with alternative responses that extend significantly beyond normally expected behavior, implies that an accurate response is not outside the realm of social acceptability. In this case, it implies that many other women may have similar histories and that having had an abortion is not necessarily aberrant behavior.

Other types of sensitive questions may pose issues more closely associated with privacy than with social acceptability, for example, when the subject of a question is income. A respondent may very well feel that his or her privacy is violated when asked, "What is your annual household income?"

Giving alternative choices in the form of income ranges will tend to mitigate such feelings and will therefore generate a much higher level of response. A question about income is much better constructed to read as follows:

Please indicate the range that best describes your annual household income:

- 1. ____ Less than \$15,000
- 2. ____ \$15,000-\$29,999
- 3. ____ \$30,000-\$44,999
- 4. _____ \$45,000-\$59,999
- 5. ____ \$60,000 and above

Finally, fixed responses are less onerous to the respondent, who will find it easier to choose an appropriate response than to construct one. Thus, use of fixed-alternative questions increases the likelihood that the response rate for particular questions, and for the questionnaire in general, will be higher.

Disadvantages of Closed-Ended Questions

Closed-ended questions have certain disadvantages that researchers should consider when developing a questionnaire. For example, there is always the possibility that the respondent is unsure of the best answer and may select one of the fixed responses randomly rather than in a thoughtful fashion. It is also possible that the simplicity of the fixed-response format may lead to a greater probability of inadvertent errors in answering the questions. For instance, an interviewer or a respondent may carelessly check a response adjacent to the one that was actually intended. The advantage of ease of response therefore comes with some potential negative consequences. In a similar vein, a respondent who misunderstands the question may select an erroneous response.

Closed-ended questions in a sense compel respondents to choose a "closest representation" of their actual response in the form of a specific fixed answer. Subtle distinctions among respondents cannot be detected within the preestablished categories. This particular drawback is frequently addressed by inserting another alternative in the fixed-response format: "Other; please specify." This alternative represents an excellent compromise between closed- and open-ended response formats in that it is an open-ended question within a closed-ended format, as shown in this example:

- Q1. What is the most important reason that your organization developed a telework policy?
 - 1. ____ Initiative by upper management
 - 2. ____ Response to employee requests
 - 3. ____ Effort to save money/attain higher level of efficiency
 - 4. ____ Effort to retain qualified staff
 - 5. ____ Other (please specify) ____

For simplicity and ease of response, however, the use of this option must be carefully controlled. The decision to include an "Other" response category for a particular question must be based on evidence obtained during the pretest of the survey instrument. If the evidence shows that a relatively large number of responses to the question do not conform to the preliminary set of fixed alternatives (a minimum of 3 percent), the researcher should formulate additional fixed categories for the responses that appear frequently and retain the "Other, please specify" category for the responses that appear less frequently. If there is no indication that an "Other" category is needed, it should not be included.

Using Open-Ended Questions

Researchers use open-ended questions in situations where the constraints of the closed-ended question outweigh the inconveniences of the open-ended question for both the researcher and the respondent. It is recommended that open-ended questions be used sparingly and only when needed. To the extent that they are used, the researcher must be aware of certain inherent problems.

First, open-ended questions inevitably elicit a certain amount of irrelevant and repetitious information. In addition, the satisfactory completion of an open-ended question requires a greater degree of communicative skills on the part of the respondent than is true for a closed-ended question. Accordingly, the researcher may find that these questions elicit responses that are difficult to understand and sometimes incoherent.

A third factor is that statistical analysis requires some degree of data standardization. This entails the interpretative, subjective, and time-consuming categorization of open-ended responses by the researchers. And finally, open-ended questions take more of the respondent's time. This inconvenience may engender a higher rate of refusal to complete the questionnaire.

Follow-Up Open-Ended Questions

As discussed, it is desirable to have relatively simple, fixed-answer questions wherever possible. However, most surveys find it necessary to seek information that cannot be fully answered within the fixed-answer format. In such cases, follow-up open-ended questions are asked in a manner that connects them to the fixed-answer question. For instance, during the study of customer satisfaction in the Applewood Water District, the following questions were asked:

Q1. How likely are you to choose to go paperless in your bill paying to the district and other regular monthly accounts within the next year or

two? Please provide your answer on a scale of 1 to 5, where 1 is very likely and 5 is very unlikely.

1.____Very likely

2.____Somewhat likely

3.____Neither likely nor unlikely

4.____Unlikely

5.____Very unlikely

Q2. [IF Q1 = 4 or 5] What is your major objection to going paperless for bill paying?

Efforts should be made to place such open-ended questions as late in the appropriate section of the questionnaire as possible, while remaining cognizant of the need to have a logical and temporal order of questions.

Open-Ended Venting Questions

At the end of the entire questionnaire, it is often beneficial to use one or more open-ended "venting" questions—ones in which the respondent is asked to add any information, comments, or opinions that pertain to the subject matter of the questionnaire but have not been addressed in it. For example, a survey about the potential development of a formal telework policy among major employers in the Syracuse region posed the following question at the end of a survey:

What, in your opinion, would motivate the management or decision makers in your organization to consider the development of a formal telework program?

Questionnaire Length

The questionnaire should be as concise as possible while still covering the necessary range of subject matter required in the study. The researcher must be careful to resist the temptation of developing questions that may be interesting but are peripheral or extraneous to the primary focus of the research project.

The purpose of being sensitive to questionnaire length is to make certain that the questionnaire is not so long and cumbersome to the respondent that it engenders reluctance to complete the survey instrument, thereby jeopardizing the response rate. As questions increase in complexity and difficulty, the questionnaire may be perceived as being tedious and longer than it actually is. Hence, the researcher must factor in such considerations as the number of questions and the time and effort required of the respondent to complete them.

As general guidelines, telephone interviews should occupy absolutely no more than twenty minutes of the respondent's time—and preferably closer to twelve to fifteen minutes; mailed questionnaires should take thirty minutes or less—preferably closer to fifteen minutes; web-based surveys should also be targeted for fifteen minutes; in-person interviews should be limited to thirty minutes; and intercepts must be accomplished in four or five minutes at most—preferably three minutes.

Editing the Completed Questionnaire

An important part of the interviewer's task is to examine finished questionnaires for accuracy, legibility, and completeness. Despite this preliminary examination, the researcher must review each questionnaire for quality control purposes, especially with regard to filtering, multiple answers, and open-ended questions. Since mail-out questionnaires receive no intermediate interviewer examination, the researcher must be particularly careful in reviewing them.

In the review, the interviewer must be sure that questions that were designed to be skipped (through a filtering process) have indeed been skipped. If the interviewer has mistakenly asked an inapplicable question or has inadvertently marked a response to that question, the response should be deleted. In the case of questions that permit multiple responses and request a ranking, the first choice should be ranked by a code of 1 and the second choice by a code of 2. Such a question should be examined for accuracy in the following way:

- If only one response was made, it should receive a code of 1.
- Two responses should be coded with a 1 and a 2. If two responses are provided but are not ranked (e.g., they are indicated with a check mark), in-person interviewers should recontact the respondent immediately. This is an important reason for interviewers to examine the accuracy of their completed interviews at the time they are given. In the mail-out format, if there are only a few such responses, follow-up telephone calls, using the cross-referenced identification code, are in order. If there are many such inaccurately coded responses, the researcher can establish a

new category for response categories that have been indicated but not ranked (see the discussion of postcoding below for the procedure for introducing new variable categories).

 More than two responses are not permitted. The intercept and in-person formats enable immediate corrections, and web-based and telephone surveys can prohibit such errors as part of the programming. The mailout can involve recontacting the respondent or creating an "Indicated but not ranked" category, and the final report should caution the reader that some respondents provided more than two responses.

After the review of the questionnaire has been completed, the researcher can begin the postcoding process.² In postcoding, responses to open-ended questions and "Other, please specify" response categories of the questionnaire are coded. To facilitate this process, the researcher should ask the interviewers to provide lists of all open-ended and "Other, please specify" responses.

With regard to "Other, please specify" responses, the researcher should first review these responses, identify those that reasonably belong to an existing, precoded category, and code them in accordance with that category by writing the code number directly on the questionnaire next to the response. This should be done in a different-colored ink from the one used to typeset the form and the one used by the respondent or interviewer to mark the questionnaire. This will permit the data entry technician to easily identify the postcoded response. The original "Other" response code should be crossed out for further clarity. "Other" responses that cannot be categorized into the precoded response categories can be treated in one of two ways, requiring a certain degree of judgment by the researcher:

1. When there is a sufficiently large number of the same or similar responses that do not belong in a precoded category, the researcher should consider creating a separate category with a new numerical code, starting with the first available number following the existing codes (the code for "Other" should also be adjusted to the highest code number so that it will print out last in the computer output). Therefore, it is not uncommon for "Other" categories to be precoded 9 if there are to be a few categories, or given a larger code number (for example, 20) if there might be a double-digit number of categories. If the frequency of any of these similar responses approaches the frequency of one of the existing categories, it is probable that a new code is warranted. This code should be marked on the questionnaire in a different color. Recoding is

a frequent necessity in survey research in order to accommodate unexpected responses.

2. All responses that have a relatively low frequency of response can remain in a "Miscellaneous" or "Other" category.

The following question and answer demonstrate this process. This completed questionnaire contains a response that has been proven to occur with great frequency on other completed questionnaires (e.g., restaurant). Consequently, another category and associated code are merited (hospitality coded as 5).

What kind of new business in Scranton do you feel would give you the best opportunity for employment?

1. ____ Retail

2. ____ Light industry

- 3. ____ Heavy industry
- 4. ____ Office/professional
- 8. 5. ____ Other, please specify: restaurant

Open-ended questions require a similar postcoding process. That is, based on a verbatim listing of all responses to an open-ended question, the researcher again uses judgment to develop categories into which these responses can be placed. The number of categories should be limited to approximately ten, with a maximum of fifteen to twenty, while adhering

	f	%
Provide improved local police protection	90	22.5
Ease traffic congestion	83	20.8
Enhance public education	74	18.5
Improve zoning/planning process	70	17.5
Provide more community funds	35	8.7
Improve communication	21	5.3
Other	27	6.7
Total	400	100.0

TABLE 2.1. WAYS IN WHICH CITY GOVERNMENT CAN SERVE COMMUNITY NEEDS.

f = frequency or number of responses.

to the guideline that each should contain a respectable percentage (3 to 5 percent) of the total responses.

Table 2.1 was derived from the categorization of responses to the openended question, "How can the city government better serve your community?" By way of elaborating on the process of categorizing open-ended responses, the category of "Improve zoning/planning process" in table 2.1 contains such verbatim responses as "fewer apartments," "more open space," "make developers pay fair share," and "protect property values."

Notes

- 1. It is important that the categories of a closed-ended question be associated with numerical codes. The codes assigned during the questionnaire development stage are necessary for data entry and useful for data analysis.
- 2. When they have been completed, edited, and coded, the questionnaires are ready for the data entry process. There are numerous statistical software programs from which to choose. The researcher selects the most appropriate program based on the size and scope of the project, the sophistication of statistical analysis envisioned, the importance of the integration of graphics into the final report, ease of operation, and program cost. A major statistical program for survey research is the Statistical Package for the Social Sciences (SPSS), which is sophisticated, comprehensive, and capable of processing large amounts of data. It can generate both the very basic and the most highly advanced descriptive and analytical statistics and graphics. Microsoft Excel is also useful as a statistical analysis tool in the conduct of survey research.



CHAPTER THREE

DEVELOPING SURVEY QUESTIONS

The previous chapter addressed overall questionnaire development. No consideration of questionnaire development would be complete, however, without a thorough analysis of the principles and potential problems involved in the actual phrasing and formatting of the individual questions themselves.

Questionnaire construction is a skill that is refined over time by experience. Each research project has its own set of conditions and circumstances, which renders the imposition of fixed and rigid rules impossible. This chapter is particularly sensitive to the need for flexibility; instead of rules, it offers a series of objectives and guidelines in the pursuit of clear questions. Two fundamental considerations are involved:

- Question phrasing
- Question formatting

The researcher must use considerable discretion in the application of the guidelines outlined in this chapter, because there is a fine line between appropriately and inappropriately constructed questions. Such appropriateness can prove to be critical to the success of a research project.

Guidelines for Phrasing Questions

The way questions are worded is critically important to the success of a survey. Injudicious phrasing can lead to results that are ambiguous and potentially biased. The following guidelines are provided to assist in the preparation of survey questions that are objective and clearly worded.

Level of Wording

The researcher must be cognizant of the population to be surveyed in terms of the choice of words, colloquialisms, and jargon used in the questions. As a general guideline, wording should be simple, straightforward, and to the point. Specifically, the researcher should attempt to avoid highly technical words or phrases, words that require or are associated with higher levels of experience or education, and words or phrases that may be insensitive to ethnic- or gender-related issues.

For example, in a questionnaire seeking to obtain information related to the use of artificial sweeteners, the following possible questions might be asked:

- Do you think soft drinks that contain aspartame, saccharine, and other such additives are dangerous to your health?
- Do you think soft drinks that contain artificial sweeteners are dangerous to your health?

The first possibility uses the terms *aspartame* and *saccharine*, which are not necessarily universally understood by the general population. Therefore, the responses to this question may not be consistent with its intent. The second possibility uses the simpler and clearer phrase *artificial sweeteners*, and the responses should consequently be more accurate.

Obviously the researcher is interested in making certain that respondents understand the questions well enough to provide accurate representations of their opinions, behavior, and characteristics for purposes of the study. If questions are not understandable, one of three problems may arise:

- Information provided may be inaccurate.
- There may be a large number of "do not know" or "no opinion" responses.
- The rate of refusal to complete the questionnaire may be inordinately high.

Developing Survey Questions

Once again, the pretest looms large in importance in the detection of language-related problems.

On occasion, the general guideline of simplicity should be modified to accommodate special population groups. In a survey among attorneys concerning attitudes about courtroom procedures, it is appropriate to include words that are recognizable to those who have been formally trained in the law. If the survey were instead administered to the general public, the level of wording would of necessity be different.

Ambiguous Words and Phrases

Effort must be devoted to avoiding ambiguity in the questions. Ambiguity can occur from the use of vague words or phrases. For example, if a survey is seeking to determine the number of people residing together in one household, an inappropriate wording of a question might be, "How many people live in your household?" It may not occur to respondents faced with this question that they should include themselves in the response. The confusion can be avoided by rewording the question in a clear and specific manner: "Including yourself, how many people live in your household?"

Similarly, in an attempt to determine household income, the question, "What is your income?" will produce a variety of unsatisfactory responses, such as the respondent's annual income, the respondent's take-home pay, the respondent's hourly wage, or the total household income. What is generally sought in most surveys is total gross annual household income, before taxes. The question, "Please indicate the category that best represents your total annual household income, before taxes," will produce the desired responses.

Words such as *affiliate, identify, involved,* and *belong* often produce ambiguous results. For instance, asking an individual with which ethnic group he or she most closely identifies can be interpreted to mean, "With which group do I best get along?" rather than, "Of which ethnic group am I a member?" In the first interpretation, a respondent may provide more than one response in order to communicate a favorable inclination toward certain ethnic groups. However, the researcher is typically interested in ascertaining the respondent's own ethnic background and would find such a response uninformative. An appropriate phrasing for obtaining such information is, "Please indicate your race or ethnicity."

Another example of ambiguous wording is demonstrated in this survey question: "Please indicate the number of organizations with which you are involved." The words *involved* and *organizations* are both sufficiently vague to be likely to generate a variety of interpretations among survey respondents. The specific organizational type (e.g., social clubs, professional organizations) should be delineated, as should the precise nature of the involvement.

Confusingly Phrased Questions

Confusing questions generate a feeling of uncertainty in the respondent as to the intent or meaning of the question. If it is a closed-ended question, the respondent frequently does not understand which of the alternative responses will express his or her opinion on the issue. Consider the question in example 3.1.

EXAMPLE 3.1

Does it seem possible or does it seem impossible to you that the Nazi extermination of the Jews never happened?

1. ____ It seems possible.

2. ____ It seems impossible.

The confusion can be eliminated with the following rewording of this question:

Does it seem possible to you that the Nazi extermination of the Jews never happened, or do you feel certain that it happened?

1. ____ It is possible that it never happened.

2. ____ I feel certain that it happened.

Another question that is illustrative of a confusing question is shown in example 3.2.

EXAMPLE 3.2

Indicate your two favorite spectator sports by placing a 1 by your favorite and a 2 by your second favorite. If you have only one favorite, place a 1 by it and do not mark any with a 2. If your two sports are equally favored, give each a 1 and do not mark a 2. If you have no favorite, go to the next question.

1. ____ Baseball 2. ____ Football

62

Developing Survey Questions

- 3. ____ Basketball
- 4. ____ Ice hockey
- 5. ____ Other, please specify (do not include figure skating because it is not a sport)

6. Next question

This question was taken from a survey about leisure activity, including spectator sports. Note the poorly worded and confusing instructions. The list of sports does not include some major spectator sports such as tennis and soccer, and the use of a category labeled "next question" is totally inappropriate. Furthermore, the bias included with reference to figure skating simply adds to the confusion and the poor quality of the question.

Another source of confusion can be found in instructions that are not explicit; for example, in relation to a question asking, "What are your favorite recreational activities from the list below?" the instructions might not inform the respondent how many activities to mark or whether these activities are to be ranked.

Double-Barreled Questions

Double-barreled questions are those that might inadvertently confuse the respondent by introducing two or more issues with the expectation of a single response. An example of a multipurpose question might be, "Are you satisfied with your instructor's organizational skills and knowledge of the subject?" To respond to this question with a yes answer, the respondent would need to have a positive opinion of both organizational skills and knowledge, thereby denying the researcher potentially valuable information about each individual issue. Hence, such wording can result in findings for which the precise meaning is uncertain. Another example is found in a questionnaire that was published in a small-town newspaper in order to determine public opinion about future land development in the community. The first question in that survey was worded as in example 3.3.

EXAMPLE 3.3

Do you believe the VISIBLE fast food restaurants planned for Jacumba's freeway entrances will affect the image and property values of our whole community?

1. ___Yes 2. ___ No

The only way to answer yes to such a question is to feel positively about both image and property values and about all of Jacumba's freeway entrances. In other words, if a respondent considers such development satisfactory at one entrance and not at another or believes that image will be affected but not property values, a negative answer is appropriate. Hence, responses to such questions are impossible to interpret accurately. Any question that contains the conjunctions *and* or *or* should be reviewed very carefully for the possibility that it may actually be composed of more than one question.

Manipulative Information

Certain questions may require some form of explanation to be presented to the respondent in order to provide necessary background and perspective. The researcher must be very careful that explanatory statements do not unduly influence the response by providing biasing or manipulative information. The objective researcher should not skew responses in one direction or another, but rather should solicit genuine opinions, behaviors, and facts from the respondents. An example of such manipulation is as follows: "Some research has shown that consuming even a small amount of alcohol will impair one's motor skills. Do you think that laws regarding drinking and driving should be stricter?" More often, manipulative information is less obvious. The question in example 3.4, adapted from a public opinion survey prior to a major local election, asked potential voters about funding for parks and recreation:

EXAMPLE 3.4

The federal government spends approximately \$1,200 per US resident on national defense. Do you believe that the federal government is adequately allocating funds for national parklands and recreational facilities by designating \$10 per resident for this purpose?

1. ____ Yes 2. ____ No 3. ____ No opinion

Whereas the researcher may have provided the information about defense spending in order to provide perspective to the potential respondent, this information may also be manipulative by characterizing the funding for parks as comparatively inconsequential and therefore inadequate. A more straightforward question, without reference to the defense budget, may well generate an entirely different response. It is frequently the case that the researcher is interested, however, in how knowledge such as the difference between defense and national park spending might affect the respondent. That is accomplished by asking the straightforward question first and then following it up with the information, as shown in example 3.5.

EXAMPLE 3.5

1. Do you believe that the federal government is adequately allocating funds for national parklands and recreational facilities by designating \$10 per resident for this purpose?

1. ____ Yes 2. ____ No 3. ____ No opinion

1a. If you were to learn that the federal government spends approximately \$1,200 per US resident on national defense, would that change your opinion about the adequacy of allocating \$10 per resident for this purpose?

1. ____ Yes 2. ____ No 3. ____ No opinion

Unfortunately, manipulative information is occasionally incorporated deliberately into a questionnaire. It is not uncommon for certain survey sponsors to want to use such surveys for publicity purposes or to influence voter opinion. Example 3.6, which can be considered to contain manipulative information, addresses the topic of national health care policy.

EXAMPLE 3.6

In your opinion, is the Affordable Care Act, passed by the US Congress in 2010, a step in the right direction in terms of providing health care insurance to the American people?

1. ____ Yes 2. ____ No 3. ____ No opinion

(The Affordable Care Act will enable millions of Americans who previously were not eligible for health insurance to obtain insurance. No one will be rejected because of pre-existing conditions. This is a revolutionary piece of legislation.)

The manipulative information is in the lengthy explanation, which can serve to bias the respondent toward an affirmative response. The information contained in that explanation may or may not be entirely correct. It is clearly subject to some interpretation. Furthermore, there may be a problem in invoking the endorsement of what might be perceived to be an organization or institution with particular expertise. Referring to the US Congress does not present such a significant biasing problem, but the researcher must be cognizant of the biasing potential involved in citing authorities such as religious organizations or highly respected public figures.

This discussion should not be construed as indicating that all explanatory information related to a question is necessarily manipulative. An appropriate use of an explanatory statement is set out in example 3.7, where the definition of *telework* is critical to an informed answer to certain questions.

EXAMPLE 3.7

Please consider the following definition of *telework*: "Telework is defined as performing your work duties during your normal work hours from home (or from a satellite facility that is closer to your home than the regular office) for the entire workday instead of commuting to the regular office."

- Q1. What type of work in your department/organization is done through telework? (check all that apply)
 - 1. ____ Routine word processing, data entry and quantitative analysis, and Internet-based work
 - 2. ____ Computer-based work requiring specialized programs/software unique to your department/organization
 - 3. _____ Telephone, FAX, e-mail communication with clients and coworkers
 - 4. ____ Videoconferencing
 - 5. ____ Other (please specify) _____
- Q2. Are there certain types of work in your organization that would not be suitable for telework?
 - 1. ____ Yes
 - 2. ____ No, just about all of the work in our department is suitable for teleworking.

Inappropriate Emphasis

The use of boldfaced, italicized, capitalized, or underlined words or phrases within the context of a question may serve to place inappropriate emphasis on these words or phrases. However, emphasis can serve a constructive purpose when the researcher needs to clarify potentially confusing nuances that may exist within the questionnaire (see exhibit 2.4) or when titles to published works are involved.

Devices for indicating emphasis are inappropriately used when they are designed to evoke an emotional response or to impose the researcher's concept of significance rather than leaving the determination of what is and is not important to the respondent. Such tactics tend to bias survey results. An example of inappropriate emphasis is found in example 3.8.

EXAMPLE 3.8

The City of Santa Fe, New Mexico. has been acknowledged as *The Jewel of the West*, with glorious sunshine and warmth. Please rate Santa Fe as a place to work and raise a family.

Very Good	Good	Neutral	Poor	Very Poor
1	2	3	4	5

Note that this survey question is simultaneously an example of the improper use of an explanatory statement, resulting in manipulative information, and the improper use of italics, resulting in inappropriate emphasis. It also violates the double-barreled-question guideline by asking the respondent to simultaneously rate Santa Fe as both a place to work and a place to raise a family. Example 3.3 can also serve to illustrate inappropriate emphasis. Its focus on the word *visible* seems to be an effort to disturb the community's rural residents by ascribing some form of visual obtrusiveness to the planned development.

Emotional Words and Phrases

Although they may be clear, simple, and otherwise acceptable, certain words and phrases carry with them the power to elicit emotions. Survey questions must be as neutral as possible to obtain accurate results and fulfill their obligation to solicit and welcome all points of view. Questions must invite true responses from the entire population and not induce the respondent into giving an answer other than the one he or she would normally give (see example 3.9).

EXAMPLE 3.9

The cornerstone of our democracy, the Bill of Rights, guarantees freedom of speech. Do you believe that subversives have the right to advocate the illegal overthrow of the US government?

1. ____ Yes 2. ____ No 3. ____ No opinion

The words *subversives* and *overthrow* evoke negative feelings in most readers, which can lead respondents to associate such negative feelings with a negative response to the question. The introductory sentence can be construed as manipulative information that also leads to the desired negative response.

In general, slanderous and prejudicial language must be avoided, as must language that conjures up specifically positive or negative images. Consider the following example:

Some city planners argue that the absence of appropriate growth controls on population will lead to devastating traffic congestion, unhealthy air pollution, and unattractive and haphazard development. Do you support growth controls in your community, or do you feel that such controls are not justified?

- 1. ____I support growth controls.
- 2. ____I do not support growth controls.
- 3. ____No opinion.

The preamble to this question deliberately slants the respondent toward affirming the need for growth controls through the use of such adjectives as *devastating*, *unhealthy*, *unattractive*, and *haphazard*. Thus, the entire preamble is manipulative. Also, these words inappropriately evoke negative emotion by emphasizing the potentially negative consequences of not instituting growth controls.

Levels of Measurement

Survey data are organized in terms of variables. A *variable* is a specific characteristic of the population, such as age, sex, or political party preference. Each variable in a survey consists of a set of categories that describe the nature and type of variation associated with the characteristic. The variable *gender*, for example, consists of two categories: male and female. Certain opinions are solicited in terms of three categories of response: yes, no, and no opinion. Some variables, such as annual income, can have numerous categories of response, depending on the researcher's purpose and focus.

Variables used in a survey project have distinct measurement properties associated with their categories. These are referred to as *levels of measurement* or *measurement scales*. Some variables can be classified only into labeled categories (*nominal scale*); other variables can be labeled but are also intrinsically capable of being ranked or ordered (*ordinal scale*); and still other variables not only imply a labeling and ranking but also are associated with certain standard units of value that determine exactly by how much the categories of the variable differ (*interval scale*).

Nominal Scale

The nominal level of measurement involves only the process of identifying or labeling the observations that constitute the survey data. In the nominal scale, data can be placed into categories and counted only with regard to frequency of occurrence. No ordering or valuation is implied. For example, a variable such as "political party preference" might be categorized into three possible responses: Republican, Democrat, and Independent. These response categories serve only the function of enumerating the number of survey respondents who indicate their respective affiliations. No ranking or ordering of the parties is specified or implied. Similarly, no valuation unit is available to permit the determination of the extent of each respondent's affiliation.

Ordinal Scale

The ordinal level of measurement goes a step beyond the nominal scale; it seeks to rank categories of the variable in terms of the extent to which they possess the characteristic of the variable. The ordinal level of measurement provides information about the ordering of categories but does not indicate the magnitude of differences among these categories. An example of the ordinal scale can be found in the variable of education—specifically, with regard to highest academic degree received. Potential responses for this variable might include doctoral, master's, and bachelor's degrees or other formal education below the level of a bachelor's degree. It is clear that these categories possess an ordinality or ranking, but they do not by themselves reveal any specific measure of the amount of difference in educational attainment.

Interval Scale

The interval level of measurement yields the greatest amount of information about the variable. It labels, orders, and uses constant units of measurement to indicate the exact value of each category of response. Variables such as income, height, age, distance, and temperature are associated with established determinants of measure that provide precise indications of the value of each category and the differences among them. Whereas ordinal levels of measurement with regard to age, for example, might include categories such as infant, child, adolescent, and adult, interval levels of measurement for age would entail precise indications in terms of established measures, such as years, months, or days.

Variables that are naturally interval but are arranged in categories with a range of values (ages: Under 10, 10–19, 20–29, and so on) are technically ordinal because the categories are not capable of precise measurement without an adjustment (discussed in chapter 5).

Formatting of Questions

Whereas open-ended questions are relatively easy to present within a questionnaire, requiring simply an ample number of lines for the respondent to write an answer in full, closed-ended questions entails a greater range of considerations. The major issues related to the layout of closed-ended questions make up the balance of this chapter.

Basic Response Category Format

In formatting response category alternatives, the primary guideline to which the researcher must adhere is clarity of presentation. The choices must be clearly delineated, so there is no confusion to the respondent or the researcher examining the responses. The researcher must be able to recognize precisely what response choice has been indicated. Of particular importance is that each question be unambiguously associated with one and only one response category, with no overlapping of categories. Generally either a check box or a line is provided next to the responses, and the responses are organized with sufficient space between categories, usually vertical space (but not necessarily if space is an issue).

There may be occasions when the researcher wishes to conserve space in order to keep entire questions and their associated response categories together on one page, for instance, or to save paper and printing costs. Questions that offer a relatively few response alternatives can be organized horizontally as long as adequate space is provided between the possible responses in order that the respondent can easily identify the appropriate place to indicate the response and not inadvertently mark the line on the wrong side of the answer. In example 3.10, the horizontal structure is acceptable and conserves space.

EXAMPLE 3.10

Do you feel that your travel needs are met by the local bus system?

1. ____ Yes 2. ____ No 3. ____ No opinion

Some questions ask the respondent to circle the appropriate response. We do not recommend this method, because circled responses tend to be less easy to read during the data entry process: they can overlap more than one response category if they are not carefully executed.

Number of Alternative Responses

As discussed in chapter 2, it is important to have as comprehensive a list of alternative responses as possible within each closed-ended question. However, the researcher must be careful that the number of fixed alternatives does not become so unwieldy that it confuses or intimidates the respondent. Ideally, in a mail-out or web-based survey, there should be fewer than ten response alternatives for each question. In some circumstances, it may be necessary to increase that number of responses to an approximate maximum of fifteen. If it is suspected (through professional judgment, previous knowledge, or the formal pretest) that there will be a large number of very distinct response alternatives to a question that will be somewhat difficult to combine and that those choices will each be represented by a respectable percentage (say, 3 to 5 percent) of the total responses, then the researcher is justified in expanding the number of alternative response categories to the maximum of fifteen. The balance of choices can be handled through the use of an "Other, please specify" category. When the number of alternative responses in an in-person or intercept survey is large, the interviewer can show the respondent a card with the choices elaborated on it. The maximum number of alternatives in such surveys can even be extended up to twenty. However, a lengthy response list becomes problematic in the telephone survey format, where fifteen to twenty response categories are far too many. The number must be held to a maximum of six for the respondent to be able to remember and choose among them as the interviewer reads them aloud. If reducing the number of response alternatives is not in the best interest of the research, the telephone question can be asked in an open-ended fashion but with the likely closed-ended categories already precoded and input into the computer for direct entry without the need for substantial postcoding.

Structure of Ordinal Categories Containing Interval Scale Variables

Interval scale variables pose special problems for structuring the alternative ordinal response categories. By the nature of their scale, nominal variables and typical ordinal variables have clearly identifiable categories in which there is generally little latitude with regard to assigning cases. For instance, a survey planned for implementation at a local zoo contained a question designed to determine exhibit preferences among zoo visitors. It could be anticipated that such a question would elicit responses such as petting corrals, reptile exhibits, or a tiger pavilion. All responses could be placed in a few possible categories that would be both reasonable and informative. In contrast, a question concerning the age of a respondent has an infinite number of possible ranges and interval sizes into which responses can be categorized. If, for example, the respondent is forty-three years of age, category alternatives for this one answer alone might include "35-44," "40-49," "40-44," "38-50," "over 40," and "under 50." Hence, deciding on the structure of categories for interval scale variables requires a greater degree of judgment and discretion on the part of the researcher.

There are several guidelines and rules of thumb to consider in this decision:

- Ideally, interval scale categories should be as equal as possible in terms of their interval sizes. In the case of age, fixed intervals such as "0–9," "10–19," and "20–29" should be considered an appropriate starting point.
- Each category should contain a reasonable number of responses. A manageable number of categories should be provided. Categories with very few respondents should be avoided, and categories with a very large number of respondents might tend to obscure details that are important to the focus of the study.

- The boundaries of the categories should conform to traditional breaking points wherever possible. It is more desirable to use income categories such as "\$10,000-\$20,000" rather than "\$11,100-\$21,100."
- Each category should consist of responses that are evenly distributed throughout its range of values. This assumption is necessary in order to avoid a skewed distribution of responses and to facilitate statistical analysis. For example, suppose that a researcher is conducting a survey in which respondents must be graduates of a four-year college in order to participate. For the variable of age, the category of "20 and less than 25" should be avoided, because most college graduates are at least twenty-one years old. Hence, the anticipated distribution within the category would be skewed toward the upper ages rather than being evenly distributed. The pretest of the survey instrument is of particular importance in helping to predict whether these preestablished categories will yield a relatively even distribution.
- The categories must be mutually exclusive so that every response has only one possible category.

It may not be possible to satisfy all of these guidelines in any given situation. A potential difficulty in the application of these guidelines occurs when traditional category boundaries conflict with the principle of nonoverlapping categories. In the case of income, for instance, categories with traditional boundaries such as "\$30,000-\$40,000" and "\$40,000-\$50,000" are not acceptable within the same question, because an individual who earns an annual income of \$40,000 applies to more than one category. An acceptable alternative is "\$30,000-\$39,999" and "\$40,000-\$49,999," which assumes that all responses are rounded to the nearest dollar (or "\$30,000-\$39,999.99") and "\$40,000-\$49,999.99" without that assumption). Observations that in theory can assume the value of any number in a continuous interval require class boundaries that are inclusive of all such possible values. The use of the terms *under* and *over* can obviate any problems in the assignment of observations to the appropriate categories in such continuous variables. In fact, it is equally valid to use this format for class boundary determination for all variables except those for which whole number values are the only possible responses (e.g., the number of children in a household). Hence, an appropriate format for these income categories would be "\$30,000 and under \$40,000" and "\$40,000 and under \$50,000," because of its clarity and simplicity and its conformity with traditional class boundaries.

Another conflict in these guidelines might arise with regard to interval sizes. Although it is desirable to maintain equal interval sizes for an income

distribution, this objective may not satisfy the guideline that each category of the variable receive a reasonable number of responses. Typically the frequency of response declines at higher income levels. Therefore, researchers often expand the size of category intervals at the higher income ranges in order to ensure that a reasonable number of responses per category is maintained rather than burdening the report of survey findings with unnecessary detail that is of minor consequence to the study. An element of proportion is also important in category construction. That is, the difference between annual incomes of \$10,000 and \$20,000 is effectively much more significant than the difference between incomes of \$150,000 and \$160,000. Furthermore, there will always be some individuals who earn enormous annual incomes. Intervals cannot reasonably be provided in anticipation of these relatively few responses. Therefore, income questions should provide an unbounded upper-income category to account for this likelihood. Age distributions and certain other socioeconomic variables also demonstrate these patterns of response and should be treated similarly. Example 3.11 shows a reasonable breakdown of income categories.

EXAMPLE 3.11

Please indicate the category that best represents your total annual household income.

- 1. ____ Under \$10,000
- 2. ____ \$10,000 and under \$20,000
- 3. ____ \$20,000 and under \$30,000
- 4. ____ \$30,000 and under \$40,000
- 5. ____ \$40,000 and under \$50,000
- 6. ____ \$50,000 and under \$75,000
- 7. ____ \$75,000 and over

Generally the most important of these guidelines to maintain are mutual exclusivity and even distribution within the categories. Next in importance is a reasonable number of responses, with traditional boundaries and evenly configured categories least necessary to maintain when they conflict with other guidelines.

Order of Response Alternatives

The list of alternative responses may possess an inherent logical order. This order must be replicated in the elaboration of these categories within the

question. Ordinal or interval data are obvious examples, as indicated in example 3.12.

Example 3.12

How would you rate your day at the San Diego Zoo?

- 1. ____ Very good
- 2. ____ Good
- 3. ____ Fair
- 4. ____ Poor
- 5. ____ Very poor

It clearly would not make sense to reorder the responses in examples 3.11 or 3.12. Nominal data categories, however, should be randomly listed so as to deliberately eliminate any potential biasing effects of a particular sequence. Therefore, the order in which these response choices are read to the respondent in telephone, intercept, or in-person interviews should be regularly shuffled. For budgetary reasons and computer coding purposes, this shuffling is frequently not feasible for mail-out and web-based surveys. However, the sequence of response alternatives in mail-out and web-based surveys is less of an issue because the respondent is able to review the choices more easily than in other interview formats.

Multiple Responses

Occasionally a question may require more than one response, as demonstrated in examples 3.13 and 3.14, which represent the two basic types of multiple-response questions. In the first, the respondent is asked to rank preferences; in the second, choices are indicated without regard to their order. In constructing the questionnaire, it should be made very clear to the respondent if more than one response is acceptable or if a ranking is requested.

EXAMPLE 3.13

What kinds of entertainment would you most like to have scheduled at the new Performing Arts Center? Indicate your highest priority with a 1, your second priority with a 2, and your third priority with a 3.

)

- 1. ____ Plays
- 2. ____ Musicals
- 3. ____ Lectures
- 4. ____ Classical music
- 5. ____ Rock music
- 6. ____ Country music
- 7. ____ "Popular" music
- 8. ____ Dance
- 9. ____ Other (please specify:______)

EXAMPLE 3.14

In what ways could police officers improve their performance? (Interviewer: If respondent indicates that no improvement is needed, check the first box.) Check the two most important.

- 1. ____ No improvement needed
- 2. ____ Concentrate on important duties such as serious crime
- 3. _____ Be more prompt, responsive, alert
- 4. _____ Be more courteous and improve their attitude toward community
- 5. ____ Be more qualified in terms of training
- 6. _____ Need more Native American police on the reservations
- 7. ____ Other (specify:_
- 8. ____ Do not know

When the researcher requests only one response in a question but there may be an inclination on the part of the respondent to supply more than one, instructions to "check only one" must be very clear, as in example 3.15.

EXAMPLE 3.15

For which of the following pool activities would you most prefer to have "adults only" time periods designated? (Check only one.) If you do not want designated "adults only" time periods, check the last choice.

- 1. ____ Lap swimming (exercise)
- 2. ____ Water aerobic exercise classes
- 3. ____ General recreational swimming
- 4. ____ Organized competitive swimming

Developing Survey Questions

- 5. ____ Instructional swimming (swimming lessons)
- 6. ____ Do not want "adults only" time periods

Scaled Responses

Some questions require the use of a scaled response mechanism, in which a continuum of response alternatives is provided for the respondent to consider. Example 3.16 demonstrates a Likert scale. A Likert scale entails a five-, seven-, or nine-point rating scale in which the attitude of the respondent is measured on a continuum from one extreme to another with an equal number of positive and negative response possibilities and one middle or neutral category.

The extremes of such scales must be labeled in order to orient respondents. It is also acceptable to label each numerical category on the scale. Generally scaled responses work best horizontally to allow respondents to perceive the continuum. Caution should be exercised to provide adequate spacing between alternatives in the layout of the question. Furthermore, the instructions should not indicate only one direction for the answer. That is, instructions should not say, "Please indicate the degree to which you agree with the following . . ." Instructions must refer to agree and disagree so that the respondent is aware that either answer is acceptable. The correct phrasing is, "Please indicate the degree to which you agree or disagree with the following . . ."

EXAMPLE 3.16

Please indicate your level of agreement or disagreement with the following quotation: "Religion, not patriotism, is the last bastion of a scoundrel."

Strongly Agree				Strongly Disagree
1	2	3	4	5

The Likert scale works particularly well in the context of a series of questions that seek to elicit attitudinal information about one specific subject matter. Example 3.17 is an example of such a series of questions that seeks to elicit the attitudes of employees about their jobs and their degrees of satisfaction. When a series of questions such as the one presented in example 3.17 has the same set of response categories, it would be prohibitively wasteful of space and monotonous to list question after question for several pages. In such circumstances, these questions can be efficiently grouped together in a matrix or gridlike format.

All scaled response series should adhere to certain principles:

- The number of questions in the series should generally consist of two to ten items, depending on the complexity of the subject matter and the anticipated tolerance of the potential respondents.
- The questions chosen for the series should cover as many relevant aspects of the subject matter under consideration as possible.
- The questions should be unidimensional; that is, they should be consistent and concerned substantially with one basic issue.
- The scale must be logical and consistent with a continuum.
- For each question in the series, the scale must measure the dimensions of response in the same order. For example, in example 3.17, the high end of the scale always measures dissatisfaction, and the low end always measures satisfaction.

EXAMPLE 3.17

Please indicate your opinion concerning the following characteristics of your present job.

	(1)	(2)	(3)	(4)	(5)
Characteristics of Present Job	Strongly Agree	Agree	Neutral	Disagree	Disagree Strongly
I feel that the contributions I make are valued.					
My ideas and opinions are taken into consideration at work.					
I have all the information I need to do my job effectively.					
I have opportunities to be innova- tive in carrying out my job.					
Overall my employee perfor- mance evaluation is fair.					

I am encouraged to provide input regarding work-related matters.

My morale at work is generally good.

I feel that I have job security.

The benefits package provided by my employer is good.

I am paid fairly for the work I do.

Although the Likert scale is quite common in survey research, it is only one of several types of scales available to researchers. For instance, when it is suspected that a great number of respondents will choose a middle response and the research requires that respondents choose among alternatives, an even-numbered scale with few choices will work well. Consider the question in example 3.18 that elicits opinions about the United States becoming independent of foreign oil. The responses do not permit the respondent to remain neutral; the question forces a positive or negative response.

EXAMPLE 3.18

How do you feel about the prospect of the United States becoming independent of foreign oil in the next ten years?

- 1. ____ Very positive
- 2. ____ Somewhat positive
- 3. ____ Somewhat negative
- 4. ____ Very negative



CHAPTER FOUR

USING FOCUS GROUPS IN THE SURVEY RESEARCH PROCESS

Chapter 2 introduced the concept of focus groups as an informationgathering technique in sample survey research. Focus groups are invaluable in this regard and are also used significantly in debriefing surveys. That is, it is possible that survey results may surprise researchers and that the reasons for results may not be entirely clear. Focus groups can serve to help inform the results regarding issues that the questionnaire itself did not or could not address, along with their role in gathering information prior to the survey. Furthermore, focus groups are valuable research tools for in-depth qualitative research apart from the sample survey process.

Focus group research is part of a broad category of research called *qualitative research*. Scientific sample survey research is *quantitative*—the distinction being that sample survey results are quantifiable to a known degree of accuracy because the sample is representative, whereas focus groups provide information without that known degree of accuracy because representativeness is not ensured. (Chapter 9 discusses sample selection and representativeness in detail.)

Focus groups generally involve eight to twelve individuals who discuss a particular topic under the direction of a moderator. The moderator promotes interaction and ensures that the discussion remains on topic. These discussions typically are conducted over a period of one to two hours. Focus groups are more formally known as *focused group depth interviews*. This more formal designation indicates several important characteristics of the focus group:

- The term *focused* implies that the discussion is a limited one that deals with a small number of fixed issues in a semistructured format.
- The term *group* indicates that individual participants share an interest in the subject matter of the discussion and that they will interact with one another during the course of the session.
- The term *depth* derives from the nature of the discussion, which is more penetrating and thorough than is possible in the sample survey research process.
- The term *interview* implies that a moderator directs and conducts the discussion and obtains information from the individuals in the group.

Inasmuch as sample survey research is the subject of this book and that qualitative research books exist that cover the topic thoroughly, this chapter's purpose is not to present every aspect of focus group research but to introduce the basics of the process and alert survey researchers to the possibility of using focus groups in conjunction with a sample survey.

Uses of Focus Groups

Focus groups were described in chapter 2 as a useful way of securing information for purposes of informing the development of the questionnaire prior to its implementation. The focus group has many other researchbased uses. The most prominent of these uses are the following:

- Deriving opinions and attitudes about products, services, policies, and institutions in the private and public sectors in order to identify consumer and user perceptions
- Obtaining background information about a subject in order to formulate specific research questions and hypotheses for subsequent use in more quantitatively oriented research techniques (such as sample surveys)
- Testing messages designed to influence or communicate with certain audiences (such as juries, consumers, or voters)
- Identifying creative and innovative ideas related to the subject of interest
- Interpreting and enriching previously obtained sample survey results

The earliest applications of focus group research were in the areas of audience response to radio broadcasts and the effectiveness of army training and morale films in the 1940s. Focus groups have since become significant contributors to research in public program evaluation, public policy development, public and private sector marketing, commercial and political advertising, communications, and litigation. These groups are qualitative in nature and do not represent scientifically drawn samples of the population (see chapters 8 and 9). Consequently, the results of focus group discussions cannot be used to make generalizations about a larger population with a known degree of accuracy, as can be done through an appropriately designed sample survey. Focus groups, however, are important tools. They can aid in gaining a deeper understanding of the subject matter prior to developing the final survey questions. They can also aid in analyzing underlying themes, patterns, and nuances that exist in the population but are difficult to delineate from the survey results.

There are four fundamental components to the focus group research process: planning the focus groups, recruiting the participants, implementing the discussion sessions, and analyzing the results. The first three components of the process are discussed in this chapter; however, inasmuch as this book is devoted primarily to sample survey research, the component involving focus group analysis is deferred to chapter 13, where it is presented in the context of how to interrelate the qualitative findings from focus groups with the quantitative findings from sample surveys.

Planning Focus Groups

The focus group planning process consists of several activities that must occur before recruitment of participants can take place. Foremost among these planning activities are the following:

- Identifying the critical characteristics of potential focus group participants
- Establishing the appropriate number of focus groups
- Choosing the most appropriate facility for conducting focus groups
- Determining the necessity for financial inducements to encourage participation
- Scheduling the focus groups at the optimal times of the day and days of the week

Identifying the Critical Characteristics of Participants

Fundamental to the focus group process is the establishment of the critical characteristics of the potential participants. As a general rule, focus groups are more effective when they consist of participants who share many of the same key characteristics. Homogeneous groups tend to exchange ideas and opinions more freely than do groups with widely divergent backgrounds. Participants in homogeneous groups have been found to relate to one another well, and they tend to generate a higher quality of conversation.

Therefore, the critical characteristics of the focus group participants must be identified early in the research process. For example, a local newspaper was interested in opinions from subscribers about possible changes in the types of features being considered for publication in various circulation areas. These key characteristics were identified:

- Subscribers to the newspaper
- Residents of particular circulation areas
- Age of subscribers

Therefore, six focus groups were scheduled, with two focus groups in each of the three circulation areas. The two focus groups in each area were to consist of newspaper subscribers—one consisting of subscribers between the ages of eighteen and thirty-five and the other consisting of subscribers over thirty-five years of age.

It is important to note that each critical characteristic can increase the number of focus groups significantly. For instance, if gender had been regarded as critical to the newspaper study, four focus groups would have been necessary in each of the circulation areas if absolute homogeneity were to be maintained. Frequently, however, budget and time constraints prohibit the luxury of maintaining strict homogeneity within each group. In a focus group study of transit use in a large urban county, for instance, the following characteristics were considered important to the composition of focus groups:

- Geographical location
- Ethnicity
- Transit users versus nonusers
- Age

In such cases, it is recommended that a maximum of two to three characteristics of the population be regarded as critical, with the other important characteristics being assigned a secondary status. In this example, three geographical areas of the county and four primary ethnic groups were identified for analysis, yielding twelve focus groups. These two characteristics (area and ethnicity) were selected because transportation research is highly location specific and because language and cultural issues are frequently paramount in effective intragroup communication. The secondary characteristics, transit use and age, were mixed into the twelve focus groups in sufficient number (at least two to three representatives of each secondary characteristic per focus group session) to evaluate those characteristics without establishing separate focus groups for them.

Establishing the Number of Focus Groups

The nature of the research, including the critical characteristics of the population, in large part dictates the appropriate number of focus groups to be conducted.

In general, the number of focus groups planned should be a minimum of two, with an upper limit in the range of ten to fourteen. At least two groups are necessary because the researcher must be certain that he or she is not simply observing a unique set of circumstances that may exist among only the participants of any one group. The upper limit of ten to fourteen groups is more flexible because of homogeneity considerations and because of the varying degree of detail that the researchers require. As a general rule, however, information becomes very repetitive and new information is rare beyond ten to fourteen focus groups.

In this planning stage, it is advisable to determine a target number of focus groups at the outset, based on key population characteristics and research requirements. However, the researcher must retain sufficient flexibility to add more groups if the originally planned groups fail to produce meaningful results.

Choosing an Appropriate Facility

Focus group discussions should be conducted in an easily accessible, convenient location. Participants are much more likely to attend when the location is near their homes or work, is easily accessible, and has convenient parking that is either free or paid for by the focus group contractor. Focus group facilities typically include conference rooms in hotels, community centers, restaurant areas, and specially designed rooms with one-way glass for viewing by the research team from a private viewing room. Each type of location has advantages and disadvantages, and the researcher must assess these in terms of the characteristics of the focus group participants:

- *Hotel conference rooms:* Conference rooms are generally quite effective as focus group locations. They are particularly well suited to mainstream, middle-income populations. The rooms generally are comfortable, climate controlled, quiet, and equipped by the hotel to suit the needs of the group, often including audiovisual capability, the necessary tables and chairs, and refreshments. On the negative side, hotel accommodations can prove to be expensive and can be uncomfortable for groups such as disabled residents and certain low-income populations who have been found to feel somewhat out of place at such locations.
- *Community centers:* Community centers include senior centers, neighborhood recreation centers, libraries, and other specialized community facilities frequently geared toward a lower-income or otherwise disadvantaged population group, such as physically disabled persons and the elderly. These groups tend to feel more comfortable close to home and in a familiar environment. Often the center does not charge the focus group researcher for use of the facility or charges only a nominal fee. On the negative side, these facilities are noisier and less adequate in terms of equipment and tables and chairs. Refreshments must be purchased outside the facility and brought to it.
- *Restaurant meeting areas:* Restaurants are particularly effective for focus group sessions among lower-income groups at mealtimes. There are a number of ethnic groups, in particular, for whom mealtimes are important socially and are therefore too significant to surrender in order to attend a focus group discussion. In our own experience, the Latino and Vietnamese populations are especially amenable to this type of location, not only because of the social importance of mealtimes but also because of the feeling that their community and its local businesses are being supported when a local restaurant is used for the session. The restaurant location can frequently prove to be very economical for the researcher. On the negative side, even with private banquet facilities, restaurants can be noisy and distracting.
- Specially designed focus group rooms: Market research firms are interested in hearing comments from the public that are not influenced by the sponsor of the focus group. Therefore, these firms frequently use a specially designed room with a one-way glass behind which the researchers and their clients can listen to and view the discussion without being seen by

the participants. Sometimes the participants are told that they are being viewed behind the glass, and sometimes they are not. Disadvantages of this type of facility are as follows: they are limited in number and hence not particularly convenient to a broad base of potential participants; the secretive nature of the one-way glass may violate certain standards concerning privacy and scientific research ethics; and when the one-way glass is disclosed to the participants, it loses a considerable amount of its original value to the researcher. Research sponsors nevertheless are particularly in favor of these facilities because they afford privacy and comfort, as well as allowing sponsors to relax, eat, and discuss issues among themselves.

Determining the Necessity for Financial Inducements

It is frequently necessary to provide a monetary incentive to potential focus group participants to secure their agreement to participate and encourage their ultimate attendance. When members of the general public are invited to discuss issues, such as transportation, that are important public affairs, we have found that an honorarium of at least \$100 plus the availability of light refreshments (such as coffee, soda, cookies, cheese and crackers) is required in order to obtain a satisfactory rate of attendance at a seventyfive- to ninety-minute session. In certain cases, it is necessary to provide meals to focus group participants. When full meals are provided, honoraria should not be reduced. In order to secure adequate attendance at private sector focus groups that test products and marketing messages, it is frequently necessary to offer honoraria of \$150 to \$175.

When focus groups consist of community leaders, government officials, corporate executives, or other highly positioned persons, honoraria are not only unnecessary but can be regarded as inappropriate. Meals are acceptable for these leaders when the focus group sessions occur at mealtimes.

Scheduling the Focus Groups

The researcher should always try to conduct more than one focus group in a day in order to achieve certain economies of scale, including the use of one facility at a single rental charge and the use of personnel with a minimum of travel and downtime. Focus groups should not be conducted on weekends (Friday, Saturday, or Sunday) or on holidays, when it is difficult to secure attendance. Therefore, groups should be scheduled on Monday, Tuesday, Wednesday, or Thursday, with Monday the least preferable among these. In order to provide the opportunity for the working population to
attend, at least one and frequently more of these sessions should be conducted outside normal working hours.

When it is expected that the focus group will consist largely of mainstream working individuals, it is advisable to hold one meeting toward the end of the workday, from 5:00 P.M. to approximately 6:30 P.M. This meeting can be attended by individuals after work, who can then proceed home for dinner. A second focus group meeting, held from 7:00 P.M. to approximately 8:30 P.M., allows other participants to go home after work, eat dinner, and then attend the discussion. For groups for whom the availability of dinner at a local restaurant, for example, is an integral part of the event, the dinner focus group should be conducted from 6:00 P.M. to approximately 8:00 P.M., with the first thirty to forty-five minutes devoted to the meal. Depending on the configuration of the room being used, discussion can begin as the participants are finishing their meals. The other meeting on that day, if scheduled at all, should be conducted in the mid- to late afternoon, the precise time depending on the characteristics of the population.

Certain groups may not find any of these times to be suitable—for instance, train commuters who must adhere to a strict schedule in order to return home at the end of the day. The researcher therefore will encounter situations that may require adapting these scheduling guidelines to the needs of the specific population groups, but for the most part, the 5:00 P.M. to 8:30 P.M. time frame satisfies most focus group research needs.

Recruiting Focus Group Participants

As in the case of sampling (see chapter 9), the researcher must use a list of potential participants from which the actual participants are selected. The characteristics of the desired focus group participants dictate how the researcher should proceed to obtain this list.

There is no overriding rule for obtaining or generating such lists, but it is important to note that in contrast to scientific sample research, the list need not be exhaustive; it must simply include members with the desired characteristics. That is, the selection of focus group participants need not conform to the formal principles of survey research. Rather, participants are selected at the convenience of the researcher so long as they possess the required characteristics.

When the focus group is to be composed of members of the general population without regard to specific population characteristics, generic sources, such as the telephone directory, various commercial directories, and computerized address and telephone directories, are usually suitable. If the requirements concerning characteristics of focus group participants are more circumscribed, then the researcher must be more creative in assembling such lists.

For example, a researcher who is conducting focus groups composed of representatives of the freight movement industry in a particular region might follow these steps in formulating the list:

- Identify the major modes of freight transportation: air, rail, truck, and ship.
- Contact various trade organizations, such as truck associations, if such trade organizations exist.
- Consult the Yellow Pages of the telephone directory for freight companies in all modes.
- Contact governmental jurisdictions, such as the Interstate Commerce Commission, for information regarding the existence of lists of freight companies.
- Ask each party contacted to identify additional potential invitees.

Sometimes the researcher's client possesses lists of customers, clients, and interested parties. These lists can prove to be quite valuable for recruiting purposes.

Once the list is developed, the researcher should begin telephoning a representative cross-section of the people or organizations on the list. The purpose of this call is to invite the potential participants to the planned focus group meetings. The potential participants are informed of the purpose and sponsor of the focus group session; the date, time, and place that it will be held; and the financial incentive or meal and refreshments to be provided.

If the person accepts the invitation, the researcher should e-mail a confirmation to the invited guest, detailing the specifications of the meeting (see exhibit 4.1). The invited guest is also to be reminded of the meeting once again one or two days prior to the focus group session by a follow-up telephone call or e-mail, or both.

The researcher should overrecruit in order to account for the fact that some guests will not show up for the appointed session. Our experience is that approximately 20 to 35 percent of confirmed guests will not attend focus group sessions. Hence, in order to satisfy the objective that eight to twelve persons participate in a discussion, for example, it is best to confirm approximately fifteen guests.

EXHIBIT 4.1. SAMPLE FOCUS GROUP CONFIRMATION E-MAIL.

October 23, 201x

Ms. Josephine Castañeda Director of Human Resources County of Santa Rita 425 Leopold Drive Saint Jerome, CA 99999

Dear Ms. Castañeda:

Thank you very much for agreeing to participate in a focus group discussion involving flexible work options for public employees in the Santa Rita County Region. The Santa Rita Council of Governments is inviting human resource professionals to discuss such options including telework policies and programs.

We look forward to your input regarding this critical issue. The focus group will be held as follows:

Date	December 1, 201x
Location	McGinnis Hotel
	450 Main Street
	Saint Jerome, CA 99999
Time	5:00-6:30 р.м.

Dinner will be served to all participants at the beginning of the session. I can be contacted at the e-mail address and telephone number listed below.

Sincerely, Gregory Kennedy, Senior Analyst ABC Research gkennedy@abc.com 555-111-1111

Implementing the Focus Group Sessions

The implementation of focus group sessions can be divided into two components: preparation for the session and the actual session.

Preparing for the Focus Group

The focus group room should be equipped with a table, preferably linen covered and rectangular. The group moderator should be positioned at

the head of a table or otherwise positioned so as to be visible by all the participants. The moderator should have a nameplate placed in front of him or her for reference by the group. Water and cups or glasses should be available to everyone at the table, and coffee, tea, and other refreshments should be available at stations against the wall or in an immediately adjacent room. Videotape equipment should be positioned as unobtrusively as possible while still providing full coverage of the session. If, in the judgment of the researchers, videotaping will interfere with the discussion in any way, it should not be used. No method of recording should be used if any participant objects to being recorded. Alternatively, the objecting participant can be excused, and then the focus group can be recorded.

Chairs should be placed against the wall, out of the primary sight line of the participants to the maximum extent possible. These chairs are to be occupied by members of the research team, who will observe and record notes on the session. The researchers' client may also wish to attend and can be seated with the researchers. If a professional focus group facility is used, researchers and other observers will sit in a separate room behind the one-way glass.

A greeting table is set up at the entrance to the room in order to compile a sign-in list and distribute nameplates to the participants to place in front of them at the focus group table. These nameplates allow the moderator and other participants to address one another by name during the session.

Conducting the Focus Group

As soon as the characteristics of the potential focus group participants are determined by the group sponsor, the preparation of a formal, semistructured focus group discussion guide can begin. The questions should flow in a logical sequence and enable the participants to engage in the discussion in a friendly, nonthreatening manner. The development of a focus group guide entails considerable research regarding the relevant subject matter and content of the information to be obtained from the focus group participants, as well as the specific methods to be used to elicit such information. The authors recommend that the discussion guide include techniques to sustain the interest of the participants, including audiovisual presentations, informational handouts, and written exercises.

Exhibit 4.2 presents an example of a moderator's discussion guide that demonstrates the guidelines and key features of such structured formats.

EXHIBIT 4.2. STRUCTURED FORMAT FOR A FOCUS GROUP DISCUSSION.

Good evening, everyone. My name is Ricardo Parkington. I'm with the market research firm of Spiro and Cheney. We are working with the Santa Rita Council of Governments (SRCOG) to conduct this discussion this evening because SRCOG wants to discuss with you certain business efficiencies and practices, especially in the area of transportation. In this discussion, SRCOG is particularly interested in your views about flexible work options for your employees. There are no right or wrong responses to the guestions in our discussion tonight, and it is perfectly acceptable and guite desirable to voice your honest opinions especially in areas where you have strong views or where you disagree with others on certain issues.

We are very appreciative that you are here, and SRCOG appreciates it very much as well. What we value the most is that you are taking the time to tell us your opinions and thoughts.

We are videotaping tonight's meeting so that we can go back and review exactly what you had to say. The video will remain in our possession for that purpose only and will not be shared with anyone else in any form outside of a summary written report. Further, information that is reported will not have your name associated with it. Your identity will remain confidential.

Background

- 1. Introduction around the table—All. Each participant will be asked to provide the following information:
 - Introduce himself/herself and indicate the company/agency he or she • represents.
 - Indicate the location of the company/agency including branches or smaller offices.
 - Indicate the approximate length of time the business has been in operation in the greater Santa Rita region.
 - What are your job responsibilities in the company/agency? How long have ٠ you been with the company/agency?
 - Indicate the approximate number of employees in your company/agency (or department if appropriate).

Discussion

1. Commuting Business Practices

Do you have any commute benefit programs for employees of your com-• pany/organization? Note: Probe for free bus and trolley passes, parking spots for vanpoolers, flexible work hours, etc.

(continued)

EXHIBIT 4.2. STRUCTURED FORMAT FOR A FOCUS GROUP DISCUSSION. (continued)

• Explain why your company implemented them and how they address issues discussed. (Moderator: These programs and options may include telework type programs, and such programs should be explored fully here.)

2. Impressions of Work Environments

Participants will be shown two PowerPoint slides. The slides will comprise images that represent various work environments. These environments are as follows:

Slide 1—Alternative home office environments Slide 2—More traditional office environments

- Participants will be asked to comment on these slides within the following framework:
 - Comment on the environments depicted on the slides (*Probe: organized, disorganized, frenetic, calm, etc.*). What is it specifically about the slide that prompts you to make these comments?
 - Comment on the suitability of the environment(s) for producing work in your business. Describe the environment that is most suitable? Describe the least suitable?
- **Employee Requests:** Have there been requests in your organization from employees who wish to have telework options?
 - If so, what have been the reasons for their request? *Probe: economic issues* such as gas prices; equality—other employees in their organization or in similar organizations are able to telecommute.
 - In general, do you know your employees' attitude toward telecommuting? How would you characterize those attitudes?
- Overall management impressions of telework:
 - How would you say middle managers in your company feel about telework options? (*Probe: embrace it, avoid it, disapprove of it.*)
 - How would you characterize the impressions toward telework from the highest decision-making levels of your company?
- **Explore options:** Have you or anyone in your organization begun to consider or explore telework options, especially with an eye toward the development of a formal program?
 - If so, how far did you get?
 - What prompted these considerations?

Moderator: Pursue operational challenges of implementing a telework program (e.g., Do supervisors/middle managers lack knowledge in how to manage teleworkers; Do they recognize who are candidates of teleworking and who are not)?

3. Benefits of Telework and Challenges to Implementation

- Benefits of telework
- Let's discuss the potential benefits of telecommuting as you see them for your organization. Some potential or possible advantages of telework will be presented to you for discussion one at a time on the screen for all to see. Please consider each statement as a potential advantage of telework for your business. Please tell us in your opinion to what extent you feel these statements are advantages, if indeed you feel they are advantages at all.
- Challenges and drawbacks to telecommuting
- Let's discuss the potential challenges or drawbacks associated with teleworking from a management standpoint. Some potential or possible disadvantages of telework will be presented to you for discussion one at a time on the screen for all to see. Please consider each statement as a potential disadvantage of telework for your business. Please tell us in your opinion to what extent you feel these statements are disadvantages, if indeed you feel they are disadvantages at all.

4. Exercise—Telework Benefits and Challenges

- I am going to hand out a list of potential benefits of telework programs. Please rank each potential benefit as it relates to your organization from 1 to 7, where 1 = most beneficial and 7 = not at all beneficial.
- Now, I am going to hand out a list of potential drawbacks/challenges to telework programs. Please rank each challenge as it relates to your organization from 1 to 7, where 1 = not a challenge and 7 = the greatest challenge.

5. Wrap-Up

- Is there anything else you would like to say about telework programs or the telework issue in general? Is there any experience, statement, or insight you would like to underscore?
- This concludes our discussion this evening. Your comments and input have been very helpful and will be very useful in informing decisions about the future of commuting needs in this region.

The exhibit is an abridged guide that elicits views and perceptions about flexible work options (including telework opportunities) for employees in the public sector in a major metropolitan area. Focus group participants include human resource personnel as well as other officials representing cities and other public entities in the region. Note that the focus group session begins with an introduction by the moderator, including reference to the use of recording equipment, if applicable. This introduction should have four segments:

- A welcoming statement
- A brief overview of the subject matter to be covered
- An explanation of the discussion rules
- Assurance that the individual comments of the participants will not be revealed

The moderator then asks each participant to introduce himself or herself and indicate particular personal attributes that are important to the discussion at hand (such as place of work, job responsibilities, number of employees supervised). Following these introductions, the moderator can proceed with the first question.

The first question should be designed to engage all of the focus group members. It should be a relatively simple yet substantive question that can be answered quickly and will quell any participants' uneasiness about speaking in front of a group of unknown people. In exhibit 4.2, the initial question is, "Do you have any commute benefit programs for employees of your company/organization?"

The moderator can then move ahead with the balance of the questions that are designed to elicit the requisite information. With each question, responses from the group are invited and discussed among the group members until the moderator determines that it is time to go forward. Following each question, the moderator should summarize the views expressed and obtain some consensus that the summary is valid.

The moderator may determine that certain answers require further elaboration and probe for this elaboration by asking follow-up questions. Some follow-up questions can be anticipated in advance and scripted for the moderator as part of the questionnaire; others require impromptu adjustments by the moderator. Exhibit 4.2 shows the suggested moderator probes in italics.

Throughout the discussion, PowerPoint slides are used to show pictures of various office environments in order to elicit comments about such office scenarios. Slides are also used to highlight certain potential advantages and disadvantages of telework programs—again for purposes of participant comment.

Near the end of the discussion, participants are handed a list of potential challenges of telework programs and a separate list of potential benefits. They are asked to rank the challenges as they relate to their organization from 1 to 7, where 1 = not a challenge and 7 = the greatest challenge. Similarly, participants ranked the benefits from 1 to 7, where 1 = most beneficial and 7 = not at all beneficial.

At the conclusion of the focus group session, seventy-five minutes to ninety minutes after it began, the moderator can briefly summarize the overall discussion, ask if there are any further comments on the subject, thank the group for their time and valuable input, and provide the participants with the cash incentive, if promised.



PART TWO

ENSURING SCIENTIFIC ACCURACY



CHAPTER FIVE

DESCRIPTIVE STATISTICS

Measures of Central Tendency and Dispersion

The balance of the book concerns the statistical basis for sampling and analyzing and presenting sample survey data. This chapter introduces the statistical concepts that are critical to the scientific sample survey process.

Measures of Central Tendency

It is often useful to include *measures of central tendency* in the frequency distribution table to augment the description of the data presented. Measures of central tendency are statistics that provide a summarizing number that characterizes what is "typical" or "average" for those data. The three foremost measures of central tendency are the *mode*, the *median*, and the *arithmetic mean*.

Mode

The mode is that category of the variable that occurs most frequently. It is the only measure of central tendency determinable from nominal scale variables, but it can also be used to describe the most common category of any ordinal or interval scale variable.

Party	Frequency (f)	Percent (%)
Conservative	200	50.0
Progressive	150	37.5
Moderate	_50	<u>12.5</u>
	400	100.0

TABLE 5.1. POLITICAL LEANING.

In the nominal scale data presented in table 5.1, there are 200 Conservatives, 150 Progressives, and 50 Moderates. The modal response for these data is Conservative. The mode is not the frequency of the most common variable category (200); rather, it is the category itself (Conservative). The mode therefore conveys to readers the category that is most typical of the population surveyed.

In retail sales, for instance, the mode takes on a particularly useful role, focusing the retailer's attention on the most popular products for purposes of identifying high-volume sizes, colors, or other characteristics. In transportation planning, traffic congestion is often described in terms of two daily peak (modal) periods: the morning and evening rush hours.

Median

The median is the category of the variable that represents the center, or midpoint, of the data. Half of the data will have values less than the median's value, and the other half will have values greater than the median. To determine the median, the data must be capable of being arranged in order, from low value to high, or vice versa. As such, the median is determinable only for variables that are on an interval or ordinal scale. The nominal level of measurement does not lend itself to the computation of the median because it does not provide the qualities of order or rank.

For example, in a class of eleven students in a graduate seminar, the final exam scores are as follows:

Student	Exam Score
А	70
В	79
С	95

Student	Exam Score
D	88
E	53
F	80
G	98
Н	93
I	76
J	85
К	82

The median for this exam score data can be identified in three steps:

- 1. Rank the data from low value to high value or vice versa.
- 2. Determine the median case location at $(n + 1) \times 0.5$.
- 3. Identify the category of the data that corresponds to the median case location.

The process therefore is as follows:

- 1. Rank the scores: 53, 70, 76, 79, 80, 82, 85, 88, 93, 95, 98.
- 2. Identify the median case location. The location of the median case is found by adding 1 to the sample size (*n*) and multiplying that sum by 0.5. Hence, in this example, the median location is $(11 + 1) \times 0.5 = 6$, or the sixth case in the ranked distribution of scores.
- 3. Identify the median category. With a median location equal to 6, the median itself is found by identifying the corresponding test score value in the ranked distribution of scores. That score is 82.

Score	53,	70,	76,	79,	80,	82,	85,	88,	93,	95,	98
Rank	1	2	3	4	5	6	7	8	9	10	11

Now suppose that a twelfth student missed the exam, took a makeup test, and received a score of 50. The median score would be that score that corresponds to a location of $6.5 = (12 + 1) \times (0.5)$. The corresponding exam score value occurs midway between 82 and 80, at a score of 81.

Table 5.2 demonstrates data that have been obtained by sample survey in the form of ordinal scale categories.

Rating	Frequency (f)	Percent (%)
Very good	100	21.1
Good	175	36.8
Fair	100	21.1
Poor	50	10.5
Very poor	<u>50</u>	<u>10.5</u>
	475	100.0

TABLE 5.2. RATING OF LOCAL LIBRARY FACILITIES.

The determination of the median for these grouped ordinal data entails the same three steps discussed above:

- 1. Ensure that the categories are ranked from high value to low value, or vice versa. (Note that the ranking in table 5.2 is on a continuum from the highest value, "very good," to the lowest, "very poor.")
- 2. Determine the median location. This is found at (475 + 1)(0.5) = 238.
- 3. Identify the category of the data that corresponds to the median location. In grouped data as in table 5.2, the identification of the median category requires the application of the concept of cumulative frequency. *Cumulative frequencies* are summations of frequencies from the frequency distribution. Each stated cumulative frequency represents the number of cases in a particular category plus those cases already accumulated. In the use of table 5.2, the cumulative frequencies are:

Rating	Frequency	Cumulative Frequency
Very good	100	100
Good	175	275
Fair	100	375
Poor	50	425
Very poor	_50	475
	475	

The median location (case 238) can be found in the "good" category, in which cases ranked 101 to 275 occur (cases 1 to 100 being in the "very good" category). The median value for these data therefore is the rating category "good."

Number of Children	f	%
None	250	41.7
1	75	12.5
2	125	20.9
3	75	12.5
4	50	8.3
5	20	3.3
6	5	0.8
	600	100.0

TABLE 5.3. NUMBER OF CHILDREN PER HOUSEHOLD IN MT. PILOT, SOUTH DAKOTA.

This same procedure can be applied to frequency distributions of data measured on the interval scale. Table 5.3 depicts such data derived from a residential sample survey.

The reader can readily identify a median location of 300.5 ([600 + 1] × 0.5) and median number of children equal to 1. (The category contains cases ranked 251 to 325.) In table 5.3, the data were measured on the interval scale with single values of the variable: number of children. Data derived from sample surveys, however, are generally grouped into ordinal categories containing ranges of interval data rather than data identified by single-value categories.

Consider the results of a sample survey of hourly consultant fees paid by cities in Pennsylvania during 2013. These results are presented in table 5.4.

When ordinal categories of a variable comprise a range of interval scale values (table 5.4), in contrast to a single value of the variable (table 5.3), the calculation of the median entails four steps rather than three:

- 1. Rank the data from low value to high value only.
- 2. Determine the median location, in the same manner specified previously: $(n + 1) \times 0.5$.
- 3. Identify the category that corresponds to the median location, in the same manner specified previously.
- 4. Estimate the value of the median within the range of values in the median category.

The procedure for estimating the median is shown in worksheet 5.1 and as equation 5.8 in endnote 1 of the chapter.

·		
Hourly Fees	f	%
\$50 and under \$75	30	7.5
\$75 and under \$100	80	20.0
\$100 and under \$150	140	35.0
\$150 and under \$200	100	25.0
\$200 to \$300	_50	12.5
Total	400	100.0

TABLE 5.4. CONSULTANTS' HOURLY FEES PAID BY CITIES IN
PENNSYLVANIA, 2013.

WORKSHEET 5.1. DETERMINATION OF MEDIAN.

- 1. Verify that the categories are arranged from low to high value.
- 2. Determine the median location = case 200.5.

Fee	f	cf
\$50 and under \$75	30	30
\$75 and under \$100	80	110
\$100 and under \$150	140	250
\$150 and under \$200	100	350
\$200 to \$300	_50	400
Total	400	

- 3. Identify the category that corresponds to the median location. Case 200.5 is in the "\$100 and under \$150" category, in which cases 111 to 250 occur.
- 4. Because these data are on an interval scale, it is possible to refine the estimate of the median further to a specific test score. To achieve this, it must be assumed that the 140 cases within the "\$100 and under \$150" category are evenly distributed throughout the category. Since there are 110 cases below the median category, 90.5 additional cases (200.5 110) must be included from the median category in order to reach case 200.5. This means that 64.64 percent (90.5/140) of the median category must be included. Under the assumption of even distribution (chapter 3), the median equals the lower limit of the median category (\$100) plus the appropriate percentage (64.64 percent) of the category width of \$50 (which represents the difference between the upper and lower limits of the category: \$150 \$100 = \$50). Thus, the median can be estimated to be equal to \$132.32, as shown below.

Lower limit of median category	= \$100.00
Width of median category	= \$50.00
Percent of median category required to attain median location	= 64.64%
\$100 + (\$50 (.6464)) = \$132.32	

The determination of category width in worksheet 5.1 depends on whether the variable being presented is discrete or continuous. The width of a continuous variable category is found by subtracting the lower limit of the category from the upper limit, as was the case in worksheet 5.1 (\$150 - \$100 = \$50). The width of a discrete variable category is found by counting all of the discrete units within the variable category, including the two end points.

In statistical analysis, variables with an infinite number of values between the end points of the categories are considered to be continuous. Examples of continuous variables are such measures as distance, time, and weight. Discrete variables, by contrast, have a limited number of possible values for the variable. Examples of discrete variables might be the number of retail establishments per city, the number of children per household, or the number of pages in a book. For discrete variables, a category such as "25 to 30 retail establishments" has a category width of 6, with the values of 25, 26, 27, 28, 29, and 30 included in the category.

An exception to these descriptions of continuous and discrete variables is money. This variable is traditionally treated as continuous despite the fact that all currencies possess minimum measures for counting purposes. Such minimum measures in essence prevent the existence of infinite values between the end points of a category. For example, in US currency, the penny effectively serves as this minimum measure.

Arithmetic Mean

The measure of central tendency that the general public most commonly uses is the arithmetic mean. It is the mean (\bar{x}) that most people refer to as the *average*. The mean is the mathematical center of the data. It takes into account not only the location of the data above or below the center (as the median does) but also the relative distance of the data from that center. The mean is, in essence, a point of equilibrium at which the sum of all distances from data points above the mean to the mean exactly equals the sum of all distances from data points below the mean to the mean. The mean requires that data be measured on the interval scale because the data are not only to be ranked but also to be measured.

In its simplest form, the mean can be demonstrated by using the same eleven test scores from the median example. It is calculated by summing all scores and dividing the total by the number of scores involved.

$$\overline{x} = \frac{\sum x}{n} \tag{5.1}$$

where

 Σ = summation of all observations x = value of each observation Scores = 98, 95, 93, 88, 85, 82, 80, 79, 76, 70, 53

$$\bar{x} = \frac{899}{11} = 81.73$$

Generally, however, sample survey research data are more likely to be encountered in the form presented in table 5.3 than listed individually. That is, in survey research, large numbers of observations are processed and organized into categories through the use of frequency distributions. When frequency distributions of interval scale data take the form of singlevalue categories, as in table 5.3, the mean is calculated as follows:

$$\overline{x} = \frac{\sum fx}{n} \tag{5.2}$$

where

 Σ = summation f = frequency of each category x = single value of the variable category

For the data in table 5.3, worksheet 5.2 demonstrates the calculation of the mean.

In survey research, interval data are more often collected into ordinal categories comprising a range of values rather than a single value. As was discussed in chapter 3, these types of categories are frequently used in closed-ended survey questions, and although they are literally ordinal in measure, they can be converted easily to interval through the application of category midpoints. Table 5.5 depicts such categories in a frequency distribution of travel time to work for a sample survey administered in Jersey City, New Jersey.

The calculation of the midpoint assumes that the data are distributed evenly throughout the various categories. For purposes of calculating the mean for interval data presented in a range of values, each category must be characterized by a specific number, which instead of *x* will be the midpoint of the category range. Each midpoint is multiplied by the frequency for the category. These products are summed, and the result is divided by the total frequency to obtain the mean:

106

$$\overline{x} = \frac{\sum fm}{n} \tag{5.3}$$

where

f = frequency of each category

m = midpoint of each category

Worksheet 5.3 can be adapted from table 5.5.

Number of Children	f	fx ^a
0	250	0
1	75	75
2	125	250
3	75	225
4	50	200
5	20	100
6	_5	_30
Total $\overline{x} = \frac{\Sigma f x}{n} = \frac{880}{600} = 1.47$ children	600	$\Sigma = 880$

WORKSHEET 5.2. CALCULATION OF MEAN NUMBER OF CHILDREN, MT. PILOT, SOUTH DAKOTA.

 $^{a}x =$ number of children indicated for each category.

Travel Time (in minutes)	f	%
Less than 5	40	8.0
5 and under 10	54	10.8
10 and under 15	90	18.0
15 and under 20	102	20.4
20 and under 30	86	17.2
30 and under 40	70	14.0
40 and under 60	33	6.6
60 to 90	_25	5.0
Total	500	100.0

TABLE 5.5. TRAVEL TIME TO WORK IN JERSEY CITY, NEW JERSEY.

107

Travel Time (minutes)	mª	f	fm
Less than 5	2.5	40	100
5 and under 10	7.5	54	405
10 and under 15	12.5	90	1,125
15 and under 20	17.5	102	1,785
20 and under 30	25.0	86	2,150
30 and under 40	35.0	70	2,450
40 and under 60	50.0	33	1,650
60 to 90	75.0	_25	<u>1,875</u>
		500	$\Sigma = 11,540$
$\bar{x} = \frac{\Sigma fm}{n} = \frac{11,540}{500} = 23.0$			

WORKSHEET 5.3. CALCULATION OF MEAN TRAVEL TIME TO WORK IN JERSEY CITY, NEW JERSEY.

^aThe midpoint for a category of interval scale data, consisting of a range of values, can be calculated by adding the lowest possible value of a category to the highest possible value for the same category and dividing that sum by 2. In the case of the category "5 and under 10," for instance,

$$m = \frac{5+10}{2} = 7.5$$

This method for determining the midpoint applies in precisely the same manner to all categories on the interval scale (including both continuous and discrete categories). Hence the midpoint for "60 to 90" is calculated in precisely the same manner as "10 and under 15."

Measures of Dispersion

Measures of central tendency yield only partial information about a variable. They summarize data by identifying an appropriate average or center for those data; however, by itself, this average can lead to an incomplete, and at times misleading or confusing, description of the data.

To demonstrate the need to supplement measures of central tendency with additional statistical measures, consider the following example. Two similar communities are engaged in studies to determine the nonemergency response times of their police departments. Table 5.6 shows the distribution of response times for Harrison and Jackson. Each community exhibits similar central tendencies with regard to response time (note that each arithmetic mean $\bar{x} = 9.3$ minutes). However, these communities also demonstrate significant differences. Specifically, Harrison demonstrates a more consistent pattern of response time and Jackson shows a wider spread of response times. Consequently, the mean response time for Harrison is more descriptive of its data than is the mean response time for Jackson. Jackson demonstrates more variability in response time or greater dispersion in responsiveness. As such, the similar means by themselves could be misleading if someone were to incorrectly conclude that these two communities were the same regarding their response times.

To demonstrate this critical concept of dispersion further, consider that San Francisco and Washington, DC, experience approximately the same mean average daily temperature in a typical year. It should be clear, however, that San Francisco's consistent, temperate climate is significantly different from that of Washington, DC, with its very warm summers and colder winters. Once again, the measure of central tendency by itself is not sufficient without additional information concerning the extent of the variability around it.

These examples demonstrate the need for an additional statistic capable of measuring the degree of dispersion associated with the central tendency. Measures of central tendency and measures of dispersion constitute the fundamental elements of what is known as descriptive statistics because they describe and summarize vast amounts of data by the use of single statistical values.

Range

The most elementary statistical measure of dispersion is the range: the difference between the highest and lowest values in the data under study. The range is calculated by subtracting the lowest value from the highest. In the police department example, Harrison has a response time range of 12 minutes (13–1), and Jackson has a range of 24 minutes (25–1).

Two points should be noted from this determination of ranges. First, there is now a measure of dispersion that shows the greater consistency of times in Harrison than in Jackson. Second, the example reflects a disadvantage of the range, in that it can be affected by extreme values. In Jackson, only 11 percent of all response times exceed the maximum response time in Harrison, yet the differential in ranges between 12 minutes (Harrison) and 24 minutes (Jackson) can convey a much greater dissimilarity between

	Hai	rrison	Jack	son
Response Time (in minutes)	f	%	f	%
1 and under 3	8	3.0	9	4.0
3 and under 5	10	5.0	20	9.0
5 and under 7	15	7.0	24	10.0
7 and under 9	32	14.0	40	18.0
9 and under 11	112	50.0	70	31.0
11 and under 13	48	21.0	38	17.0
13 and under 15	0	0.0	13	6.0
15 and under 20	0	0.0	9	4.0
20 to 25	0	0.0	_2	1.0
	225	100.0	225	100.0
	$\bar{x} = 9.3$		$\bar{x} = 9.3$	
	Median $=$ 9	9.9	Median = 9.6	
	Mode = 10)	Mode = 1	0

TABLE 5.6. POLICE NONEMERGENCY RESPONSE TIMES.

these two communities than actually exists. Therefore, the concept of the range can be modified by eliminating from the analysis some portion of the low and high ends of the database.

In statistical analysis, there are several measures of dispersion that treat extreme scores similarly. The most common of these measures are the interquartile range (the difference between the values of the 25th and 75th percentiles) and the decile deviation (the difference between the values of the 10th and 90th percentiles).¹

Standard Deviation

The interquartile range and other such modifications of the range improve on it by reducing the influence of extreme values. However, for most statistical research, these modifications are too severe because they eliminate a considerable amount of data. A measure of dispersion is desired that does not eliminate any values yet is not overly influenced by extreme values. The standard deviation is that measure. Rather than eliminating values, the standard deviation weights all values of the variable by their frequency of occurrence, thereby including extreme values but tempering their mathematical importance.

The standard deviation represents a version of a mean distance from each value of the variable to the arithmetic mean. The more dispersed the data are, the greater is the standard deviation.

Since the arithmetic mean represents that value at which the sum of the deviations from all data points above the mean and all data points below the mean balance out to zero, it becomes necessary to square these deviations in order to convert all deviations to positive values and eliminate the offsetting effect of equal positive and negative values. The squared distances are then divided by n in order to obtain a mean distance. The result of this process is the variance (s^2):

$$s^2 = \frac{\Sigma(x - \overline{x})^2}{n} \tag{5.4}$$

To convert the squared units of the variance back to units consistent with the data set, the square root of the variance must be calculated. This calculation yields the standard deviation (s):

$$s = \sqrt{\frac{\Sigma(x - \overline{x})^2}{n}} \tag{5.5}$$

Equation 5.5 is applicable to data consisting of individual cases. For data presented in frequency distributions with variables consisting of single values only, as in table 5.3, the equation for the standard deviation is adjusted:

$$s = \sqrt{\frac{\sum f(x - \overline{x})^2}{n}}$$
(5.6)

In the case of interval data presented in ordinal categories consisting of a range of values, *m* (the category midpoint) is substituted for *x*:

$$s = \sqrt{\frac{\sum f(m - \bar{x})^2}{n}}$$
(5.7)

It is important to note that for small samples (generally fewer than 120), n - 1 must be substituted for n in equations 5.5, 5.6, and 5.7. (The purpose of this adjustment is explained more thoroughly in chapter 7.)

For the example of the eleven exam scores presented earlier in this chapter, the standard deviation can be calculated by operationalizing equation 5.5, as shown in worksheet 5.4.

Worksheet 5.5 demonstrates the use of equation 5.6 (single-value category frequency distributions) in the calculation of the standard deviation pertaining to the number of children in Mount Pilot, South Dakota (table 5.3).

In the example of police department response times (table 5.6), the standard deviations for Harrison and Jackson can be calculated as demonstrated in worksheet 5.6. The ranges (Harrison = 12, Jackson = 24) and standard deviations (Harrison = 2.43; Jackson = 3.60) also demonstrate that Jackson is characterized by less consistent response times for its police than Jackson.

The utility of the standard derivation with regard to its integral role in the explanation of sampling theory is discussed in detail in chapter 6. As a descriptive statistic, the utility of the standard deviation derives from its indication of how closely the mean represents its data set. The lower the standard deviation is, the better the mean reflects its data. Conversely, the greater the standard deviation is, the more dispersed the data and the less representative the mean.

WORKSHEET 5.4. CALCULATION OF STANDARD DEVIATION FOR INDIVIDUAL CASE DATA.

1.	Exam score	$\frac{1}{x} = 53, 70, 76, 79, 8$ $\overline{x} = 81.73$	80, 82, 85, 88, 9	3, 95, 98
2.	$s = \sqrt{\frac{\Sigma(x - n)}{n}}$	$\frac{(-\overline{x})^2}{(-1)}$ (<i>Note:</i> number of	cases, <120)	
3.	x	$(x-\overline{x})$	$(x-\overline{x})^2$	
	53	-28.73	825.41	
	70	-11.73	137.59	
	76	-5.73	32.83	
	79	-2.73	7.45	

x	$(x-\overline{x})$	$(x-\overline{x})^2$
80	-1.73	2.99
82	0.27	0.07
85	3.27	10.69
88	6.27	39.31
93	11.27	127.01
95	13.27	176.09
98	16.27	<u>264.71</u>
		$\Sigma =$ 1,624.15

4.
$$s = \sqrt{\frac{1,624.15}{10}} = \sqrt{162.415} = 12.74$$

WORKSHEET 5.5. CALCULATION OF STANDARD DEVIATION FOR A FREQUENCY DISTRIBUTION WITH SINGLE-VALUE CATEGORIES.

•	$s = \sqrt{\frac{\Sigma f}{\Sigma}}$	$\frac{f(x-\overline{x})^2}{n}$ (N	<i>ote:</i> number of	cases \geq 120)	
	<i>x</i> =1.4	7			
2.	x	f	$(x-\overline{x})$	$(x-\overline{x})^2$	$f(x-\overline{x})^2$
	0	250	-1.47	2.16	540.00
	1	75	-0.47	0.22	16.50
	2	125	0.53	0.28	35.00
	3	75	1.53	2.34	175.50
	4	50	2.53	6.40	320.00
	5	20	3.53	12.46	249.20
	6	5	4.53	20.52	102.60
	600				$\Sigma = 1438.80$
	14	38.80 -			

3. $s = \sqrt{\frac{1438.80}{600}} = \sqrt{2.398} = 1.55$

Harrison ($\overline{x} = 9.3$)					
x	f	m	$m - \overline{x}$	$(m-\overline{x})^2$	$f(m-\overline{x})^2$
1 and under 3	8	2	-7.3	53.29	426.32
3 and under 5	10	4	-5.3	28.09	280.90
5 and under 7	15	6	-3.3	10.89	163.35
7 and under 9	32	8	-1.3	1.69	54.08
9 and under 11	112	10	0.7	0.49	54.88
11 and under 13	48	12	2.7	7.29	349.92
13 and under 15	0	14	4.7	22.09	0
15 and under 20	0	17.5	8.2	67.24	0
20 to 25	0	22.5	13.2	174.24	0
	n = 225				$\Sigma = 1,329.43$

WORKSHEET 5.6 CALCULATION OF STANDARD DEVIATIONS FOR TABLE 5.6.

$$s = \sqrt{\frac{\sum f(m - \overline{x})^2}{n}}$$
$$= \sqrt{\frac{1,329.45}{225}}$$

= 2.43 minutes

Jackson ($\overline{\mathbf{x}} = \mathbf{9.3}$)

x	f	m	$m-\overline{x}$	$(m-\overline{x})^2$	$f(m-\overline{x})^2$
1 and under 3	9	2	-7.3	53.29	479.61
3 and under 5	20	4	-5.3	28.09	561.80
5 and under 7	24	6	-3.3	10.89	261.36
7 and under 9	40	8	-1.3	1.69	67.60
9 and under 11	70	10	0.7	0.49	34.30
11 and under 13	38	12	2.7	7.29	277.02
13 and under 15	13	14	4.7	22.09	287.17
15 and under 20	9	17.5	8.2	67.24	605.16
20 to 25	2	22.5	13.2	174.24	348.48
	<i>n</i> = 225				$\Sigma = 2,922.5$

$$s = \sqrt{\frac{\sum f(m - \bar{x})^2}{n}}$$
$$= \sqrt{\frac{2,922.5}{225}}$$
$$= 3.60 \text{ minutes}$$

USING SPSS TO GENERATE FREQUENCY DISTRIBUTIONS/TABLES

The tables shown throughout this chapter are known as frequency distributions and can be generated by SPSS. Two data sets have been prepared for use in conjunction with this book and are found at www.wiley.com/college/rea. One data set is called Transportation.sav and the other is Water.sav. Each is modified from actual survey research projects we have conducted.

The steps used to generate frequency distribution tables are presented below.

- 1. Open the Transportation.sav data file and click the Data View tab in the lower left portion of the screen.²
- 2. Click the Analyze drop-down menu at the top, and select Descriptive Statistics and then Frequencies

 day hour ✓ superdist ▲ Language 	<u>C</u> harts <u>E</u> ormat <u>B</u> ootstrap
<pre>satis_overall_q1 change_service satis_cost_q3a satis_ontime_q3b satis_freq_q3c satis_transfer#_q</pre>	

3. At this point, make certain that the list in the Frequencies window appears as shown.³

- 4. Highlight a few variables—passby_q12, purpose_q14, transfers#_q15, and satis_ traveltime_q3n. This is accomplished by highlighting these variables and clicking the arrow to move them to the Variable(s) list or by dragging and dropping them into the Variable(s) list.
- 5. By switching the bottom tab from Data View to Variable View and looking for these variables, researchers will see the that Labels for these variables tell us that passby_q12 is a variable that allows the respondent to tell researchers whether a bus has ever passed the respondent by without stopping. Under the Values column, it can be seen that the answer choices were yes or no.
- 6. The variable purpose_q14 indicates where (other than home) the respondent was going on his or her bus trip, transfers#_q15 indicates the number of bus transfers that were made on that trip, and satis_traveltime_q3n is how satisfied the respondent is with how long it takes to complete his or her trip.

The Output file now reveals the frequency distributions (tables). It can be seen that out of 1,954 respondents, 876 (44.8 percent) have had a bus pass them by without stopping, 1,025 (52.5 percent) have not, and 53 (2.7 percent) are missing. Missing responses are often simply people who did not answer that question. There are other reasons for declaring certain values missing, some of which will be dealt with in this book.⁴ In almost all cases, however, the reason for a missing declaration is that the response or nonresponse should not enter into the analysis; therefore, missing responses are not taken into account in the Valid Percent column. As such, it is better to report that 46.1 percent of respondents have been passed by and 53.9 percent have not.⁵

		Frequency	Percent	Valid Percent	Cumulative Percent
	Yes	876	44.8	46.1	46.1
	No	1025	52.5	53.9	100.0
	Total	1901	97.3	100.0	
Missing	System	53	2.7		
Total		1954	100.0		

Bus Ever Pass by Without Stopping for You?

From the other tables, it can be seen that a substantial plurality of trips were made for work purposes (43.6 percent), that more people make two bus transfers (36.0 percent) than any other number of transfers (including no transfers), and that the most frequent response to travel time satisfaction is somewhat satisfied (32.9 percent)—slightly ahead of very satisfied (30.7 percent). These are the modes for these three variables.

		Frequency	Percent	Valid Percent	Cumulative Percent
	Work	818	41.9	43.6	43.6
	Shopping	144	7.4	7.7	51.3
	School (K–12)	67	3.4	3.6	54.8
	College/trade school	236	12.1	12.6	67.4
Valid	Recreation/social visit	117	6.0	6.2	73.6
	Personal errands	313	16.0	16.7	90.3
	Health/doctor	167	8.5	8.9	99.2
	Other	15	.8	.8	100.0
	Total	1,877	96.1	100.0	
Missing	System	77	3.9		
Total		1,954	100.0		

Where Going (Other Than Home)—Trip Purpose

Number of Transfers—Subject Trip						
		Frequency	Percent	Valid Percent	Cumulative Percent	
	0	302	15.5	17.3	17.3	
	1	358	18.3	20.5	37.8	
	2	630	32.2	36.0	73.8	
Valid	3	269	13.8	15.4	89.2	
	4	143	7.3	8.2	97.4	
	5	22	1.1	1.3	98.6	
	6	24	1.2	1.4	100.0	
	Total	1748	89.5	100.0		
Missing	System	206	10.5			
Total		1,954	100.0			

Number of Transfers—Subject Trip

Satisfaction: Travel Time

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Very satisfied	573	29.3	30.7	30.7
	Somewhat satisfied	613	31.4	32.9	63.6
	Neither satisfied nor dissatisfied	339	17.3	18.2	81.8

(continued)

		Frequency	Percent	Valid Percent	Cumulative Percent
	Somewhat dissatisfied	209	10.7	11.2	93.0
	Very dissatisfied	131	6.7	7.0	100.0
	Total	1865	95.4	100.0	
Missing	System	89	4.6		
Total		1,954	100.0		

USING SPSS TO GENERATE MEASURES OF CENTRAL TENDENCY AND DISPERSION

With the bus passing by and trip purpose being nominal, the mode is the only measure of central tendency that can be properly used. It is important to note that the SPSS program will calculate a median and mean for these two variables if asked to do so, and it is the responsibility of the researcher to recognize the appropriateness of making such a request (ordinal and interval levels of measurement only). The SPSS calculation of a mean or median for these two nominal variables would be nonsensical in a statistical context.

However, satisfaction with travel time is ordinal and number of transfers is scale (interval), which permits the use of the median for satisfaction and the median and mean for transfers. As will be discussed later in the chapter, there is a good case for calculating a mean for satisfaction also, despite its being an ordinal variable.

- 1. In order to have SPSS determine the medians for number of transfers and satisfaction with travel time, return to Analyze—Descriptive Statistics—Frequencies and remove all of the nonordinal variables from the list that included the nominal variables of passby_q12 and purpose_q14 and the scale (interval) variable, transfers#_q15, leaving only the satisfaction variable.
- 3. Select Median under Central Tendency. Click Continue and then OK.
- 4. Output will now show the frequency distribution again but will also show the following:

Ν	Missing	89
Median	Wiissing	2.00

The median = 2 refers to the category coded as 2—(not an actual 2 (e.g., 2 minutes, 2 hours, and so forth), which can be found in a number of ways, the simplest

being clicking on the Variables icon (ninth from the left at the top) or clicking on the Utilities drop-down menu and then selecting Variables. The category with a code of 2 is Somewhat Satisfied, and this is the median response for travel time satisfaction.

Variables	×
 Variable Satis_treleinto_q satis_transferre satis_drivecourt satis_driveknow satis_safetybus satis_safetybus satis_safetybus satis_clean_q3m satis_traveltime satis_infostops satis_busbook satis_busbook import_cost_q4a import_free_odc 	Variable Information: satis_traveltime_q3n Label: Satisfaction: Travel Time Type: F8 Missing Values: none Measurement Level: Ordinal Value Labels: 1 Very Satisfied 2 Somewhat Satisfied 3 Neither Satisfied nor Dissatisfied 4 Somewhat Dissatisfied 5 Very Dissatisfied 5 Very Dissatisfied 60 T0 Paste Cancel Help

The reader may have noticed that means and standard deviations are also available in this manner; however, a more straightforward way is available for means and standard deviations.

- 1. Select Analyze—Descriptive Statistics—Descriptives.
- 2. Place the transfers#_q15 variable in the Variable(s) box and then select Options, where mean and standard deviation are already selected by default, along with minimum and maximum values.
- 3. The resulting output shows a mean of 1.86 and a standard deviation of 1.30.

Descriptives		×
 othermodes_Met othermodes_Amt othermodes_oth othermodes_oth travtime_hours_q travtime_minutes travtime_total payment_q18 paycash_q19a 	Variable(s):	Options Bootstrap
Save standardi <u>z</u> ed values as vari	iables Reset Cancel Help	

	N	Minimum	Maximum	Mean	Std. Deviation
Number of transfers—subject trip	1,748	0	6	1.86	1.300
Valid N (listwise)	1,748				

USING SPSS TO CALCULATE A MEAN AND STANDARD DEVIATION FOR AN ORDINAL VARIABLE WITH CATEGORIES THAT CONTAIN RANGES OF INTERVAL/SCALE DATA

From Transportation.sav, select the variable income_q35 (Annual Household Income Category). The Value column in Variable View and the frequency distribution for income_q35 both indicate that income is measured in ordinal categories for this variable. Worksheets 5.3 and 5.6 used the substitution of a midpoint to represent the categories' values when calculating means and standard deviations under these circumstances.

SPSS can apply midpoints through its recoding process.

- 1. Click on Transform and then Recode into Different Variables.⁶
- 2. Move income_q35 into the Numeric Variable-Output Variable Box.
- 3. Give the new variable that is being created a new name. We recommend that the new name be similar to the original variable for ease of reference in an alphabetical search for the new variable. The original and new names will now be close to one another and easier to locate in that search. Therefore, increc was selected and labeled to show that it is a recoded variable using midpoints. Click on Change, and the new name should appear next to the existing variable.
- 4. The next step in the recoding process is to select the Old and New Values button and then proceed to instruct SPSS that the coded income categories should replace the codes with the corresponding midpoints. Were this not done and SPSS were to be asked to generate a mean for income_q35, SPSS would find the mean of the code numbers. As such, the instruction must be presented to SPSS not to use the code numbers but the midpoints instead. Once the Old and New Values button has been selected, a new window appears that allows this recoding process to take place.
- 5. The first step in assigning new values to the original codes, as earlier in this chapter, is to manually determine the midpoints of each category. These midpoints would be \$5,000 (Under \$10,000), \$15,000 (\$10,000 to less than

\$20,000), \$27,500 (\$20,000 to less than \$35,000), \$42,500 (\$35,000 to less than \$50,000), \$62,500 (\$50,000 to less than \$75,000), \$87,500 (\$75,000 to less than \$150,000), \$125,000 (\$100,000 to less than \$150,000), and \$175,000 (\$150,000 or more). Note that the \$150,000 or more category is open-ended and that no midpoint can therefore be calculated. The researcher must use some judgment here. Recognizing that the number of people with higher incomes declines as the income decreases, he or she should choose a number closer to the lower end of the category while still allowing for some upper incomes to exist.

- 6. In the Old Value section, select the Value button and type in 1. In the New Value section, select value and type in 5000. Next click the Add button, and the window will show that 1 is being replaced by 5000.
- 7. Next, type 2 in the Value window of the Old Value section, and place the corresponding midpoint (15000) in the value window of the New Value section.
- 8. Repeat this for each midpoint and finally select the System- or user-missing button in the Old Value section and System-missing in the New Value section. This will instruct SPSS that anything that was left out or declared missing in the original variable is to be left out in the recoded variable.

Old Value	New Value
● <u>V</u> alue:	Value: 5000
1	O System-missing
System-missing	Copy old value(s)
◎ System- or <u>u</u> ser-missing ◎ Range:	Ol <u>d</u> > New:
through	Add 1> 5000 2> 15000 3> 27500 4> 42500
Range, LOWEST through value:	Change 5> 62500 6> 87500 7> 125000
© Rang <u>e</u> , value through HIGHEST:	8> 175000 MISSING> SYSMIS
⊘ All <u>o</u> ther values	Output variables are strings Width: 8
	Convert numeric strings to numbers ('5'->5)

- 9. Press Continue and then OK, and verify that a new variable (increc) has been created at the end of the data set in both Data View and Variable View.⁷
- 10. Request a mean for increc (Analyze—Descriptive Statistics—Descriptives) and SPSS Output will provide the following, indicating a mean of \$19,718.68 and standard deviation of \$25,541.97.

	Ν	Minimum	Maximum	Mean	Std. Deviation
recoded income-midpoints	1,804	5,000.00	175,000.00	19,718.6807	25,541.97312
Valid N (listwise)	1,804				

USING SPSS TO FIND MEASURES OF CENTRAL TENDENCY AND DISPERSION FOR INDIVIDUAL CATEGORIES OF A VARIABLE

The discussion has covered finding the measures of central tendency and dispersion for an entire variable. It is also possible to find measures of central tendency and dispersion for each category of another variable. That is to say, using transfers#_q15, it was found that the mean number of transfers was 1.86; however, if a researcher wanted to determine the mean number of transfers for each supervisorial district (superdist), that can be accomplished using Analyze—Compare Means—Means.

- 1. Place the variable that is to have these measures calculated (transfers#_q15) in the Dependent List box and the variable for which category measures are to be calculated (superdist) in the Independent List box.
- 2. Click on Options and add the Median to the list of default measures (mean, standard deviation, and number of cases).
- 3. Click OK. The output displayed below shows that the Central and West districts have the most average transfers (means = 1.93) and the East district has the fewest (1.64).

ta Means		×		
effectiveinfo_tele passby_q12 busroute_q13 purpose_q14 othermodes_bicy othermodes_driv othermodes_car othermodes_van othermodes_van othermodes_van	Dependent List: Transfers#_q15 ayer 1 of 1 Previous Next Independent List: Superdist Paste Reset Cancel Help	Options Bootstrap		
Supervisorial District	Mean	N	Std. Deviation	Median
------------------------	------	------	----------------	--------
Central	1.93	673	1.250	2.00
West	1.93	273	1.243	2.00
East	1.64	215	1.332	2.00
North	1.84	496	1.345	2.00
South	1.79	91	1.465	2.00
Total	1.86	1748	1.300	2.00

Report: Number of Transfers—Subject Trip

The Normal Distribution

Most features or characteristics (variables) of a population tend to be distributed in accordance with the commonly understood concept of the bellshaped curve. For instance, most adult American men stand between 5 feet 6 inches and 6 feet 2 inches tall, with far fewer less than 5 feet 6 inches or more than 6 feet 2 inches. If the heights of all American men were recorded and their frequencies plotted, the distribution would most likely resemble the bell shape depicted in figure 5.1, which is known statistically as the *normal distribution* or *normal curve*.



FIGURE 5.1. THE NORMAL CURVE.

In the normal distribution, the mean is located at the exact center and peak of the curve, dividing the curve into two symmetrical halves, each the mirror image of the other. That is, the mean, median, and mode are equal to one another in a perfectly normal distribution of data. The normal curve is also asymptotic to the *x*-axis. In other words, in both directions, the curve moves closer and closer to the *x*-axis but in theory never touches it. The possibility therefore always exists that there are findings that might be made outside the realm of existing knowledge.

Most cases in the normal distribution are clustered around the mean. In the example of the heights of adult American men, if the mean height were 5 feet 10 inches, more men would be 5 feet 9 inches tall than 5 feet 6 inches. Similarly, more men would stand 5 feet 11 inches tall than 6 feet 0 inches.

Selecting the Most Appropriate Measure of Central Tendency for Describing Survey Data

When the three measures of central tendency are relatively close in value, it would be redundant to include more than one in a frequency distribution table. Moreover, when these measures differ in value substantially, it would be inappropriate and confusing to include more than one. Hence, the researcher should select only one measure of central tendency for inclusion in the frequency distribution table. This section outlines considerations that can aid the researcher in selecting the most appropriate measure of central tendency.

In the ultimate hypothetical normal distribution, the mode, median, and mean are equal. If, however, the distribution has a few extreme values, either high or low, the three measures of central tendency will begin to deviate from one another. Such a distribution is said to be *skewed*. Skewed distributions generally assume one of two forms:

- 1. The *positive skew* (figure 5.2), which has a mean higher than the median and the mode because there are a few rather high values pulling the mean (and to a lesser extent, the median) toward these higher values
- 2. The *negative skew* (figure 5.3), which has some relatively low values pulling the mean and median toward the low values

In the case of a normal distribution, with the three measures of central tendency equal or approximately so, any one of these measures should



FIGURE 5.2 DISTRIBUTION WITH POSITIVE SKEW.

FIGURE 5.3 DISTRIBUTION WITH NEGATIVE SKEW



suffice as a summary statistic for the data. In normal distributions, the rule for choosing the appropriate measure revolves around the level of measurement associated with the variable. Specifically, the mean should be used for normally distributed interval data, with the median being most appropriate for ordinal data and the mode remaining as the only measure of central tendency for nominal data. The choice between median and mean under conditions of skewed interval data depends on how heavily the data are skewed. When the researcher determines that the mean is overly affected by the skew and therefore is a poor representation of the central tendency of the distribution, the median becomes the chosen statistic. Consider the following simple example.

The annual salaries of ten employees of a small private business are indicated in exhibit 5.1. The mean income for these employees is \$76,400 per year. It can be readily observed that this mean is not an appropriate representation of the central tendency of this income distribution, because nine employees out of the total of ten earn less than the mean income. A much more informative measure in this case is the median of \$26,000. Skewness is often encountered with such variables as household income, age, and housing prices. The rule of thumb for such skewed variables therefore is that the best summary measure of central tendency, or average, is the median, not the mean.

To explain this concept of skewness further, consider table 5.7, in which the modal category is \$5.00 and under \$10.00 (often designated by the category's midpoint, 7.5), the median is \$9.37, and the mean is \$23.00. A positive skew is strongly suggested by these measures of central tendency, insofar as the mean is substantially higher than the median, which is in turn higher than the mode (see figure 5.2). Such a skewed distribution is

Employee	Salaries	
А	\$19,000	
В	\$21,000	
С	\$21,500	
D	\$22,500	
E	\$25,000	
F	\$27,000	
G	\$33,000	
Н	\$40,000	
I	\$55,000	
J	\$500,000	

EXHIBIT 5.1. SALARIES OF EMPLOYEES OF A SMALL, PRIVATE BUSINESS.

Note: $\bar{x} =$ **\$76,400**; median = **\$26,000**.

ANNUAL AMOUNT	FREQUENCY	
Nothing	37	
More than \$0 but less than \$5	38	
\$5 and under \$10	95	
\$10 and under \$25	50	
\$25 and under \$50	40	
\$50 and under \$75	30	
\$75 to \$100	_25	
Total	315	

TABLE 5.7. WILLINGNESS TO SPEND FOR STREET AND HIGHWAY BEAUTIFICATION.

caused by a few respondents who are willing to spend a substantial amount of money on street and highway beautification.

An indicator of skewness is the nature of dispersion, as measured particularly by the standard deviation. As discussed in chapter 6, in a normal distribution, three standard deviations cover nearly the entire distribution. Therefore, the researcher can strongly suspect a skewed distribution if the mean, plus or minus three standard deviations, does not approximate the range of the data. In the case of the data in table 5.7, note that the mean of \$23.00 is associated with a standard deviation of \$26.37 and that $23.00 \pm 3(\$26.37) = -\56.11 to 102.11; this is outside the range of the data on one side (the lower side) and roughly coincident on the other (higher side). The standard deviation will be sufficiently large to reach the farthest data extreme, and the result will be that one side will be substantially outside the existing data range when the data are not normally distributed.

Finally, the most rudimentary way of determining whether a skew exists in a frequency distribution is to prepare a frequency polygon of a distribution. In particularly skewed distributions, the frequency polygon will demonstrate the characteristics of skewness indicated in figures 5.2 and 5.3.

USING SPSS TO HELP EVALUATE IF A DISTRIBUTION IS NORMAL

In a normal distribution, interval data are best described by the mean as the most appropriate measure of central tendency, but the median serves to describe skewed data better than does the mean. There is no clear statistic that tells whether a distribution is approximately normal. It is more a preponderance of information that will lead the researcher to his or her ultimate conclusion. An especially helpful tool in SPSS is found in Analyze—Descriptive Statistics—Explore. Explore not only helps make the determination about whether a distribution is approximately normal; it also combines many of the measures of central tendency and dispersion that have been discussed in this chapter into one single output. On the negative side, Explore provides a huge number of data that are not needed just to identify measures of central tendency and dispersion and requires a bit more configuration than the tools presented thus far. We therefore recommend that Explore be reserved for determination of a normal distribution and for setting confidence intervals that will be covered in chapter 7.

To show its use in determining if a distribution is normal:

- 1. Go to Analyze—Descriptive Statistics—Explore and insert the interval variable age_q32 in the Dependent List box.
- 2. Select Statistics, and make certain that Descriptives are checked. Then click Continue.
- 3. Select Plots and change Stem and Leaf to Histogram. Then click Continue.
- 4. In Options, change Exclude Cases Listwise to Exclude Cases Pairwise⁸ and click Continue again.
- 5. Click OK, and Output will contain the following descriptive statistics and graph.

It is known that a normal distribution will have a mean and median that are close together. In this case, the mean age is 40.32 and median is 40, which are very

			Statistic	Std. Error
	Mean		40.32	.375
	95% confidence interval for	Lower bound	39.59	
	mean	Upper bound	41.06	
	5% trimmed mean		40.07	
	Median		40.00	
Age Category	Variance		255.968	
	Std. deviation		15.999	
	Minimum		13	
	Maximum		75	
	Range		62	
	Interquartile range		28	
	Skewness		.096	.057
	Kurtosis		-1.162	.115

DESCRIPTIVES



FIGURE 5.4. HISTOGRAM.

close. Another useful statistic is the 5% trimmed mean. This deletes the top 5% of the data and the bottom 5% of the data and recalculates a mean. If the data are seriously skewed, the trimmed mean will move away from the mean because, in a positive skew, the upper values removed will have greatly increased the mean, and the same effect is in reverse for a negative skew. In this case, the mean (40.32) and trimmed mean (40.07) are close together, indicating a likely normal distribution.

Skewness (.096) is also a potentially helpful statistic. It can be positive or negative to reflect the direction of the skew, but the closer the value is to 0, the more the distribution approximates a normal distribution. Unfortunately, there is no firm cut-off point to delineate skewed from normal; however, some statisticians consider that a skewness value that is less than two times the standard error (to be explained in chapter 6) indicates a normal distribution. Applying that rule-of-thumb to the age data standard error of .375 reveals a skewness (.093) that is considerably less than .75 (double the standard error). All of these statistics would cause a researcher to conclude that the age data are essentially normally distributed. The histogram (figure 5.4), however, is not as clear. If the data were perfectly normal, the histogram would resemble a bell curve. Although the rise and fall on the extremes seem normal, the multimodal (several peaks to the curve) nature of the distribution is less normal in nature. Another way to examine the histogram besides simply looking at its shape is to determine if the distribution is consistent with 99.74% of all cases being within +/- 3 standard deviations of the mean. Obviously, inasmuch as these are sample data, it will not conform precisely to that; therefore, once again, the researcher uses his or her best judgment by asking two questions:

- 1. Does +/- 3 standard deviations from the mean reach to approximately the end points of the data distribution—not too far beyond and not too far short?
- 2. Is +/-3 standard deviations relatively equal on both ends of the data? That is, is the overage or shortage that +/-3 standard deviations represents in relation to the end points of the data very close to the same on both ends—indicating symmetry?

In the case of these age data, the histogram indicates a mean of 40.32 and a standard deviation of 15.999. The range of (+/-3)(15.999) would reach to a low age of -7.677 and a high age of 88.317. These values extend beyond the ends of the data—13 minimum and 75 maximum by approximately twenty years on the low end and thirteen years on the high end, which are outside the +/-3 standard deviations but are relatively symmetrical. This is where the researcher must use some independent judgment. We would conclude that although it is not completely clear, the closeness of the mean, median, and trimmed mean, the small skewness value, and the histogram symmetry are indicative more of a normal distribution than the counter-findings of multimodality and the overage of the +/-3 standard deviations are of a skewed distribution. Therefore, the mean age would represent a more appropriate measure of central tendency than would the median.

Scaled Frequency Distributions

Another issue concerning the use of an appropriate measure of central tendency involves variables measured using a scale. The following is an example of a Likert scale question:

What is your general impression of how the Montgomery city government affects your business?

Highly positive	Positive	Neutral	Negative	Highly negative
1	2	3	4	5

Impression	Value	f	%
Highly positive	1	100	16.7
Positive	2	175	29.2
Neutral	3	200	33.3
Negative	4	75	12.5
Highly negative	5	<u>50</u>	<u>8.3</u>
Total		600	100.0

TABLE 5.8. IMPRESSION OF EFFECT OF MONTGOMERY CITY GOVERNMENT ON BUSINESS.

Note: $\bar{x} = 2.67$.

The responses to this question are shown in table 5.8.

The reader may have noted that an arithmetic mean has been calculated for the data in table 5.8, although the data are clearly ordinal in nature. As indicated above, the median, rather than the mean, is the appropriate measure of central tendency for ordinal data. In the case of table 5.8, the median is found at case 300.5, which is contained within the category "Neutral." However, the scale, which generally associates numerical values with ordinal data (in this case, 1 to 5 for "Highly positive" to "Highly negative"), permits the researcher to calculate an arithmetic mean. In table 5.8, this mean is calculated to be 2.67, which is slightly to the positive side of neutral. Hence, the mean has provided more information than the median. It has told the researcher that the center of the distribution is close to "Neutral" (the median) but is weighted somewhat toward positive response categories.

The calculation of the mean in this case makes important assumptions about the nature of the ordinal data by treating them as if they were interval data. First, it requires that the question be asked numerically. That is, it asks the respondent to reply with the scaled number, not words, and then assumes that all respondents have a common understanding of the meaning of each response category. For example, in asking individuals how old they are, it is properly assumed that there is a common understanding of the concept of years in the measurement of age. Technically this cannot be the case in terms of what respondents understand to be "Highly negative" versus "Negative," and so forth. Second, it assumes that there is an equal distance between each category of the variable that is measurable in accordance with the numerical values assigned to these categories. No such conclusion can be properly drawn with regard to ordinal data, inasmuch as ordinality affords the researcher the ability only to rank the data, not the ability to manipulate the data arithmetically. However, such manipulation has become accepted because the power of the information obtained is considered to far outweigh the costs associated with relaxing these technicalities. Hence, it is recommended that in the case of scaled responses, the proper measure of central tendency should be considered to be the arithmetic mean, and in the case of a series of such responses, an arithmetic mean is an acceptable summary measure of the subject matter under study.

Example 3.17 (in chapter 3) presented a scaled response series of questions administered to employees of a public agency. The responses are on a five-point scale where 1 = strongly agree and 5 = strongly disagree. The answers to those questions produced a mean score for each job characteristic, as follows:

Job Characteristic	Mean
I feel that the contributions I make are valued.	2.81
My ideas and opinions are taken into consideration at work.	2.82
I have all the information I need to do my job effectively.	2.85
I have opportunities to be innovative in carrying out my job.	2.94
Overall my employee performance evaluation is fair.	3.10
I am encouraged to provide input regarding work-related matters.	3.12
My morale at work is generally good.	3.17
I feel that I have job security.	3.18
The benefit package provided by my employer is good.	3.19
I am paid fairly for the work I do.	3.29
Overall mean	3.05

These responses, if described solely as ordinal data, would have yielded medians of "Neutral." However, the relaxation of a strict adherence to the ordinality of these data has allowed the researcher to identify important information not apparent in the use of the median. For instance, it is clear that these employees feel that their contributions are valued and that their ideas and opinions are taken into consideration at work more so than they feel they are fairly compensated for what they do and that the benefit package provided by their employer is a good one.

Notes

1. Persons who prefer to calculate the median using an equation may use the following formula for this purpose:



The median is also known as the 50th percentile, at which value 50 percent of the observations in the data set are greater in value and 50 percent are less. In general, it is possible to calculate any percentile (k) by determining the value of the variable at which k percent of total values are lower. The method for determining the kth percentile is a four-step process similar to the method used for the calculation of the median:

- a. Ensure that the categories are ranked low value to high value.
- b. Determine the location of the *k*th percentile: location of *k*th percentile = (n + 1) (*k*/100) where *k* = percentile to be calculated.
- c. Identify the category that corresponds to the *k*th percentile location, using cumulative frequencies.
- d. Estimate the value of the *k*th percentile within the range of values in the *k*th percentile category.

Applying these steps to the data contained in table 5.6 can generate the determination of the 75th percentile, for example. Worksheet 5.6 depicts the process of the 75th percentile calculation for Community A.

Persons who feel more comfortable calculating percentiles using an equation may use the following formula for this purpose:

kth perceptile -	(n+1)(k/100) – number of cases below <i>k</i> th percentile category		
number of cases in k th percentile category		nber of cases in <i>k</i> th percentile category	
	width of		
	kth percentile		(5.9)
	category		. ,

- 2. Refer to SPSS manuals for instruction in the creation of a data file, including rules for naming files, using labels, inserting data values, and declaring missing values.
- 3. If the list appears in a different format, the reader can change that format by clicking on the Edit drop-down menu and then selecting Options. Once in Options, go to the General tab. In the upper left, under Variable Lists, are two sets of buttons one to select either labels or names. The lists in this book use names, which are the shorter variable indicators. Labels are the more detailed variable description that

will be printed in the output tables. The other buttons are for the order in which the variables will be displayed. They can be in alphabetical order; file order, which is the order that the variables were created when the data file was initially set up (in surveys, that will generally be the order of the survey questions); or level of measurement (grouping nominal, ordinal, and *scale*—the SPSS term for interval—together). In this book, file order is used because we are familiar with the variable names and order and find searching for variables easier in file order. When we are using files created by others and do not possess that familiarity, we find alphabetical searching easier. The selection of names or labels and the order of display are optional; the reader can use whatever he or she finds to be most convenient when doing his or her own studies. By selecting those options, the reader should have in view the Frequencies window displayed here.

- 4. For a more complete listing of all the reasons for declaring missing variables, consult a basic SPSS manual that discusses the creation of data files.
- 5. If the output that the reader generates differs from the above, once again go to Edit—Options and click the Output tab. For purposes of this book, Output labeling and Pivot Table labeling are set as labels only. The reader can experiment by changing these options to names and labels and values and labels and choose the output that he or she prefers.
- 6. Using Recode into Different Variables preserves the original variable categories for future use. Recode into Same Variable changes the original variable and should be used only when it is absolutely certain that the original categories are no longer needed.
- 7. Note that SPSS default calls for variables to be coded with two decimal places. If the researcher prefers a default with a different number of decimal places (we recommend 0 for most codes), this can be accomplished through Edit—Options—Data. Change the number of decimals in Display Format for New Numeric Variables to 0. If the researcher wishes to retain the default of two decimals but to change this variable only, that can be accomplished by changing the decimal column for that one variable to 0 in Variable View.
- 8. Exclude cases listwise: Cases with missing values for any dependent or factor variable are excluded from all analyses. This is the default. Exclude cases pairwise: Cases with no missing values for variables that are part of the particular research request submitted to SPSS are included in pairwise, even if other variable values are missing. Pairwise admits more data into the data set.



CHAPTER SIX

THE THEORETICAL BASIS OF SAMPLING

This chapter provides an introduction to the statistical concepts that underlie the essential theoretical foundation to understanding the scientific basis of survey research.

The purpose of sampling is to be able to make generalizations about a population based on a scientifically selected subset of that population. Sampling is necessary because it is generally not practical or feasible to seek information from every member of a population. A sample therefore is intended to become a microcosm of a larger universe. However, chapter 1 asked how a relatively small subset of cases can be used to represent the much larger population from which the subset has been selected. To address and fully understand the implications of this question, it is important to establish the theoretical basis of sampling and its associated assumptions.

Properties of the Normal Distribution

There are certain standard properties of the normal curve that convey how values of the variable are distributed around the mean. The measurement of distance from the mean is calculated in terms of the standard deviation. The *standard deviation* is, as the name implies, a measurement of dispersion around the mean in standardized units. Consequently, no matter what the

variable is (e.g., weight, IQ scores, or income), a constant proportion of the total area under the normal curve will lie between the mean and any given distance from the mean as measured in units of standard deviation.

Chapter 5 demonstrated the manner in which the standard deviation is calculated for data derived from a sample. The calculation of the true population standard deviation (σ) is similar, but is shown with different symbols for the mean and standard deviation. The formula for the standard deviation of the true population is

$$\sigma = \sqrt{\frac{\Sigma(x-\mu)^2}{N}} \tag{6.1}$$

where

 σ = true population standard deviation

 $\mu =$ true population mean

N = population size

These symbols for the mean and standard deviation are known as *population parameters* when they refer to the true population, in contrast to the formulas from chapter 5 that contained symbols for a sample and are known as *sample statistics*.

For any particular normal distribution, regardless of the mean or the calculated standard deviation, the proportion of cases between the mean (μ) and one standard deviation (σ) always turns out to include 34.13 percent of the total cases (see figure 6.1). Furthermore, since the normal distribution is symmetrical, the identical proportion of cases will lie below the mean (between μ and -1σ). Hence, 68.26 percent of all cases in the entire population will be found within one standard deviation of the mean in either direction. Similarly, 95.44 percent of all cases are to be found within two standard deviations and 99.74 percent within three standard deviations. It should be clear, therefore, that a very small number of cases exists beyond three standard deviations from the mean.

For example, in a population of twenty-year-old men who have recently completed military basic training, assume a true population mean weight of 170 pounds and an associated standard deviation of 15 pounds. It can be expected that in that population, 68.26 percent of the males would weigh between 155 and 185 pounds, 95.44 percent would weigh between 140 and 200 pounds, and almost all men (99.74 percent) would weigh between 125 and 215 pounds (see figure 6.2).





The Standardized Z Score

Suppose one individual in this population of military personnel weighs 176 pounds. It can be readily observed from figure 6.2 that his weight would lie between the mean and one standard deviation above the mean. Figure 6.2, however, does not provide any greater specificity than that regarding his relative weight. For instance, if he wished to know the exact proportion of such military personnel who weigh more than he does, figure 6.2 would not suffice because his weight is other than 125, 140, 155, 170, 185, 200, or 215 pounds (one, two, or three full standard deviations above or below the mean).

It is possible, however, in the normal distribution, to calculate the relative position of any score by converting it into fractional units of standard deviations, known as standardized Z scores. This conversion can be accomplished through the following formula:

$$Z = \frac{x - \mu}{\sigma} \tag{6.2}$$

where

x = individual score

 μ = mean of the population distribution

 σ = standard deviation of the population

Z = standard deviation unit scores



FIGURE 6.2. PROPORTIONATE AREAS UNDER THE NORMAL CURVE FOR MILITARY WEIGHTS.

To convert *Z* scores, the population mean (in this case, $\mu = 170$) is represented by a *Z* score of 0 [(170 - 170)/15 = 0]. Applying equation 6.2 to the individual's weight of 176 pounds generates a *Z* score as follows: (176 - 170)/15 = .40.

For example, a weight of 176 pounds is .40 standard deviation units to the right (positive) of the mean (Z = 0) on the normal curve. To comprehend this score in the context of relative position in the distribution, it is necessary to find the percentage of cases above or below .40 standard deviations. That is accomplished by consulting the Table of Areas of a Standard Normal Distribution (given in Resource A at the back of the book). A Z score of .40 (column A) represents the point on the curve at which 65.54 percent of all weights are lower than the subject weight of 176 and 34.46 percent are greater. The determination of the percentage of all scores below Z = .40 is derived from column B, which shows 15.54 percent of scores existing between the mean and the Z under consideration. Since the properties of the normal distribution stipulate that 50 percent of all cases are on each side of the mean (see the discussion in chapter 5), adding the 50 percent of cases below the mean to the 15.54 percent of cases above it (column B) yields 65.54 percent.

Another military man who recently advanced from basic training weighs 152 pounds. His relative position in this population is:

$$Z = \frac{152 - 170}{15} = -1.20$$

Negative Z scores reflect the fact that the individual weight under analysis is less than the mean. The determination of the relative position of this Z score also uses resource A, with an understanding of the symmetrical nature of the normal distribution. A Z score of -1.20 can be evaluated by referring to Z = 1.20 and noting that there is a standard percentage of 38.49 percent of cases between the mean and either 1.20 standard deviations above or below the mean. Hence, with the negative Z of -1.20, 88.49 percent (38.49 percent + 50.00 percent) of all individuals in this military population weigh more than 152 pounds and 11.51 percent weigh less. Figure 6.3 provides a graphic illustration of this example.

If researchers need to identify the proportion of military men in this population who weigh between these individual weights of 152 pounds

FIGURE 6.3. STANDARDIZED PROPORTIONATE AREAS UNDER THE NORMAL CURVE: AN EXAMPLE.



and 176 pounds, that proportion is the sum of the two percentages from resource A: 15.54 percent + 38.49 percent = 54.03 percent.

Another valuable application of the standardized Z score is found in the comparative evaluation of the relative position of members of separate, normally distributed populations. Continuing with the military example, assume that twenty-year-old women in the military who have recently advanced from basic training have a mean weight of 120 pounds, with a standard deviation of 10 pounds. If a particular woman in this population weighs 127 pounds, it is possible to identify which individual from all of those discussed in this section weighs more relative to his or her own population. The woman's Z score is

$$Z = \frac{127 - 120}{10} = .70$$

This Z score exceeds the Z scores of both men (.40 and -1.20) and is therefore indicative of the conclusion that although she weighs the least among these three people in absolute terms, she weighs more relative to her own population than do either of the men.

The Theoretical Basis of Sampling

So far, the discussion has focused on the normal distribution, the true mean, and the true standard deviation of every case in the population. It should be evident that such complete information is rarely available. Gathering data from every member of a population is either logistically impossible or economically infeasible in most cases. Therefore, it has become practical for samples of the population to be selected so that generalizations can be inferred from the sample to the total population. These generalizations find their statistical basis in the characteristics of the normal distribution.

As stated in chapter 1, the average person with little statistical background is quite skeptical about the prospect of making generalizations from a single, relatively small sample. Therefore, let it be assumed that in order to determine the mean weight of recently trained twenty-year-old military personnel and to simultaneously appease those who are skeptical, the researcher suggests that 100 separate, mutually exclusive samples be conducted from the same population and that the mean of each of the 100 sample mean weights be calculated in order to estimate the total population's mean weight. The skeptics agree, feeling somewhat more confident of the accuracy of these results compared to those of a single sample. (They would be even more comfortable were 1,000 samples used.)

The researcher would first select 100 samples (according to principles that will be established in chapters 8 and 9); he or she would then calculate the mean weight from each of the 100 samples. Table 6.1 presents such sample data, and figure 6.4 plots these sample means.

The distribution of sample means presented in figure 6.4 has certain properties that give it a critical role in the sampling process. These properties can be stated as follows:

- Property 1: The value of the mean of sample means ("the mean of means") approaches the true population mean. The larger the number of samples, the closer the approximation is to the population mean. This property is referred to as the *Central Limit Theorem* and is the concept that was agreed to by the skeptical persons above.
- Property 2: The distribution of sample means will approximate a normal curve as long as the sample size of each individual sample is sufficiently large (100 or more at a minimum). This remains true whether or not the raw data are normally distributed.
- Property 3: The standard deviation of the distribution of sample means (called the standard error of the mean or standard error) is smaller than the standard deviation of the total population. There can be a great deal of heterogeneity in the total population. Some men may weigh 100 pounds, others 300 pounds or more. However, when sample means are used, the variation among the mean weights will be significantly less than with the raw data because of the summarizing nature of the mean (figure 6.5). The standard error is estimated to be

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \cdot \sqrt{\frac{N-n}{N-1}}$$
(6.3)

where

 $\sigma_{\bar{z}} = \text{standard error}$

 σ = population standard deviation

n = size of sampling distributions (number of cases in each sample)

N = true population size

Sample Means (pounds)	f
175	1
174	1
173	9
172	11
171	16
170	22
169	16
168	12
167	9
166	2
165	1
Total	100

TABLE 6.1. DISTRIBUTION OF 100 HYPOTHETICALSAMPLE MEAN WEIGHTS.

FIGURE 6.4. DISTRIBUTION OF SAMPLE MEANS.





FIGURE 6.5. HYPOTHETICAL NORMAL DISTRIBUTIONS FOR SAMPLE MEANS COMPARED TO RAW DATA.

Equation 6.3 takes into account the true population size (*N*) in the calculation of the standard error. As the population size increases, $\sqrt{(N-n)/(N-1)}$ approaches 1. Hence, with large populations, the standard error approaches

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \tag{6.4}$$

The expression $\sqrt{(N-n)/(N-1)}$ has come to be known as the finite population correction. The distinction between the population sizes of large and small populations will be defined in chapter 7. For now, assume that the population being analyzed in this chapter is large.

Generalizing from a Single Sample

Sampling theory invokes the three properties discussed above for purposes of justifying the use of one sample to make inferences about a larger population rather than conducting many separate samples, as the researcher agreed to do above. The process of many samples is quite costly and time consuming—hence, the desirability of being able to generalize from a single sample.



FIGURE 6.6. PROPERTIES OF NORMAL CURVES APPLIED TO HYPOTHETICAL DISTRIBUTION OF SAMPLE MEAN WEIGHTS.

With regard to the weights of military personnel (table 6.1), the mean weight (\bar{x}) of the sample means is 170. The Central Limit Theorem (Property 1) stipulates that this mean approaches and is a good estimate of the true population mean (μ) . Property 2 (assumption of normality) permits us to adapt this information into the context of the normal curve (see figure 6.6) through the calculation of the standard error (Property 3). The example postulated a population standard deviation of 15 pounds, and the size of the samples was 100; hence, the standard error of the distribution of the 100 sample means equals $\sigma_{\bar{x}} = \sigma / \sqrt{n} = 15\sqrt{100} = 1.5$.

In this example, 68.26 percent of all sample means can be expected to fall between 168.5 and 171.5 pounds. In the case of the 100 sample means conducted, therefore, it is expected that 68 or 69 samples would indicate mean weights between 168.5 pounds and 171.5 pounds. Similarly, 95.44 percent of all sample means should lie between 167 pounds and 173 pounds, and 99.74 percent of all sample means should be found between 165.5 and 174.5 pounds. Another way to look at these sample means is in probabilistic terms. In other words, given a true population mean of 170 pounds and true standard deviation of 15 pounds, if only a single sample of 100 persons were to be conducted, the chances of the sample's mean weight being within the 168.5 to 171.5 pound range is 68.26 percent (.6826 in probability terms), and there is a .9544 probability of the sample mean being within the 167 to 173 pound range. This idea of assessing a single sample in probabilistic estimates of its proximity to the true mean is the essence of sampling.

Sampling affords researchers the ability to estimate the true mean and standard deviation by understanding the probability of any one sample's likelihood of approximating the true sample mean. It does so through using the normal distribution to understand that any one sample has a 68.26 percent chance of its mean coming within one standard error of the true population mean, a 95.44 percent chance of coming within two standard errors, and a 97.44 percent chance of being within three standard errors. Chapter 7 applies these probabilities to sample data by determining the range of likely values for the true mean based on the single sample; these ranges are known as *confidence intervals*.



CHAPTER SEVEN

CONFIDENCE INTERVALS AND BASIC HYPOTHESIS TESTING

The discussion in chapter 6 made use of population parameters, such as the population mean (μ) and the population standard deviation (σ) . However, researchers rarely, if ever, are actually in possession of such information. Probabilistic determinations of the true mean were made using sample sizes and the true population standard deviation, and therein lies a hole in sampling theory. The procedure discussed for estimating *m* from one sample requires that σ be known, and if μ is not known, then σ will also be unknown.

Equation 6.4 can be adapted to best accommodate the results of a single sample through the process of substitution. In the absence of σ , the researcher can substitute the standard deviation from a prior comparable study, if available, or, in the absence of a prior comparable study, the standard deviation from the single sample (*s*) for the population standard deviation (σ). The standard deviation of the prior study or of the sample itself is used as the best available estimate of σ since it is known probabilistically that it is likely that the sample standard deviation will be close to the true population standard deviation, and since any differences are mitigated, in large part, by the fact that the standard deviation and the sample standard deviation. Equation 6.4 can be expressed in its single-sample form as follows:

$$s_{\bar{x}} = \frac{s}{\sqrt{n}} \tag{7.1}$$

where

- n = sample size (number of cases in a single sample)
- $s_{\bar{x}}$ = standard error of the mean for the single sample
- s = sample standard deviation

By mere chance alone, some difference between a sample and the population from which it is drawn must always be expected to exist. In all likelihood, the population mean (μ) will rarely be the same as the sample mean, and the population standard deviation (σ) is highly unlikely to be the same as the sample standard deviation (σ) . These differences are known as *sampling error*, and they can be expected to result regardless of how scientifically the sample has been selected and implemented. The sampling error that exists between the sample and its population can be formally depicted through the use of *confidence intervals*.

Confidence Intervals

Suppose that in a single sample of 400 university undergraduates, it is found that the mean sample grade point average (GPA) is 2.75 with a standard deviation (*s*) of .4. Applying equation 7.1, the standard error is obtained as follows:

$$s_{\bar{x}} = .4 / \sqrt{400} = .4 / 20 = .02$$

It is known that there is .6826 probability that the mean of any sample drawn from a population will be within one standard error of the true mean (figure 7.1). Hence, the researcher is 68.26 percent confident that μ is in the interval bounded by $\overline{x} \pm s_{\overline{x}}$.

In terms of the specific example:

There is 68.26 percent confidence that $2.75 - .02 \le \mu \le 2.75 + .02$. (one standard error).

There is 95.44 percent confidence that $2.75 - .04 \le \mu \le 2.75 + .04$. (two standard errors).

There is 99.74 percent confidence that $2.75 - .06 \le \mu \le 2.75 + .06$. (three standard errors).

The researcher is 68.26 percent confident that with a sample mean of 2.75 and a sample standard error of 0.02, the true population mean GPA lies between 2.73 and 2.77. There is 95.44 percent confidence that the true

population mean is found within the interval bounded by 2.71 and 2.79 and 99.74 percent confidence that it is between 2.69 and 2.81.

In most scientific investigations, a confidence level of 68.26 percent is not satisfactory. It is common for a researcher to seek a 95 percent or 99 percent level of confidence. The choice of a confidence level is often a tradeoff among economy, precision, and risk of error. (The factors associated with the choice of a confidence level are discussed in chapters 8 and 9.) Referring to figure 7.1 and resource A, it can be shown that a standardized Z score of \pm 2.575 encompasses approximately 99 percent of all cases and that a standardized Z of \pm 1.96 includes 95 percent of the cases. Thus, the 95 percent and 99 percent confidence intervals take on the following configurations:

$$(95\%)\overline{x} - 1.96_{s_{\overline{x}}} \le \mu \le \overline{x} + 1.96_{s_{\overline{x}}}$$
(7.2)

$$(99\%)\overline{x} - 2.575_{s_{\overline{z}}} \le \mu \le \overline{x} + 2.575_{s_{\overline{z}}}$$
 (7.3)

The researcher can be 95 percent confident that μ is located in the range expressed by $\bar{x} \pm 1.96s_{\bar{x}}$ and 99 percent confident that μ is found within $\bar{x} \pm 2.575s_{\bar{x}}$.

In this example of undergraduate GPAs, we can be 95 percent confident, based on the single sample, that the true mean undergraduate grade point average is between 2.711 and 2.789 because 95 percent is the probability that the sample mean will be within 1.96 standard errors of the true population mean. Similarly, we can be 99 percent confident that the true mean is between 2.699 and 2.801.

Hence, through the use of confidence intervals, the researcher is able to determine that the true population mean can be estimated within a fixed interval range based on one sample mean. Notice should be taken that for any given sample size, the more rigorous the level of confidence that is demanded, the more broadly delineated the confidence interval must be. By broadening the confidence interval, the researcher can mitigate the risk of making an error in generalizing from the sample to the population at large.

Confidence Intervals Expressed as Proportions

Readers are more likely to be familiar with the concept of confidence intervals in the context of percentages (proportions). Almost everyone has been exposed to political public opinion polls. In general, the results, when given in terms of proportions, can be expressed as follows:

$$(95\%) p = \bar{p} \pm 1.96 \ \sigma_{\bar{p}} \tag{7.4}$$

$$(99\%) p = \bar{p} \pm 2.75 \ \sigma_{\bar{p}} \tag{7.5}$$

where

p = true population proportion

 \overline{p} = sample proportion

 $s_{\overline{p}}$ and the standard error of the population proportion is estimated by the sample mean proportion $(s_{\overline{p}})$ as follows:

$$s_{\overline{p}} = \sqrt{\frac{\overline{p}(1-\overline{p})}{n}} \tag{7.6}$$

It is important to note the parallel between equation 7.6 (standard error for proportions) and equation 7.1. The equation for the standard deviation of a proportional distribution is $\sigma_p = \sqrt{p(1-p)}$. Just as with interval data, the equation for standard error requires the standard deviation to be divided by \sqrt{n} . Hence, the standard error for a distribution of sample proportions is $\sigma_p = \sqrt{(p(1-p))/n}$. Equation 7.6 represents the standard error for a single sample proportion, where \overline{p} is substituted for p in the same manner as s is substituted for σ in interval data.

The typical results of a public opinion poll might appear as follows:

Twelve hundred scientifically selected respondents were asked to state their preference between Senator Ramos and Governor Cooper for the office of president of the United States. The results of the survey are as follows:

Sen. Ramos	47 percent
Gov. Cooper	45 percent
Undecided	8 percent

The survey contains a margin of error of ± 3 percent.

Such reports rarely contain specific references to confidence intervals and confidence levels, but these results fit precisely into the interpretive context discussed above with regard to interval scale data. The true meaning of this survey finding is that the researcher is 95 percent confident that Ramos has between 44 percent and 50 percent of the vote (47 ± 3 percent) and that White has between 42 percent and 48 percent. This is comparable to the interval scale example above, when the researcher was 95 percent confident that the mean GPA was between 2.711 and 2.789.

Let it be assumed that it is much later in the presidential campaign than when this poll was initially taken, and Senator Ramos would like to know where he stands now. He commissions a sample survey of 2,000 registered voters, which finds his support to be 52 percent. Encouraged by this finding but understanding the concept of sampling error, Senator Ramos would like to know the margin of error associated with this poll. In other words, can the campaign staff really be confident that he possesses majority support? Senator Ramos has asked his statistician to respond to that question at a confidence level of 99 percent. Equations 7.5 and 7.6 yield the following result:

$$p = \overline{p} \pm 2.575_{s_{\overline{p}}}$$
$$= .52 \pm 2.575 \left(\sqrt{\frac{(.52)(.48)}{2,000}} \right)$$
$$= .52 \pm 2.575 (.011)$$
$$= .52 \pm .028$$

Thus, Senator Ramos can be 99 percent certain that his campaign has between 49.2 and 54.8 percent of the vote. He may be somewhat disappointed that majority support cannot be claimed with 99 percent confidence, but it is better not to be misled by the poll that indicated 52 percent support than it would be to proceed as if majority support were certain. The reader should verify that the 95 percent confidence interval is $.52 \pm .022$, which might alarm Senator Ramos in that it does not seem to indicate that he has even 95 percent confidence of a majority, but this conclusion is premature and will be called into question in the "Single- and Dual-Direction Research Questions" section of this chapter.

USING SPSS TO GENERATE CONFIDENCE INTERVALS

As we mentioned in the SPSS section of chapter 5, Analyze—Descriptive Statistics— Explore is especially valuable in having SPSS determine confidence intervals. A researcher who is accessing Transportation.sav to find the 95% confidence interval for total travel time on all buses, including waiting time, should follow these steps:

- 1. Access Analyze—Descriptive Statistics—Explore.
- 2. Insert travtime_total in the Dependent List box.
- 3. Inasmuch as there is no reason to ask for the Histogram, the default button labeled Both at the bottom can be changed to Statistics in order to reduce extraneous output.¹ Click on Continue.
- 4. In Options, again change Listwise to Pairwise. Click on Continue.
- 5. Click on OK.
- 6. Output should be as below.²

DESCRIPTIVES				
			Statistic	Std. Error
	Mean		1.6724	.02918
	95% confidence interval	Lower bound	1.6152	
	for mean	Upper bound	1.7297	
	5% trimmed mean		1.5361	
Travel time in hours	Median		1.5000	
	Variance		1.576	
	Std. deviation		1.25545	
	Minimum		.07	
	Maximum		10.00	
	Range		9.93	
	Interquartile range		1.17	
	Skewness		2.073	.057
	Kurtosis		6.227	.114

It can be seen that the 95% confidence interval is 1.6152 hours to 1.7297 hours. The 99% confidence interval is accomplished by changing the default 95% in Statistics— Descriptives to 99%, revealing a 99% confidence interval of 1.5972 hours to 1.7477 hours.

Hypothesis Testing

The question Senator Ramos posed is more commonly known as *hypothesis testing* based on a sample. The sampling error associated with any sample is used to determine if the sample permits certain conclusions to be drawn from a sample against some standard or hypothetical objective under conditions of 95 percent and 99 percent confidence. That is, a confidence interval identifies the range of the likely location of the true mean and can be used in many cases to answer a question such as the one Senator Ramos asked. However, there is a variation on confidence intervals that is better able to address the different types of hypotheses than can confidence intervals. Hypothesis testing is that variation.

Recall the example in chapter 6 in which the mean weight of a 400person sample of military personnel was reported as 170 pounds, with a standard deviation of 15 pounds. If the Department of Defense had established a mean weight of 165 pounds as indicative of a healthy military population (established standard mean weight), is a researcher 95 percent confident that the population sampled could be characterized healthy or unhealthy because of its sample mean of 170 pounds?

Rather than determining a confidence interval, a researcher can calculate the number of standard errors that separate the sample mean from the hypothesized or standard mean. That is done by determining the number of standardized Z units between the two means. The formula for Z would be as follows:

$$Z = \frac{\overline{x} - \mu}{s_{\overline{x}}} \tag{7.7}$$

where

 \overline{x} = sample mean

 μ = established standard mean

 $s_{\overline{x}}$ = standard error of the sample mean

$$\frac{s}{\sqrt{n}}$$

where

s = sample standard deviation and

n = sample size.

Confidence Intervals and Basic Hypothesis Testing

The denominator s / \sqrt{n} is the standard error of the sample mean.

This Z test permits the researcher to assess the health of the military personnel more easily than with confidence intervals. First, the researcher poses a *null hypothesis*, as follows: Null Hypothesis (H_0): There is no difference between the sample mean and the standard, which means that the population is healthy. Second, the researcher applies equation 7.7:

$$Z = \frac{\overline{x} - \mu}{s_{\overline{x}}} = \frac{\overline{x} - \mu}{s / \sqrt{n}}$$
$$= \frac{170 - 165}{15 / \sqrt{400}}$$
$$= \frac{5}{0.75}$$
$$= 6.67$$

Third, the calculated Z of 6.67 must be compared to the two "critical" Z scores that represent 95 percent and 99 percent confidence: 1.96 and 2.575. The comparison is made in absolute value terms in that calculated Z scores with absolute values that equal or exceed these critical values indicate that the null hypothesis can be rejected and that the sample and standard are not the same. This indicates that the sample population is statistically different from a standard healthy population and therefore is unhealthy. This difference is also known as *statistical significance*. Statistical significance indicates a sample finding that is sufficiently different from the hypothesized result that sampling error is not able to explain the difference. Under such circumstances, the apparent result can be applied to the population that the sample represents (in this case, military recruits). That is, the sample finding that sampled military recruits do not meet the Department of Defense standard can be "generalized" to all military recruits if the sample has been drawn appropriately (see chapter 9). Chapters 10 and 11 explore additional tests of statistical significance in greater detail.

There is also a form of this test regarding standards and hypothesized true populations that applies to data generated in the form of proportions. As in the case of the interval data, a *Z* score is calculated and compared to the appropriate critical *Z* in order to determine if the null hypothesis

can be rejected and thereby ascertain a statistically significant finding. The equation for proportions is as follows:

$$Z = \frac{\overline{p} - p}{\sqrt{\frac{\overline{p}(1 - \overline{p})}{n}}} \tag{7.8}$$

where

 \overline{p} = sample population mean proportion

p = established standard proportion

n = sample size

To illustrate further, suppose that a sample of 150 faculty members at a midsized liberal arts college in Arkansas was polled to find out if the faculty who teach nonquantitative courses incorporate a critical writing component in their courses. The State Accreditation Board indicates that writing should be a component in approximately 50 percent of all nonquantitative courses. The survey at the liberal arts college indicated that 90 of the 150 faculty members (60 percent) in the sample have a writing component in their classes. Is this finding indicative of a lack of balance in the curriculum (95 percent confidence)? That may appear to be the case, but sampling error must be ruled out. Equation 7.8 can be applied to this situation as follows:

$$Z = \frac{0.60 - 0.50}{\sqrt{\frac{0.60 - (0.40)}{150}}} = \frac{.1}{\sqrt{0.0016}} = 2.50$$

Because the calculated Z of 2.50 exceeds the critical Z of 1.96 (95 percent confidence), the researcher can conclude with 95 percent confidence that the college is out of balance with the state's accreditation board. However, were 99 percent confidence required (2.575), that same conclusion could not be drawn.

Single- and Dual-Direction Research Questions

Generally a research question is posed not in terms of an overall difference between the sample and a hypothesized finding (as were the above questions about lack of balance and unhealthy recruits) but rather in terms of a single direction of difference. In the case at hand, instead of posing the research question in the context of general differences between the college and the state standard, the researcher is much more likely to examine the survey results and hypothesize that the college teaches too much ethics and may be shortchanging other disciplines.

A single-direction research hypothesis such as this one requires a reconfiguration of the confidence interval. When the question is one of difference only, the researcher must allow the possibility of sampling error in both directions. That is, a researcher who is testing to determine if there is a significant mean weight difference between military recruits and the standard can reach a conclusion of significant difference if recruits' mean weight is either significantly less than or greater than the standard. Since the researcher can err in either direction, he or she must allocate the possibility of error evenly between these two directions. Figure 7.1 illustrates this allocation of error for the 95 percent confidence level where the confidence interval is formed at 1.96 standard errors from the mean, with the 5 percent error divided equally (2.5 percent in either tail of the diagram).

When the research question is oriented in only one direction, the 95 percent confidence interval shifts totally toward one side of the distribution. Inasmuch as the question is posed in one direction only, the error can be made in one direction only. Therefore, the entire 5 percent is allocated to one tail at 1.645 standard errors from the mean. Figure 7.2 illustrates







FIGURE 7.2. SINGLE-DIRECTION RESEARCH QUESTION (ONE-TAIL TEST AT 95 PERCENT CONFIDENCE LEVEL).

the confidence interval in the case of a single-direction, 95 percent level of confidence.

Statistical significance for the single-direction hypothesis test therefore can be established at a lower absolute value of Z, but the researcher must be careful to structure the question in a manner that reflects the results of the data. For example, the researcher in the military weight example must pose the question in either of the following formats:

- Are recruits overweight? Look only for a positive *Z* score.
- Are recruits underweight? Look only for a negative Z score.

With sample data indicating a mean weight of 170 pounds, in contrast to the standard of 165, the appropriate *research hypothesis* is that the recruits are overweight and the null hypothesis is that their weight is not different from the standard—that is, they are not overweight.

Hence for the military recruits, a calculated Z of 6.67 is found to be greater than the single-direction critical Z of 1.645, the null hypothesis can be rejected, and the total population of military recruits can be said to be overweight on average. In the dual-direction situation, critical Z is found at a higher absolute value, and because there is an interest in difference only, the structure of the question need not address which group has a higher or lower measure than the other.

The single-direction critical Z of 1.645 has been derived from resource A (Table of Areas of a Standard Normal Distribution). You should verify that a Z of 1.645 indicates that 95 percent of the data is on one side

of it and 5 percent on the other. For 99 percent confidence levels, the reader should further verify that the single-direction critical Z equals 2.33. Single-direction questions are generally referred to as *one-tail questions* and dual-direction questions as *two-tail questions*.

As such, whereas we could not be 99 percent confident that the college in Arkansas was out of balance with the Accreditation Board's standards (either too many critical writing courses or too few), the Z score of 2.50 is sufficient to declare with 95 percent confidence that this college spends an overabundance (in contrast to either an overabundance or underabundance) of time in this line of instruction according to state standards.

For Senator Ramos's question about a majority, the question really is, "Does Senator Ramos have more than 50 percent support?" Using a confidence interval did not account for the one-tail nature of the question. Using hypothesis testing, we can use a one-tail application of equation 7.8 as follows:

$$Z = \frac{.52 - .50}{\sqrt{\frac{.52(.48)}{2000}}} = \frac{.02}{\sqrt{.000125}} = 1.789$$

Senator Ramos is 95 percent confident of a majority ($Z \ge 1.645$), but he has not achieved the 99 percent level of confidence that he desired ($Z \le 2.33$).

In sum, statistical significance in hypothesis testing can be established when calculated *Z* equals or exceeds the critical *Z* scores in exhibit 7.1.

TO RESEARCH HYPOTHESIS DIRECTION).			
Level of Confidence	Two Directions (Two Tail)	One Direction (One Tail)	
95 percent			
Positive Z score	Right of 1.96	Right of 1.645	
Negative Z score	Left of 1.96	Left of -1.645	
99 percent			
Positive Z score	Right of 2.575	Right of 2.33	
Negative Z score	Left of 2.575	Left of -2.33	

EXHIBIT 7.1 VALUES OF CALCULATED Z SCORES TO DETERMINE SIGNIFICANCE (ACCORDING TO RESEARCH HYPOTHESIS DIRECTION).

The t Test

Recall from chapter 6 that the properties of the normal distribution assume that the sample sizes in the distribution of sample means are sufficiently large—120 or more. When the sample size is fewer than 120, as might be the case, for instance, in a random sample of city departmental directors or a random sample of teachers from public schools in one neighborhood, the application of the properties of the normal distribution is problematic. Hence, *Z* scores are not applicable for such small samples. The *t* distribution, sometimes called Student's *t*, is an adjustment to the normal distribution to account for small sample size, and it should be used in place of *Z* when the sample size is small (fewer than 120).

In hypothesis testing, when the sample size is below 120, the equation used for determining a calculated Z is applied in almost the same format to produce a calculated t, with the only change being the substitution of n-1 for n in the denominator.

$$t = \frac{\overline{p} - p}{\sqrt{\frac{\overline{p}(1 - \overline{p})}{n - 1}}}$$
(7.9)

The calculated *t* is then compared to the table of critical values of *t* in exhibit 7.2. The level of confidence, directional properties of the question, and degrees of freedom must be known in order to find critical *t*. Degrees of freedom is a statistical concept that most often represents a derivative of *n*. It can be explained as the number of individual data that are free to vary when only the sum of all data is known. That is, knowing the total number of years of education among parties to a conference of thirty-five people, how many of those individual years of education are free to vary until all others become known? The correct answer is 34, or n - 1. In the case of hypothesis testing, therefore, degrees of freedom equal n - 1. Note that when the degrees of freedom exceed 120, the critical *t* and critical *Z* effectively coincide. Level of confidence is determined as before in conjunction with the particular research requirements, and the direction of the research question must be determined from the data and the research objectives.

As with Z, if calculated t equals or exceeds the one- or two-tail critical t (whichever is applicable), the researcher can conclude that differences between the groups are statistically significant or genuine.

To illustrate the use of the *t* distribution, suppose that 45 percent of a sample of fifty-five teachers at a local high school have indicated that they
are dissatisfied with the administration of their school. The school district has established that when 30 percent or more of the teachers in a school vote their disapproval of the administration, the school district is required to embark on a costly review and evaluation of the administrative staff at that school. Can it be concluded that the percentage of teachers who feel dissatisfied is high enough to require the district to embark upon the investigation? That may appear to be the case, but as always, sampling error must be ruled out by applying the appropriate test of statistical significance. Equation 7.9 can be applied to this situation as follows:

$$t = \frac{0.45 - 0.30}{\sqrt{\frac{0.45(0.55)}{54}}}$$
$$= \frac{0.15}{\sqrt{0.0046}} = \frac{0.15}{0.068} = 2.21$$

	One-Tail		Two	-Tail
df	t _{.os} (95 percent confidence)	t _{.01} (99 percent confidence)	t _{.05} (95 percent confidence)	t _{.01} (99 percent confidence)
1	6.314	31.821	12.706	63.657
2	2.920	6.965	4.303	9.925
3	2.353	4.541	3.182	5.841
4	2.132	3.747	2.776	4.604
5	2.015	3.365	2.571	4.032
6	1.943	3.143	2.447	3.707
7	1.895	2.998	2.365	3.499
8	1.860	2.896	2.306	3.355
9	1.833	2.821	2.262	3.250
10	1.812	2.764	2.228	3.169
11	1.796	2.718	2.201	3.106
12	1.782	2.681	2.179	3.055
13	1.771	2.650	2.160	3.012
				(continued)

EXHIBIT 7.2. CRITICAL VALUES OF THE *t* DISTRIBUTION.

_	One	-Tail	Two-Tail	
df	t _{.05} (95 percent confidence)	t _{_01} (99 percent confidence)	t _{.05} (95 percent confidence)	t _{_01} (99 percent confidence)
14	1.761	2.624	2.145	2.977
15	1.753	2.602	2.131	2.947
16	1.746	2.583	2.120	2.921
17	1.740	2.567	2.110	2.898
18	1.734	2.552	2.101	2.878
19	1.729	2.539	2.093	2.861
20	1.725	2.528	2.086	2.845
21	1.721	2.518	2.080	2.831
22	1.717	2.508	2.074	2.819
23	1.714	2.500	2.069	2.807
24	1.711	2.492	2.064	2.797
25	1.708	2.485	2.060	2.787
26	1.706	2.479	2.056	2.779
27	1.703	2.473	2.052	2.771
28	1.701	2.467	2.048	2.763
29	1.699	2.462	2.045	2.756
30	1.697	2.457	2.042	2.750
40	1.684	2.423	2.021	2.704
60	1.671	2.390	2.000	2.660
120	1.658	2.358	1.980	2.617
∞	1.645	2.330	1.9860	2.575

Because the calculated t of 2.21 exceeds the one-tail, critical t of 1.676 (95 percent confidence, interpolated between 40 and 60 degrees of freedom), the researcher can conclude with 95 percent confidence that the district should begin to implement the investigation. Were 99 percent confidence required, that same conclusion could not be drawn (critical t = 2.402).

For interval data, the *t* test equation is

$$t = \frac{\overline{x} - \mu}{\frac{s}{\sqrt{n-1}}} \tag{7.10}$$

USING SPSS FOR HYPOTHESIS (ONE-SAMPLE) TESTING

SPSS can test hypotheses related to confidence intervals for interval data. For example, in the case of travel time in the Transportation.sav file, let it be hypothesized that if the average travel time for bus passengers exceeds 1.625 hours, the system needs to be improved. As was shown previously using Explore, the mean travel time is 1.6724 hours; therefore, it may well be that the transportation provider is facing an expensive improvement program.

The reader has already seen that confidence intervals are 1.6152 to 1.7297 hours at 95% confidence and 1.5972 to 1.7477 hours at 99% confidence, which might lead to the conclusion that the system is not facing this expenditure because 1.625 hours is within these confidence intervals. It is important to note, however, that, as is the case under most such hypotheses, this is a one-tail question that is more appropriately answered using hypothesis testing.

The steps necessary for SPSS to make the calculation are as follows:

- 1. Go to Analyze—Compare Means—One-Sample T Test.
- 2. Move the variable travtime_total into the Test Variable(s) box.
- 3. Look at Options and note that 95% confidence is the default and that the Missing Values are not Listwise (which can be left as is for both).
- 4. Set the Test Value at the hypothesized time that will trigger the improvement program and press OK.

	Test Variable(s):	Options
ID day day superdist Language satis_overall_q1 change_service	travtime_total	Bootstrap
satis_cost_q3a	Test <u>V</u> alue: 1.625	

Output is as below, showing the mean of 1.6724, t = 1.626 (with n = 1,851, t and Z are equal), which would imply a critical Z of 1.645 and a conclusion that the mean of 1.6724 does not exceed the standard of 1.625 sufficiently for the researcher to be 95% confident that a new program is necessary.

	N	Mean	Std. Deviation	Std. Error Mean
Travel time in hours	1,851	1.6724	1.25545	.02918

One-Sample Statistics

One-Sample Test

	Test Value = 1.625					
			Sia	Mean Difference	95% Confide of the D	ence Interval ifference
	t	df	(2-tailed)		Lower	Upper
Travel time in hours	1.626	1,850	.104	.04744	0098	.1047

It is not even necessary to compare the calculated t to a critical t or Z because SPSS has already done that. The Sig (2-tailed) indicates a confidence of 89.6% (1 – Sig.). SPSS uses a two-tail default. As such, the 95% two-tail confidence level leaves 5.2% error in each tail. A one-tail test requires that less than 5% be in the tail that is in the direction of the question. As such, the significance must be less than .100 and the sign of the t in this case must be positive. The value of t is positive, but the significance exceeds .10; therefore, the conclusion is that the researcher is not 95% confident that the travel time exceeds the standard and the necessity for the improvement program has not been proven.

The reader might wish to verify that were the point of requiring the improvement program to be implemented at 1.62 hours instead of 1.625 hours, t would be 1.797, with a significance of .072. In that case, the positive t and significance under .100 indicates that there is 95% confidence that the program is necessary.

In the event that the research question is a two-tailed question, the significance that is required is .05 and the sign of the t is of no consequence.

Notes

- 1. This is not absolutely necessary. The default can be left as is and the graphical output simply ignored.
- 2. It might be of interest to examine these statistics and notice that this is much more likely to be a positively skewed distribution than was the example of age in chapter 5.



CHAPTER EIGHT

DETERMINING THE SAMPLE SIZE

A crucial question at the outset of a survey research project is how many observations are needed in a sample so that the generalizations discussed in chapter 7 can be made about the entire population. The answer to this question is by no means clear-cut; it requires the careful consideration of several major factors. Generally the greater the level of accuracy desired and the more certain the researcher would like to be about the inferences to be made from the sample to the entire population, the larger the sample size must be.

Determinants of Sampling Accuracy

Researchers must address two interrelated factors with specificity before proceeding with the selection of a sample size: *level of confidence* and *confidence interval*. The level of confidence is the risk of error the researcher is willing to accept in the study. Given time requirements, budget, and the magnitude of the consequences of drawing incorrect conclusions from the sample, the researcher will typically choose either a 95 percent level of confidence (1 percent chance of error) or a 99 percent level of confidence interval determines the level of sampling accuracy that the researcher obtains. In this chapter, it will be shown that selection of the sample size is a primary contributor to the researcher's success in achieving a certain degree

of sampling accuracy. In other words, sample size is directly related to the accuracy of the sample mean as an estimate of the true population mean.

Recall that the equations for the standard error for samples containing interval scale variables (equation 7.1) or proportions (equation 7.6) included a factor (n) representing sample size. For any given sample standard deviation, the larger the sample size is, the smaller is the standard error. Conversely, the smaller the sample size is, the larger is the standard error. For instance, if a sample of 100 respondents indicates a mean income of \$20,000 per year with a sample standard deviation of \$3,000, the standard error and associated confidence intervals, with 95 percent or 99 percent levels of confidence, would be calculated as

$$s_{\bar{x}} = \frac{s}{\sqrt{n}} = \frac{3,000}{\sqrt{100}} = \frac{3,000}{10} = \$300$$

95 Percent Confidence Interval	99 Percent Confidence Interva
$\overline{x} \pm 1.96 \mathrm{s}_{\overline{x}}$	$\overline{x} \pm 2.575 \mathrm{s}_{\overline{x}}$
$20,000 \pm 1.96 (300)$	$20,000 \pm 2.575(300)$
$20,000 \pm 588$	$20,000 \pm 773$

Hence, the researcher can be 95 percent certain that the true mean income for the population is between \$19,412 and \$20,588 or 99 percent certain that the true mean is between \$19,227 and \$20,773. To phrase it another way, the researcher will have achieved a *margin of error* of \pm \$588 with 95 percent confidence and \pm \$773 with 99 percent confidence.

If the available sample contains 400 responses, the standard error and associated confidence intervals would be calculated as

$$s_{\bar{x}} = \frac{\$3,000}{\sqrt{400}} = \$150$$

95 Percent Confidence Interval	99 Percent Confidence Interval
\$20,000 ± 1.96(\$150)	\$20,000 ± 2.575(\$150)
\$20,000 ± \$294	\$20,000 ± \$386

Therefore, with a sample size of 400 rather than 100, the researcher has been able to narrow the margin of error by 50 percent for each level

of confidence, respectively, but this 50 percent narrowing has required that the sample size be quadrupled. If the sample size were to be increased to 1,000, the 250 percent increase in sample size from 400 would reduce the margin of error by only an additional 37 percent. It is noteworthy that such reductions in confidence intervals can in fact be achieved, but at the potentially high cost of a substantially larger sample size.

In terms of proportions, a sample of approximately 1,050 persons provides 95 percent confidence of a margin of error of ± 3 percent but 1,850 respondents for 99 percent confidence (more than a 75 percent increase in sample size).

The process of selecting a sample size requires that the researcher determine an acceptable range of uncertainty or margin of error, given the time and cost constraints of the study. In the example, for instance, a researcher who determines that the study can tolerate no more than a \$300 margin of error in either direction from the sample mean would not be satisfied by the 100-person sample at either level of confidence but would be served by the 400-person survey with 95 percent confidence only. In the case of the latter example, the researcher must decide whether increasing the cost and time involved in the study would be justified by an increase in confidence from 95 to 99 percent.

There are no fixed criteria by which to make this choice. The researcher must make this determination on a case-by-case basis and in accordance with the goals and objectives of the study. This interrelationship among level of confidence, margin of error, and the effect of sample size on these parameters makes the determination of sample size an absolutely vital component of the sample survey process.

The researcher should consider the following guidelines in the selection of sample size:

- The researcher will generally find a margin of error of ±3 to ±5 percent to be satisfactory for proportional data. Interval data margins of error must be established on a case-by-case basis, depending on the unit of measurement, magnitude, and range of the particular variable.
- The greater the consequences are of generating data that might lead to incorrect conclusions, the greater the level of confidence the researcher should establish. In practical terms, this is a choice between the 95 and 99 percent levels of confidence.
- In most cases, the researcher can be satisfied by choosing the 95 percent confidence level, which implies a 5 percent risk that the confidence interval is incorrect.

This choice between 95 percent and 99 percent confidence levels involves a variety of factors, including cost, time, and difficulty of achieving required sample sizes. Assuming, however, that no such constraints exist, one might ask why 99 percent should not be the optimal confidence level choice. To address this question, the concepts of Type I and Type II errors must be discussed.

A *Type I error* is the error associated with making a decision based on the data from the sample. It is the error that this book has discussed as the confidence level. The other form of error associated with sample data is known as a *Type II error*; it derives from being overly conservative and not acting on the results, only to find out later that the researcher should have acted.

To illustrate, suppose that a researcher wanted to administer a new serum to 2,000 randomly selected individuals suffering from a communicable disease. Also suppose that this researcher requires a 99 percent confidence level to conclude that the serum is beneficial and to recommend its mass production and distribution (Type I error = 1 percent). The results of the research showed that there was only 95 percent confidence that patients benefited from the serum; consequently, the researcher could not conclude with 99 percent confidence that the serum should be marketed, and its development was not pursued.

Over the ensuing years, thousands of individuals continued to die of the disease. Eventually another researcher decides to repeat the test on another sample population, this time using a decision rule of 95 percent confidence (Type I error = 5 percent). The results again showed a 95 confidence, but under the new confidence level, this researcher could advocate production of the serum.

Deaths from the disease dropped dramatically over the next few years, demonstrating that a Type II error (not acting on the data after the first experiment) had been made by being overly conservative in setting the original confidence level at 99 percent.

It is now fairly well accepted in the research community that in most instances, the 95 percent level of confidence represents a reasonable balance between the risks associated with Type I and Type II errors.

Determination of Sample Size for Variables Expressed in Terms of Proportions

Determination of sample size for data given in terms of proportions is somewhat more straightforward than when the variable is on an interval scale. Hence, we first introduce this methodology. Determining the Sample Size

Large Populations

The relationship among the confidence interval, the level of confidence, and the standard error of sample proportions can be expressed by the following equation:

$$ME_p = \pm Z_a(\sigma_p) \tag{8.1}$$

where

 $ME_{h} =$ margin of error in terms of proportions

 $Z_{a} = Z$ score for various levels of confidence (α)

 $\sigma_{\rm p} = {\rm standard\ error\ for\ a\ distribution\ of\ sample\ proportions}$

The formula for the standard error of the true population proportion is $\sigma_p = \sqrt{(p(1-p))/n}$. Substituting it into equation 8.1, we can rewrite the equation as follows:

$$ME_p = \pm Z_a \sqrt{\frac{p(1-p)}{n}}$$
(8.2)

Solving for n yields

$$n = \left(\frac{Z_a \sqrt{p(1-p)}}{ME_p}\right)^2 \tag{8.3}$$

To proceed with the calculation of specific sample sizes (*n*), the values of Z_a , ME_p , and *p* must be established. As discussed, Z_a is most commonly set at 1.96 for the 95 percent level of confidence or 2.575 for 99 percent. The margin of error ME_p is typically set not to exceed 10 percent and is much more frequently set in the 3 to 5 percent range, depending on the specific degree of accuracy to which the findings must conform. As we have seen, the true proportion (*p*) is unknown, so we have been using the sample proportion, which is unknown prior to actually conducting the survey. The most conservative way of handling this uncertainty for purposes of setting a sample size is to set the value of *p* at the proportion that would result in the highest sample size. This occurs when *p* = .5; equation 8.3 therefore can be further refined:

$$n = \left(\frac{Z_a(.5)}{ME_p}\right)^2 \tag{8.4}$$

because $\sqrt{.5(1-.5)} = .5$.

Now suppose that a government decision maker is in the process of determining an appropriate sample size for a study of public opinion concerning community service system adequacy. The question to be posed is whether the respondents find community services to be adequate. Percentages responding yes and no are to be tallied and presented for review. For purposes of this study, the decision maker feels that it is important for the sample proportion to be accurate within ± 4 percent of the true proportion, and it is felt that 95 percent confidence in these findings would be satisfactory in order for the information to be effectively used. To obtain the appropriate sample size for this study, the researcher can substitute numbers into equation 8.4 as follows:

$$n = \left[\frac{(1.96)(.5)}{.04}\right]^2 = 600.25$$

The calculated *n* must be rounded to the next highest whole number, so in this case, a sample size of 601 persons is required.

Keep in mind that $Z_a = 2.575$ can be substituted for $Z_a = 1.96$ when 99 percent confidence is required and that the confidence interval (ME_p) also can be varied according to the researcher's requirement for sampling accuracy.

For each level of confidence (95 or 99 percent), required sample sizes can be calculated for various confidence intervals in terms of proportions by operationalizing equation 8.4. Table 8.1 portrays the calculated required sample sizes under these conditions.

It should be noted, once again, that there is an important inverse relationship between the sample size and the standard error, as manifested in the margin of error. To narrow the margin of error, an increase in the sample size is required—but that increase can quickly become prohibitively expensive.

Small Populations

As discussed in chapter 6, sampling theory and the equations derived from it assume a large population size. Therefore, the assumption underlying the calculation of sample sizes in table 8.1 is that the general population from which the sample or samples are taken is large. If, however, the population is not large, the standard error must be recomputed with the finite

	Sample Size			
Margin of Error	95% Confidence	99% Confidence		
±1	9,604	16,577		
± 2	2,401	4,145		
± 3	1,068	1,842		
± 4	601	1,037		
± 5	385	664		
± 6	267	461		
±7	196	339		
± 8	151	260		
± 9	119	205		
±10	97	166		

TABLE 8.1. MINIMUM SAMPLE SIZES FOR VARIABLES EXPRESSED AS PROPORTIONS.

population correction included. The formula for sample size in this case becomes

$$n = \left(\frac{Z_a \sqrt{p(1-p)}}{ME_p} C_v \sqrt{\frac{N-n}{N-1}}\right)^2$$
(8.5)

Having introduced a factor for n on each side of the equation, we must solve for n again. This yields

$$n = \frac{Z_a^2 [p(1-p)]N}{Z_a^2 [p(1-p)] + (N-1)ME_p^2}$$
(8.6)

When p is replaced with .5, as discussed previously, the general equation for sample size in all populations, large and small, becomes

$$\frac{Z_a^2(.25)(N)}{Z_a^2(.25) + (N-1)ME_p^2}$$
(8.7)

In practice, since the finite population correction approaches 1 in large populations, this adjusted sample size determinant is used only when populations are not large, and we continue to use equation 8.4 and table 8.1 when the population is large.

The distinction between "large" and "not large" will be addressed shortly, but prior to doing so, an example of the required sample size from a small population is in order. If a researcher seeks to determine the political party preferences of the 2,500 professors at a large state university and does not have the time or financial resources to interview them all, a sample of professors can be taken. The researcher must decide how many professors to survey and establishes that a 95 percent level of confidence will be satisfactory along with a margin of error that does not exceed ± 3 percent. Applying equation 8.7, the following is obtained:

$$n = \frac{(1.96)^2 (.25)(2,500)}{(1.96)^2 (.25) + 2,499(.03)^2}$$
$$= \frac{(3.8416)(.25)(2,500)}{(3.8416)(.25) + 2,4999(.0009)}$$
$$= \frac{2401}{.9604 + 2.249}$$
$$= 749$$

If the researcher wished to be 99 percent confident of the ± 3 percent margin of error, the following sample size would be required:

$$n = \frac{(2.575)^2 (.25)(2,500)}{(2.575)^2 (.25) + 2,999 (.03)^2}$$
$$= 1.061$$

Comparing these results to those in table 8.1 shows that in the case of this relatively small population, the researcher can obtain 95 percent confidence interviewing 749 professors instead of 1,068 in a large population, as indicated in table 8.1, and 99 percent confidence can be achieved with 1,061 interviews instead of 1,842. These differences arise from the logical fact that fewer interviews are needed from a very small population in order for that population to be adequately represented by the sample.

Table 8.2 reflects the application of equation 8.7 for various population sizes for the 95 percent and 99 percent levels of confidence and for margins of error of ± 3 percent, ± 5 percent, and ± 10 percent. Notice that as the population size (*N*) reaches 100,000, the required sample size approaches

those listed in table 8.1. Hence, a population size of 100,000 or greater can be considered large, and populations of fewer than 100,000 could be considered small. For particularly conservative confidence intervals and very small population sizes (designated by note a in table 8.2), the assumption of normality does not apply. Very small populations are most accurately characterized in terms of a hypergeometric distribution, an advanced concept not addressed in this book. In certain of these hypergeometric cases, equation 8.7, which is derived from properties of the normal distribution, does not yield appropriate sample sizes. In lieu of this equation, a sample size of 50 percent of the population has been determined to provide the required accuracy as long as the sample appears to be relatively representative of the general population (Yamane, 1967). In sum, the survey administrator will never require a sample size in excess of 50 percent of the total population that the sample represents. This rule has particular significance when it is necessary to undertake internal surveys of organizations with a small population base. Examples of such surveys are employee satisfaction polls and job performance evaluations. It must be established only that these very small samples do an adequate job of representing the full population base.

	Sample Sizes						
	95% Lo	95% Level of Confidence			99% Level of Confidence		
Population Size (<i>N</i>)	± 3%	± 5%	±10%	± 3%	\pm 5%	± 10%	
500	250ª	218	81	250ª	250ª	125	
1,000	500ª	278	88	500ª	399	143	
1,500	624	306	91	750ª	460	150	
2,000	696	323	92	959	498	154	
3,000	788	341	94	1,142	544	158	
5,000	880	357	95	1,347	586	161	
10,000	965	370	96	1,556	622	164	
20,000	1,014	377	96	1,687	642	165	
50,000	1,045	382	96	1,777	655	166	
100,000	1,058	383	96	1,809	659	166	

TABLE 8.2. MINIMUM SAMPLE SIZES FOR SELECTED SMALL POPULATIONS.

Note: The choice of ± 3 percent, ± 5 percent, and ± 10 percent for confidence intervals is based on the tendency of researchers to use these intervals or a similar range of intervals in the design of their surveys. ^aPopulation sizes for which the assumption of normality does not apply; in such cases, the appropriate sample size is 50 percent of the population size.

Determination of Sample Size for Interval Scale Variables

If some sample data are in the form of interval scale variables, adaptations to the equation are necessary.

Large Populations

For large populations, equation 8.1 must be adapted as follows:

$$ME_i = \pm Z_a \left(\sigma_{\bar{x}} \right) \tag{8.8}$$

where

 ME_i = confidence interval in terms of interval scale

 $Z_{a} = Z$ score for various levels of confidence (α)

 σ_{r} = standard error for a distribution of sample means

Using equation 6.4 ($\sigma_{\bar{x}} = \sigma / \sqrt{n}$) and substituting it into equation 8.8 results in the following equation:

$$ME_i = Z_a \left(\frac{\sigma}{\sqrt{n}}\right) \tag{8.9}$$

Solving for *n* yields

$$n = \frac{Z_a^2 \sigma^2}{M E_i^2} \tag{8.10}$$

To proceed with the calculation of specific sample sizes (n), the values of Z_a , ME_i , and σ must be established. As discussed, Z_a is most commonly set at 1.96 or 2.575; ME_i is generally set in the context of the variable under study, as the research study dictates, and σ is estimated from the sample data themselves by *s*, the standard deviation of the single sample, as discussed in chapter 6. Hence:

$$n = \frac{Z_a^2 s^2}{M E_i^2} \tag{8.11}$$

Suppose that a researcher is interested in obtaining a sample from a large population of households in the County of Rosebud to determine the mean household income. The goals are to select a sample size that will yield a margin of error of no more than \pm \$1,000 and to be 95 percent confident of this result.

A problem exists, however, that is similar to the one that was addressed with regard to proportions by setting p = .5 because the value for the sample standard deviation is unknown before the sample is conducted. Furthermore, there exists an additional difficulty for interval data from which proportional data are exempt. That is, there is no simple parameter to use in advance for *s* that would maximize the determination of *n* as there is with proportions (namely, p = .5). Since it is not at all likely that accurate information about the population parameters will be known before the completion of the survey, only a reasonable estimate of *s* can be made. There are two alternative methods of making this estimate:

- 1. The researcher may wish to use any previous information that is available (e.g., another survey of this population that can provide the necessary mean and standard deviation for a key variable).
- 2. A pilot survey or pretest (see chapter 2) conducted on the population will yield a standard deviation that can be used as estimates for the proposed sample.

Because a pretest should be conducted as a critical part of the survey process in any case, it is generally more feasible and efficient to use this method for the estimate of the sample standard deviation.

Based on a pretest of households in the County of Rosebud, a preliminary estimated mean of \$30,000 and standard deviation of \$6,000 are determined. The researcher may now use this information to operationalize equation 8.11 to yield the following sample size:¹

$$n = \frac{(1.96)^2 (6,000)^2}{(1,000)^2} = \frac{138,297,600}{1,000,000} = 139$$

Small Populations

As is the case when calculating sample sizes for variables in the form of proportions, an adjustment must be made to equation 8.11 when the general population is small (under 100,000). The finite population correction

regarding proportions, $\sqrt{(N-n)(N-1)}$, is also applicable to interval scale variables. Equation 8.11 becomes

$$n = \left(\frac{Z_a^2 s^2}{M E_i^2}\right) \left(\frac{N-n}{N-1}\right)$$
(8.12)

which can be written as

$$n = \frac{Z_a^2 s^2}{M E_i^2 + \frac{Z_a^2 s^2}{N - 1}}$$
(8.13)

In the example regarding household income in the County of Rosebud, suppose that the administrator wishes to research household income in only one small community within the county. This community has a population of 5,000. The researcher, as before, is interested in determining a sample size that will yield a margin of error of no more than \$1,000 and wishes to be 95 percent confident of the result. A pretest has estimated a mean of \$30,000 and a sample standard deviation of \$6,000. The researcher may now use this information to operationalize equation 8.13 to yield the following sample size:

$$n = \frac{(1.96)^2 (6,000)^2}{(1,000)^2 + \frac{(1.96)^2 (6,000)^2}{4,999}}$$
$$= \frac{(3.8416)(36,000,000)}{1,000,000 + \frac{(3.8416)(36,000,000)}{4,999}}$$
$$= \frac{138,300,000}{1,000,000 + 27,665.05}$$
$$= 135$$

Note that as with proportions from small populations, fewer respondents are required from the smaller community than from the entirety of the County of Rosebud to attain the same level of confidence and margin of error. Assuming the requisite characteristics of the normal distribution, this difference in sample size should be intuitively clear. Because interval

		Margin of Error	
		Proportion	Interval
Population Size	Large	Equation 8.4 or Table 8.1	Equation 8.11
	Small	Equation 8.7 or Table 8.2	Equation 8.13

FIGURE 8.1. DETERMINING SAMPLE SIZE.

scale variables tend to provide an enormous and unbounded range of possible values, tables of minimum sample size, such as tables 8.1 and 8.2 for proportions, are not feasible for the interval scale. Equations 8.11 and 8.13 must be applied in each case. Figure 8.1 summarizes these four distinct determinations of sample size.

Determination of Sample Size When Both Proportional and Interval Scale Variables Are Present

In most surveys, both proportional scale variables and interval scale variables are present. For example, the researcher is usually interested in determining the proportion of respondents who are identified as male or female and may also be interested in knowing what proportion of respondents intends to vote for a particular candidate. At the same time, he or she often wishes to obtain knowledge regarding respondents' household income, age, or years of formal education—variables that are expressed in the form of the interval scale. It may be difficult to ascertain which equation or equations to use in order to determine the appropriate sample size for such a survey. The researcher must make certain that the largest sample size requirement is satisfied. Hence, for all interval scale variables and proportional variables with varying margins of error or levels of confidence, the researcher must calculate the required sample size by using the appropriate equations or tables and establish a survey sample size equal to the largest required individual variable sample size.

Let's say that the researcher calculates that the age variable requires 300 respondents to satisfy the research needs, the income variable requires 350 respondents, and the proportional variables all require a total sample size of 385 respondents. The researcher would then conclude that at least 385

respondents must be secured and that this represents the overall survey sample size.

To avoid this repetitive and laborious process, a survey administrator in most instances will use the proportional variables to select a sample size with an overall margin of error and level of confidence. This sample size generally satisfies the most stringent requirement of the interval scale variables. Be aware, however, that if a particular interval scale variable is crucial to the focus of the survey, it would be prudent to make certain that the general sample size also meets the requirements for that particular variable.

Note

1. The interval scale sample size of 139 respondents may strike some people as a bit small in comparison with the sample sizes discussed in this chapter. A sample of 139 respondents would correspond to a margin of error of around ± 8 percent. The \$1,000 margin of error in this example would seem to be less than 8 percent based on the \$30,000 mean. Be aware, however, that the relationship of importance is not between the margin of error and the mean but, rather, between the margin of error and the standard deviation. In terms of proportions, the standard deviation is assumed to equal .5. The ratio of the margin of error can be assessed similarly, generating the same ratio of margin of error to standard deviation:

 $\frac{\text{margin of error}}{\text{standard deviation}} = \frac{1000}{6000} = .167$



CHAPTER NINE

SELECTING AND OBTAINING A REPRESENTATIVE SAMPLE

A lthough sample size is very important, it is by no means the only determinant of what constitutes adequacy of the sample. It is critical that the sample be selected according to well-established, specific principles. The purpose of this chapter is to discuss these principles and the methods of sample selection that have been adapted and derived from them.

Probability Sampling

Sampling methods can be categorized into probability sampling and nonprobability sampling. In *probability sampling*, the probability that any member of the working population will be selected to be part of the eventual sample is known. This implies extensive and thorough knowledge of the composition and size of the population. In *nonprobability sampling*, the selection process is not formal; knowledge of the population is limited, and hence the probability of selecting any given unit of the population cannot be determined.

Probability samples have these two characteristics:

- The probability of selection is equal for all members of the population at all stages of the selection process.
- Sampling is conducted with elements of the sample selected independently of one another (one at a time).

For example, consider a population that consists of 100 persons whose names are written on equal-sized pieces of paper and placed in a hat for selection. The pieces of paper are thoroughly mixed and selected one by one, without being seen, until the sample size is obtained. This is a probability sample. Of course, when a piece of paper is selected from the hat to become part of the sample, the remaining pieces of paper no longer have the same probability of being selected as the previous one did. For instance, each member of a population of 100 has a 1-in-100 chance of being selected. After 25 have been selected, however, those remaining have a 1 in 75 chance of being selected. This might seem to violate the first rule of probability sampling (equal probabilities), but as long as the chances for selection are equal at any given stage in the sampling process, the rule is not violated. This is called *sampling without replacement*, and it is particularly acceptable when the population is relatively large because the probability differences from stage to stage are negligible.

Identification of the Sampling Frame

The first consideration in deriving a sample is specifying the unit of analysis. The *unit of analysis* is the individual, object, or institution (or group of individuals, objects, or institutions) that bears relevance to the researcher's study. This relevance relates to the concept of a general universe or population. The *general population* is the universe to which the research findings will be applied. In other words, it is the theoretical population to which the researcher wishes to generalize the study's findings. This population is composed of units, which become the units of analysis for purposes of the study. Specifically, a unit of analysis can be a person, a household, a social organization, a political jurisdiction, a corporation, an industry, a hospital, or a geographical entity, among others, depending on the nature of the conceptual general population. If the objective of the research, for example, is to generalize findings to residents of California, then the units of analysis are individuals. If the study seeks to determine information on a county-wide basis about crime, counties are the units of analysis.

The designation of a general population, although critical for conceptualizing the purpose and objectives of a study, is generally not conducive to the actual selection of a sample. Let us suppose that a study intends to generalize its findings to the residents of an entire metropolitan area. The general population can be considered to be the people who reside within the political boundaries of the area under study, and the units of analysis are the individual residents. The process of selecting a representative sample requires, at its theoretical optimum, that the researcher know where and how to contact each person in the population. From a practical standpoint, it is highly unlikely that all members of the general population can be identified and contacted. People die and are born every minute; some people live in unrecorded accommodations (e.g., the homeless), and others cannot be contacted for various personal reasons. Herein lies the essence of the statistical necessity for establishing an intermediate step between the general population and the actual sample: the development of a *working population*. The working population is an operational definition of the general population that is representative of the general population and from which the researcher is reasonably able to identify as complete a list as possible of members of the general population through the use of a working population is known as the *sampling frame*.

Establishing a Representative Working Population

It is useful to illustrate the relationship between the general population and the working population. Suppose that a researcher is studying factors associated with economic disadvantage and poverty in New York City. The general population is to consist of all the economically disadvantaged people in New York City. Because there are no lists of such a population, the researcher must substitute a working population for this general population. He or she therefore must find some other identifiable population that can be claimed to correspond to the general population closely enough to be considered its surrogate. The designation of a working population requires the researcher to operationalize the concept of economic disadvantage, which has a variety of definitions. The federal government has established poverty-level criteria; poor families receive government aid from various federal, state, and local programs; and other agencies have their own measure of economic disadvantage. Based on any one or any combination of these definitions, a list of persons that constitutes the sampling frame can conceivably be obtained.

None of these lists, alone or in combination, will be complete and exhaustive. Recognizing this, the researcher will want to select a list, or combination of lists, that maximizes representativeness and minimizes systematic omissions (frequently referred to as *systematic biases*). *Systematic omissions* are absences from the working population of groups that clearly differ from those in the working population. For example, in political polling, people who do not have landlines (e.g., those with only cellular phones), people who screen their incoming phone calls, and people without phones are generally not included in the poll. To the extent that these groups possess significant differences from the people who are contacted, there is a systematic omission in the sample that must be corrected. If the individuals who are omitted are very similar to those who are included in all or most of the important categories (e.g., in ethnicity, age, and sex), their omission would not be considered to be systematic and would require no corrective action.

It is usually impossible to eliminate all omissions from the working population. Ideally the working population would be a complete list of members of the general population. However, no list can be expected to be perfectly complete; therefore, it is important for the researcher to be reasonably satisfied that the working population and derivative sample represent the general population as much as possible.

In constructing the sampling frame, therefore, the researcher should attempt to determine the extent to which members of the working population have been excluded from this list and decide whether these excluded members differ in any significant manner from those who are on the list.

In the economically disadvantaged example, the researcher may legitimately choose to define the economically disadvantaged as consisting of households with annual incomes below the federal poverty standard. This definition identifies the working population. The researcher might then identify a working population consisting of individuals who receive funds through the Temporary Aid to Needy Families (TANF) program, which requires, for qualification, a household income below the federal poverty level. The researcher is likely to encounter a systematic omission in this list, however, that is sufficient to render it unrepresentative: individuals who may be economically disadvantaged but do not have children are not eligible for the TANF program. However, in this case, since TANF serves only families with children, the list does not include individuals or families without children whose income is sufficiently low to otherwise qualify. Such an omission is clearly systematic in the sense that there is a clear and important difference between those who are included and those who are excluded. A systematic omission of this nature renders the working population unrepresentative of economically disadvantaged persons and therefore unacceptable. The researcher must search for an alternative working population-another list (or combination of lists) of people with incomes below the poverty level that does not contain such a systematic omission or weight the responses (as discussed later in this chapter) in order to establish satisfactory representativeness.

Selecting and Obtaining a Representative Sample

Examples of Sampling Frames

The ultimate accuracy of a sample depends in large part on how well the sampling frame is constructed. This concept is so important that we provide several examples of appropriate sampling frames:

• A survey of women of childbearing age is to be conducted. Since no such list exists, the researcher must operationalize this general population. This is likely to be accomplished by defining *childbearing age* in some reasonable manner (e.g., fourteen to forty-five years of age) and then identifying women in this age bracket for the research. The sample could be drawn from such lists or screened by qualifying respondents through a set of introductory questions.

• A survey of people affected by noise generated by a local airport is to be implemented. The sampling frame can be established by identifying officially designated noise-impact areas and obtaining a list of households within those boundaries. The sample would be drawn from this list.

A researcher is interested in criminal behavior and plans to survey individuals who have committed a felony. Initially he or she obtains permission to interview inmates in jails and prisons. If this jail and prison population were used to represent the entirety of the sampling frame, systematic bias could be suspected because of the omission of criminals not currently incarcerated (those who have been released or were never imprisoned in the first place). The researcher can address this suspected bias by including in the sample individuals currently on parole or probation to represent the released criminal population. The researcher may also be able to obtain a list of former convicts whose parole or probation has been successfully completed. However, the researcher cannot readily identify individuals who have committed felonies but have not been identified by the criminal justice system. The researcher can conclude that the absence of such individuals from the working population represents a systematic bias and can terminate the study, redefine the general population to include convicted criminals only, or proceed with the study with the appropriate disclaimer. In the alternative, however, it might not be unreasonable for the researcher to proceed without any disclaimer if it can be satisfactorily established that it is unlikely that this systematic omission is significant. In the example, the researcher may conclude that there is no major difference, in terms of identifying aspects of criminal behavior, between those who have been apprehended and those who have not.

Simple Random Sampling

There are several methods of drawing probability samples from the working population. The method chosen depends on a variety of factors, such as the manner in which the sampling frame is constructed and the focus of the study. The best-known form of probability sample is the *simple random sample*. The usual procedure is to assign a number to each potential respondent, or *sampling unit*, in the working population. Numbers are then chosen at random by a process that does not tend to favor certain numbers or patterns of numbers, and the sampling units selected become part of the sample itself. A common procedure for accomplishing this random process is to use computers to generate a random sample.

USING SPSS TO GENERATE RANDOMLY SELECTED CASES

SPSS will randomly choose cases from a data file by using these steps:

- 1. Select Data—Select Cases.
- 2. Click on the Random Selection of Cases button and then on Sample.
- 3. The following window then appears:

-Sample Size-		
<u>Approximately</u> <u>Exactly</u>	% of all cases cases <u>f</u> rom the first	cases
Cor	ntinue) Cancel Help)

- 4. Choose the exact number needed out of the total data set or an approximation. If 400 were needed out of the Transportation.sav data set, the Exact button would be pressed and 400 out of 1,954 would be inserted in the boxes. If a sample of approximately 1,000 were needed out of an unknown population that the researcher knows to contain approximately 1 million cases, the researcher could use the first box to select .1% of all cases at random.
- 5. The data view now shows the cases excluded from the random sample with a diagonal line through the case number, and all calculations will include only those cases without that line.

Systematic Random Sampling

Systematic random sampling is an adaptation of the simple random process used when the working population list is quite large and the sampling units cannot be conveniently or feasibly numbered. If the working population consisted of 3 million people and a sample of 1,500 people was required, the process of numbering and generating random numbers from a computer program would be prohibitively burdensome, as would the inputting of 3 million cases simply in order to select 1,500 of them.

A systematic sample assumes that the working population list is randomly distributed; therefore, the researcher can systematically choose sample members by selecting them from the list at fixed intervals (every *n*th entry). For instance, the 1,500 sample members represent 1 out of 2,000 people on the working population list (3,000,000/1,500 = 2,000), so if the selection process were to count to every 2,000th sampling unit from a starting point on the list selected at random from the first 2,000 cases, it would be expected that a random sample would result. If the starting point were selected outside of the first 2,000 cases, the list could be exhausted before 1,500 respondents were chosen. The researcher is encouraged to use the SPSS feature for generating random numbers, explained above, to select a random starting point between 1 and 2,000.

It is rare for the systematic process to yield fixed intervals that are whole numbers without a fractional remainder. In the example, if the working population were 3,100,000, the requisite fixed interval would be 2,066.67, which would require truncating the decimal. As an example, suppose that a working population consists of a list of 250 students and that the researcher requires that a sample of 9 students be selected. Dividing the working population of 250 by the desired sample size of 9 yields 27.8. This decimal is truncated (not rounded), leaving the whole number 27; hence, every twenty-seventh person on the list would be selected. The starting point would be any number between 1 and 27, selected randomly. If the starting point happens to be 7, the sample of 9 would consist of the following sampling units: 7, 34, 61, 88, 115, 142, 169, 196, and 223. If the decimal were rounded instead of truncated, the sample would include student number 252, which does not exist in the population of 250 students. Truncation does tend to eliminate certain sampling units from the possibility of being selected. In this case, students 244 through 250 would not have a chance of being selected. However, since the working population list is assumed to be random, this tendency to eliminate a small number of possible sample members is also random, and therefore it is far more acceptable than choosing nonexistent sampling units.

Taking this process one step further, let it be supposed that the persons numbered 34, 61, and 115 refuse to respond or cannot be reached for an interview. The researcher is then faced with having to elect three alternate respondents. Systematic sampling proceeds as follows. The remaining unselected working population sampling units are renumbered from 1 to 241 to account for the 9 sampling units previously selected. This remaining number is divided by the required three respondents. Thus, 241 is divided by 3 to yield 80.3. Then every 80th person is selected, starting with any number (selected at random) between 1 and 80. If the starting number is 60, the sample of three would include the 60th, 140th, and 220th person on the list. This procedure is repeated, if necessary, until all respondents have been successfully interviewed.

Cluster (Multistage) Sampling

Another variation on simple random sampling is known as *cluster* or *multistage sampling*. A cluster sample is a sample in which there is a hierarchy of sampling units. The primary sampling unit is a grouping (or cluster) of the individual elements that are the focus of the study. This grouping must be a well-delineated subset of the general population that is considered to include characteristics found in that population. Such groupings typically consist of counties, cities, census tracts, census blocks, and so forth. A random sample of these units is selected. Secondarily, a subset of smaller units within the primary units is randomly selected. This process continues at various stages until the researcher has selected a random sample of the actual units of analysis. Cluster sampling arises predominantly from situations in which the population is so prohibitively cumbersome that traditional random sampling techniques cannot be easily employed.

To illustrate a situation in which cluster sampling can be used, suppose that a mail survey of 1,000 residents of the United States is to be conducted. Traditional sampling methods would require that a list of US addresses be available. Such a list would be very difficult to obtain and, at the very least, highly cumbersome to process. Cluster sampling can provide a multistage procedure that will alleviate this problem. This procedure might entail randomly selecting counties, then census tracts within the counties, and finally individual households in those census tracts.

In certain situations, clusters are known to be substantially different from one another in terms of size or homogeneity of their populations. For instance, in most states, the population tends to be concentrated in a handful of counties. If clustering is to be performed by county, it is possible to underrepresent or even completely overlook these large counties. To avoid this lack of representativeness, it is necessary to establish a list of large counties and smaller counties based on population size (e.g., 1 million residents and above versus fewer than 1 million residents). Clusters can then be selected from each of these groups to ensure adequate representation.¹

A further illustration of cluster sampling is an onboard survey of bus riders in a major metropolitan county that we have performed for several transit agencies. Inasmuch as there is no efficient way to obtain a comprehensive list of bus riders, it is necessary to consider a cluster sampling procedure, as in this example of a 1,200 rider sample:

- 1. List all bus routes in the system by direction of travel, day of the week, and time of day. This list will take the form of a matrix of route-and-time cells containing one bus route for a specific direction, at a certain time of day, on a particular day of the week.
- 2. Each cell is weighted according to the known average volume of bus passengers. These weighted cells constitute the primary sampling frame for sample selection.
- 3. Select a random sample of 400 weighted cells. These cells indicate the buses earmarked for onboard interviews.
- 4. To obtain the 1,200 personal interviews, randomly select 3 passengers on each of the 400 selected buses.

Stratified Sampling

The example of selecting clusters from both large and small counties leads to a discussion of stratified sampling. *Stratified sampling* consists of separating the elements of the working population into mutually exclusive groups called *strata;* random samples are then taken from each stratum. For example, a researcher may be interested in voter opinion concerning the issue of gun control. It is considered important to analyze the population by ethnic group. Accordingly, the sampling frame is separated into strata based on ethnicity. The primary purpose in this sample selection process is to make certain that each stratum is represented by an adequate sample size in order to analyze the stratum both separately and as part of the total population. This is much more likely to occur when selection is performed by stratum than by an overall population random sampling procedure. Table 9.1 presents a hypothetical overall breakdown of the working population by

Ethnic Group	Population Size (in thousands)	%	Expected Proportionate Sample Representation	%
White	6,000	60.0	360	60.0
Black	1,500	15.0	90	15.0
Hispanic	1,500	15.0	90	15.0
Asian	1,000	10.0	60	10.0
Total	10,000	100.0	600	100.0

TABLE 9.1. PROPORTIONATE SAMPLE REPRESENTATION FOR A HYPOTHETICAL ETHNIC DISTRIBUTION.

ethnic group and the expected random sample representation of the various groups proportionate to their number in the overall population, based on a sample size of 600.

If the researcher were to proceed with random sampling procedures, it could be expected that the sample sizes of the ethnic strata would approximate the proportions they represent in the overall population. Hence, the researcher might consider, in advance of ultimate sample selection, that the sample is likely to have approximately 360 whites, 90 blacks, 90 Hispanics, and 60 Asians. However, since ethnicity has been established as an important criterion in the study, the researcher must now recognize that the number of blacks, Hispanics, and Asians to be sampled in the study is probably going to be too small to achieve certain requisite margins of error. Chapter 8 indicates that these expected group sizes do not meet sample size requirements for even a margin of error of ± 10 percent (95 percent level of confidence), thereby calling into question the researcher's ability to make reasonably accurate generalizations concerning these groups. A practical rule of thumb is that a 10 percent margin of error is the maximum that should be tolerated for any group or subgroup within the overall sample. Hence, a sample size of approximately 100 is required for all strata and substrata. Therefore, if the researcher wants to attempt to achieve at least the ± 10 percent margin of error for each stratum, the overall sample size will have to be increased accordingly to reasonably ensure that each group will meet that threshold. For example, since Asians are the smallest ethnic group in terms of numbers and they represent 10 percent of the population, a total sample size of a minimum of 1,000 persons will be required to anticipate approximately 100 Asian respondents within that sample. Increasing the sample size can have serious cost considerations, however, and may not be feasible within the researcher's budget or time frame.

Selecting and Obtaining a Representative Sample

Stratified sampling offers to the researcher a method by which the margin-of-error requirement of a maximum of ± 10 percent for each stratum can be satisfied while still keeping the overall sample size at 600, as long as at least 100 persons are interviewed in each stratum. The recommended disproportionate sample sizes by stratum for this particular example are

300
100
100
<u>100</u>
600

This disproportionate sampling distribution is obtained by expanding the sample size of strata that would otherwise not attain the requisite minimum size and correspondingly reducing the group or groups that would achieve the requisite minimum through the standard random sampling process. The samples must now be selected randomly, at least to some extent, from four separate sampling frames within the total population. At this point, in the case of small populations for any stratum, equation 8.7 and table 8.2 should be used to determine the sample size of each stratum. This will reduce the requisite sample sizes.

The difficulty with disproportionate stratification is that the overall sample is now skewed toward the smaller strata. In the case at hand, African Americans, Latinos, and Asians are overrepresented and whites are underrepresented. A weighting procedure must be employed to analyze the data with regard to the total population.

Weighting Disproportionate Samples

In situations where the survey has produced disproportionate results either deliberately stratified or due to systematic omissions that can be corrected by weighting—an adjustment is required to make certain that the total sample is a proportionate representation of the population rather than a summation of disproportionately sampled groups within that population. This adjusted total sample is to be used when analyzing the total population, whereas the individual disproportionate strata will be used for the analyses for each of those subgroups.

Planning Area	Population	%
North Central	2,026	3.7
West Davis	8,510	15.4
Central	19,971	36.1
East Davis	13,360	24.1
East Davis–Mace	939	1.7
Core	1,258	2.3
South Davis	8,268	14.9
South Davis–County	997	1.8
Total	55,329	100.00

TABLE 9.2. POPULATION OF DAVIS PLANNING AREAS.

This corrective weighting process is best demonstrated by an example of disproportionate sampling in the City of Davis, California. For planning purposes, Davis and its environs are divided into a series of eight planning areas that were specifically targeted for study. These planning areas are listed in table 9.2, along with their populations and corresponding percentages.

It was determined that the importance of these planning areas was sufficiently significant to require that each planning area be separately analyzed in accordance with a maximum margin of error of ± 10 percent (95 percent confidence level). Budget and time considerations limited the overall sample size to approximately 800 persons. In large populations (100,000 or more), this margin of error is achievable with a sample size of approximately 100 persons. For smaller populations, the requirement is reduced somewhat (see chapter 8). Therefore, appropriate sample sizes were obtained to fulfill this objective; these are indicated in exhibit 9.1 as "Actual Sample Size."

It was equally important in this study to analyze the results on a citywide basis. This required that the sample from each planning area be weighted to reflect that area's relative population within the city. Exhibit 9.1 demonstrates the calculation of the weights to be applied to the data from each planning area to generate a citywide total. The calculation of these weights entails identifying the expected number of survey respondents in each planning area if the survey had been conducted randomly without stratification. Hence, for example, since the Central planning area represents 36.1 percent of the population of Davis, it is expected that approximately 36.1 percent of 833 randomly selected citywide participants (301 persons) would be residents of that area in that survey. The expected proportionate sample size

Planning Area	Expected Proportionate	Actual Sample Size	Weight (Expected/Actual)
North Central	31	103	.30
West Davis	128	106	1.21
Central	301	110	2.74
East Davis	201	110	1.83
East Davis–Mace	14	104	.13
Core	19	105	.18
South Davis	124	103	1.20
South Davis–County	_15	_92	.16
Total	833	833	

EXHIBIT 9.1. SAMPLE SIZES AND WEIGHTS FOR THE CITY OF DAVIS PLANNING AREAS.

for each area is then compared to the sample size actually obtained, and a weight is calculated by dividing the expected proportionate sample size by the actual sample size.

Table 9.3 depicts the outdoor recreation facilities used most often by Davis residents, cross-tabulated by planning area. The column totals for each individual planning area represent the actual sample data, and the "Weighted Citywide Total" column reflects the application of the weights from exhibit 9.1. This calculation is depicted in worksheet 9.1, which selects categories of the recreational facility variable and applies the weight for each planning area to the actual sample data for those categories. The worksheet then sums these weighted data to obtain a weighted citywide total, which is rounded to whole numbers to reflect the discrete nature of the data: individual respondents. The process continues through each category of the dependent variable. Worksheet 9.1 depicts this process for two categories of the dependent variable. You can verify that this process has been applied to the other eight categories in table 9.3. When reporting about the resident opinions and behavior concerning the recreational facilities in the City of Davis as a whole, it is appropriate to use the percentages associated with the weighted data. For example, 21.3 percent of Davis residents use greenbelts most often as a recreational facility (see table 9.3). When the researcher wished to report on individual planning areas, the unweighted data should be used. For example, 28.9 percent of north central residents use greenbelts most often as their recreational facility.

	No Cei	orth ntral	Wes	t	Cer	ntral	East	Davis	Ea Davis	ast –Mace	C	ore	Sout	h Davis	Sc Davis-	uth County	Weig Citywid	hted e Totalª
Outdoor Facility	f	%	f	%	f	%	f	%	f	%	f	%	f	%	f	%	f	%
Greenbelts	56	28.9	31	16.6	45	26.8	35	18.8	54	31.4	26	14.8	28	17.2	10	9.6	288	21.3
Athletic fields	34	17.5	39	21.0	26	15.5	27	14.5	23	13.4	30	17.0	24	14.7	19	18.3	218	16.1
Children's play areas	26	13.4	19	10.2	18	10.7	33	17.8	24	13.9	15	8.5	22	13.5	18	17.3	175	13.0
Lawn areas	23	11.8	14	7.5	21	12.5	14	7.5	13	7.5	32	18.2	15	9.2	8	7.7	134	9.9
Swimming pools	12	6.2	2614.0	9	5.3	24	12.9	19	11.0	16	9.1	15	9.2	8	7.7	128	9.5	
Picnic areas	10	5.2	7	3.8	8	4.8	18	9.7	18	10.5	15	8.5	17	10.4	13	12.5	94	7.0
Tennis courts	14	7.2	17	9.1	12	7.1	8	4.3	8	4.7	6	3.4	10	6.1	7	6.7	88	6.5
Basketball courts	9	4.6	17	9.1	10	6.0	9	4.8	8	4.7	16	9.1	6	3.7	7	6.7	79	5.9
Other ^b	10	5.2	12	6.5	10	6.0	12	6.5	5	2.9	16	9.1	20	12.3	12	11.6	97	7.2
None used	0	0.0	4	2.2	9	5.3	6	3.2	0	0.0	4	2.3	6	3.7	2	1.9	49	3.6
Total ^c	194	100	.0186	100.0	168	100.0	186	100.0	172	100.0	176	100.0	163	100.0	104	100.0	1,350	100.0

TABLE 9.3. OUTDOOR RECREATION FACILITIES USED MOST OFTEN BY DAVIS RESIDENTS, BY PLANNING AREA.

*Because of the nature of the weighting process, the sum of the planning area frequencies does not equal the weighted citywide total.

^b"Other" includes such outdoor recreation facilities as skate parks, golf courses, parks, the arboretum, bicycle trails, and community gardens.

"Because respondents were provided the opportunity to give more than one response, column totals may exceed sample size.

	Facility					
Planning Area	Weight	Greenbelts	Athletic Fields ^a			
North Central	0.30	56 imes 0.30 = 16.80	$\textbf{34} \times \textbf{0.30} = \textbf{10.20}$			
West Davis	1.21	31 × 1.21 = 37.51	39 × 1.21 = 47.19			
Central	2.74	$45 \times 2.74 = 123.30$	$26 \times 2.74 = 71.24$			
East Davis	1.83	$35 \times 1.83 = 64.05$	27 imes 1.83 = 49.41			
East Davis–Mace	0.13	$54 \times 0.13 = 7.02$	$23 \times 0.13 = 2.94$			
Core	0.18	$26\times0.18=4.68$	$30\times0.18=5.40$			
South Davis	1.20	$28 \times 1.20 = 33.60$	24 imes 1.20 = 28.80			
South Davis–County	0.16	$10 \times 0.16 = 1.60$	$19\times0.16=3.04$			
Weighted citywide total ^b		288	218			

WORKSHEET 9.1. CALCULATION OF WEIGHTED CITYWIDE TOTAL FOR OUTDOOR RECREATION FACILITIES USED MOST OFTEN.

^aContinue the same procedure for each outdoor recreation facility. ^bSummation of weighted planning area totals.

SAMPLE FOR TABLE 9.1.					
Ethnic Group	Expected Proportionate Sample Size	Actual Sample Size	Weight (Expected/Actual)		
White	364	304	1.197		

91

91

_61

607

100

102

<u>101</u>

607

.910

.892

.604

EXHIBIT 9.2. SAMPLE SIZES AND WEIGHTS FOR STRATIFIED SAMPLE FOR TABLE 9.1.

As another example, suppose that the survey that was proposed in Table 9.1 was performed and the ultimate distribution of responses was as follows:

White	304
African American	100
Hispanic/Latino	102
Asian	101
	607

African-American/Black

Latin/Hispanic

Asian

Total

Exhibit 9.2 shows the calculation of weights for this sample survey.

Supervis District	orial	Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Central	757	38.7	38.7	38.7
	West	297	15.2	15.2	53.9
	East	244	12.5	12.5	66.4
	North	553	28.3	28.3	94.7
	South	103	5.3	5.3	100.0
	Total	1,954	100.0	100.0	

USING SPSS FOR WEIGHTING

SPSS can calculate and apply weights to the data set. For example, in Transportation .sav the frequency distribution for supervisorial district (superdist) is as below.

For purposes of this illustration, assume that the true population of bus riders is distributed as follows:

	Expected (True) Percentages
Central	35.0%
West	20.0%
East	15.0%
North	20.0%
South	10.0%

Also assume that it is important that the survey sample be distributed by supervisorial district as closely as possible to the true population. By using the Transform process (as was used in recoding midpoints in chapter 5), weights can be calculated and assigned to each respondent:

- 1. Go to Transform—Recode into Different Variable.
- 2. Rename old variable "superdist" into the new variable "supertrue" (label: true supervisor %).
- 3. Replace codes 1 through 5 in the old variable with the expected (true) percentages above in the new variable.
- 4. Press Continue and then OK, and a new variable (supertrue) will have been created at the end of the data set in both Data View and Variable View. Check both views to make certain that this has occurred.

Selecting and Obtaining a Representative Sample

Next, do the same for the actual percentages in the survey (rename as superact and label as "actual sample percentage"), recoding 1 as 38.7, 2 as 15.2, and so on. Note that supertrue and superact must be comparable, so that if supertrue is input as whole numbers, so must superact. If supertrue is input as proportions (e.g., 1 was recoded as .387), then so must superact (.35).

The final step in the weighting process follows these procedures:

- 1. Return to Transform, but this time select Compute Variable.
- 2. Name the target variable Weight.
- 3. Input the function that is required to weight the data—in this case, dividing the expected/true percentage (supertrue) by the actual percentage obtained (superact).
- 4. Select OK, and a new variable (Weight) will appear in the data set. If the researcher wishes to have more than the default decimal places, all that is needed is to change the number of decimal places for Weight in Variable View.

The weights are now:

- a. Central .90
- b. West 1.32
- c. East 1.20
- d. North .71
- e. South 1.89

Compute Variable			X				
Compute Variable Target Variable: Weight Type & Label ethnicity_q34 ethnicity_q37a english_q37ab estimation_g37 ethnices_q37c ethnices_q38e ethnices_q3	=	Numeric Expression: supertrue / superact + > - > 4 5 * = 1 2 / 8 0 . * ~ () Delete * ~ () Delete	Function group: All Arithmetic CDF & Noncentral CDF Conversion Current Date/Time Date Arithmetic Date Creation				
	OK Paste Reset Cancel Help						

Applying these weights to obtain a weighted frequency distribution involves these steps:

- 1. Click on Data and then Weight Cases.
- 2. Select Weight by Weight (moving Weight from the variable list).
- 3. Click on OK.

🙀 Weight Cases	
tarsi_q3/t	 <u>D</u>o not weight cases <u>W</u>eight cases by
Sign_language	Frequency Variable:
<pre>prefer_lang_q38 papeder</pre>	
increc	
superact	Current Status: Do not weight cases
OK Paste	Reset Cancel Help

The data are now weighted (the lower right corner of the data set shows "Weight On"). Requesting a new, weighted frequency distribution for superdist shows the following (note the small, inconsequential rounding error in the South district):

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Central	685	35.0	35.0	35.0
	West	391	20.0	20.0	55.1
	East	293	15.0	15.0	70.0
	North	391	20.0	20.0	90.1
	South	194	9.9	9.9	100.0
	Total	1953	100.0	100.0	

SUPERVISORIAL DISTRICT

Any frequency distributions or other statistics run by SPSS will now be weighted until the weights are taken off by changing Data—Weight Cases from the "Weight cases by" button to "Do not weight cases."
Nonresponse Bias

In statistical primary data gathering, where people are involved in answering questions, the response rate to the process will rarely be 100 percent. There are almost always some members of the initial sample who cannot be contacted, refuse to participate, or for some other reason fail to complete the data collection process.

Nonresponse bias is the departure of the sample statistics from their true population values owing to the absence of responses from some portion of the population that differs systematically from those portions of the population that did respond.

To the extent that responses are not received from 100 percent of the sample, questions arise about using the results from the ultimate sample to provide a reliable estimate of the true population. This concern results from the fact that if the sampled cases from which data are not received are different in some important way from those who responded, then the estimates from the sample can be potentially biased. A low cooperation or response rate can do more damage in rendering a survey's results questionable than does a small sample, because there may be no valid way to scientifically infer the characteristics of the population represented by the nonrespondents.²

In order to make proper use of inferential statistical techniques (generalizing findings from a sample to the general population from which the sample was drawn) and have an acceptable degree of confidence in the representativeness of the data obtained, the sample itself must be random in both selection method and the actual distribution of participant characteristics. If those who do not respond differ in a significant way from those who do respond, then the sample becomes nonrandom even if it was randomly selected. A significant nonresponse bias can convert a randomly selected sample into one that is nonrandom in actual participation and respondent distribution. Even if a researcher chooses a truly random target sample, the final sample cannot be assumed to be representative (as randomness assumes) unless those who respond are random.

Random (probability) sampling entails letting chance determine the selection of members of the population for the sample. However, to the extent that randomly selected respondents do not participate in a manner consistent with that randomness, the randomness of those who ultimately respond grows more and more suspect and problematic, and possible nonresponse bias grows.

The overall response rate is one critical factor in assessing the representativeness of a randomly selected sample. If a high response rate is achieved, there is a lower chance of significant response bias than with a low rate. A low response rate is a potential problem because the nonrespondents are increasingly likely to differ from the respondents in ways other than just their willingness to participate in the survey, and the more nonrespondents there are, the more likely and diverse this problem can be.

So what is a high or low response rate? A word of caution about response rates is in order. The term *response rate* is not used uniformly by all authors and researchers. The American Association for Public Opinion Research has long sought to achieve unanimity in the use of this term in the correct belief that as long as differing definitions are in use, the full comprehension and comparability of articles and studies cannot be achieved to a satisfactory degree.

In researching this issue over many years and testifying in courts of law about sample representativeness, we have generally found that when cooperation rates (the percentage of persons contacted who complete the survey process) in a survey decline below 50 percent, that survey should be regarded with some caution as a basis for precise quantitative statements about the population from which the sample was drawn. When the cooperation rate exceeds 80 percent, even if the nonrespondents differ from those who respond, the overall estimates will not be badly biased.

Efforts to Compensate for Nonresponse Bias

The existence of nonresponse bias in a sample is not necessarily indicative of the need to ignore the results from a survey with a lower response rate. It is, however, to be understood as cautionary and that to maintain the potential to be reliable, samples with low response rates must undertake additional steps. Because the issue of nonresponse infiltrates almost every survey research project that is undertaken, statisticians have developed ways to adjust for this effect.

Response rates are best addressed during the design and data collection phases of research, including the use of cash or prize incentives to secure responses from those otherwise less inclined to participate, repeated efforts to contact nonrespondents by telephone and e-mail, well-structured and attractively appointed mail-out, intercept, and web-based survey questionnaires, and well-trained and professional interviewers.

During analysis, however, researchers can follow a number of best practices to help account for nonresponse and guard against nonresponse bias. These efforts will not improve the response rate, but they may help compensate for nonresponse and increase confidence in data quality: Selecting and Obtaining a Representative Sample

1. Compare response rates across subgroups. Compute and compare response rates across key subgroups of the target population to make certain that they match these groups' representation in the total population. These subgroups might include age, gender, and region, for example, for a survey of the general public. This method does not help determine the extent of nonresponse bias, but it can indicate whether there might be this bias. If the response rates are quite similar across these key subgroups, nonresponse bias might have a limited impact on the data. The problem with this approach is that there are other causes of nonresponse aside from such subgroup variables. That is, it is unlikely that response propensity is affected only by the subgroup variables. Comparing these response rates is a good place to start, but doing so should not take the place of other

methods of addressing survey nonresponse. 2. Weight the survey data if possible. Statistical adjustment, or weighting by observable variables, is one of the most common approaches used to address survey nonresponse. Weighting the data can help the researcher present results that are representative of the target population, where it can be assumed that no or little nonresponse bias exists in variables other than those being weighted. With this method, demographic, socioeconomic, and other descriptive parameters of the survey sample are weighted to account for nonrespondents.

Compare respondents and nonrespondents. Nonresponse bias is specific 3. to each survey. It is important, then, to try to assess nonresponse bias on key survey variables, not just demographics. Even if the sample composition closely matches the demographic distribution of the total population, there may be bias on other substantive, topic-related survey variables other than demographics. A key to dealing with nonresponse is knowing the extent to which it might affect the data. One method used to quantify the difference between respondents and nonrespondents is to randomly survey a subsample of nonrespondents when the fieldwork is complete. These responses can then be compared to those of the respondent group. If no statistical differences are observed between the respondents and the random sample of nonrespondents across these key survey measures, then the overall results might more confidently be generalized to the target population. The drawback of a nonrespondent follow-up is that it stands to reason that if nonrespondents were difficult to reach or reluctant during the fieldwork, they will be equally difficult to reach or reluctant during a follow-up exercise. Such follow-ups involve callbacks or a change of data collection mode, and they may require the use of substantial incentives or elite interviewers to persuade nonrespondents to complete interviews in a sufficiently large enough number of cases.

4. Compare late respondents to early respondents. Making a relatively large leap of faith, some researchers assume that late respondents to the survey are more typical of nonrespondents than they are typical of early respondents. If the late respondents (or ones that require several callbacks) do not differ from the early respondents on the key variables, these researchers will be more comfortable that the sample is not affected by nonresponse bias.

Nonprobability Sampling

The essential characteristic of nonprobability sampling is that the researcher does not know the probability that a particular respondent will be selected as part of the sample. Therefore, there is no certainty that the probability of selection is equal among the potential respondents. Without such equality, the researcher cannot analyze the sample in the context of the normal distribution. Therefore, the sample data cannot be used to generalize beyond the sample itself, because the degree of sampling error associated with the sample cannot be estimated without an assumption of normality. Intercept surveys in particular are likely to be nonprobability without significant preparatory work and careful planning to ensure representativeness.

Nonprobability samples do not provide the researcher with the ability to generalize survey data with a known degree of accuracy. That is, the information obtained, although useful, does not allow a specific margin of error to be identified, which interferes with any attempt to scientifically generalize the findings.

In spite of this shortcoming, nonprobability sampling can be helpful to researchers. It is considerably less complicated in terms of strict adherence to the tenets of random sample selection and is therefore much less costly and time consuming than probability sampling. The primary advantage of nonprobability sampling rests in its usefulness in the preliminary stages of a research project or in a postsurvey debriefing phase to elaborate on certain survey findings that are particularly difficult to explain. In stage 3 of the survey process (see chapter 1), for instance, the researcher must ensure that there is adequate knowledge of the investigative area before constructing specific questions. The use of a nonprobability sample can quickly generate a preliminary understanding of some of the key issues underlying the research study. It is also the primary means by which researchers pretest and refine their survey instrument, as discussed in chapter 2.

The most common example of a nonprobability sample is a *sidewalk survey*, where interviewers interview passersby at, for example, a shopping center. The general population in this example is shoppers. Strict adherence to the principles of probability sampling would require the compilation of a count of all such shoppers and the ability to access all shoppers as the working population. In the case of a sidewalk survey, this working population is typically not enumerated; consequently, the probability of selecting any particular passerby from that working population cannot be determined. Furthermore, under these circumstances, there can be a significant element of individual interviewer discretion in the selection of interviewees, which might compound the existing uncertainty about whether the sample truly represents the general population.

There are several types of nonprobability samples. The sidewalk survey is an example of convenience sampling in which interviewees are selected based on their presumed resemblance to the working population and their ready availability. Frequently students are interviewed in their classrooms, enabling the researcher to contact large numbers of respondents in a relatively short period of time and at minimal cost. It is important to reemphasize that the researcher cannot generalize the findings in such cases beyond the sample itself. These findings can be used only as an informal base of knowledge, in preparation for a survey research project based on probability sampling or for purposes of elaborating on the otherwise undetected nuances, themes, and patterns of the population already quantitatively surveyed. Note that convenience sampling and *snowball sampling* are at the core of obtaining participants for focus groups (see chapter 4).

Snowball sampling is particularly beneficial when it is difficult to identify potential respondents. Once a few respondents are identified and interviewed, they are asked to identify others who might qualify as respondents. Suppose that a researcher has initiated a study that requires interviewing drug abusers who have not sought medical or social assistance. Obviously such respondents could not be easily identified. However, the researcher may be able to identify and interview a small number of drug abusers using personal reconnaissance. Snowball sampling could then be invoked by relying on these initial respondents to provide access to other drug users.

Another type of nonprobability sample is known as the purposive sample. In the purposive sample, the researcher uses his or her professional judgment instead of randomness in selecting respondents. For example, she may be interested in gathering information about problems related to juveniles in a particular community. *Key respondents*, whom the researcher considers to be particularly knowledgeable about the subject, may be selected for interviews. These respondents may include such people as the directors of social service agencies, law enforcement personnel, judges, attorneys, and educators. Responses to a set of questions may then be summarized as part of a larger study concerning juvenile problems.

Notes

1. The most extensive treatment of a method of selecting the number of clusters to be sampled is contained in Schaeffer, Mendenhall, and Ott (1986). They present a complex and innovative formula for determining the number of clusters to be included in a cluster sample, which can be shown at the 95 percent level of confidence, as follows:

$$n = \frac{N\sigma_c^2}{N\left(B^2\bar{M}^2\right) + \sigma_c^2}$$

where

- n = number of clusters selected in a simple random sample
- N = number of clusters in the whole population
- σ_c^2 = population variance associated with the sizes of the clusters in the population
- B = margin of error (confidence interval) in terms of either proportions or interval data
- \overline{M} = mean cluster size of the whole population

The formula will tend to yield a number of clusters to be sampled that is somewhat lower than the number generated by tables 8.1 and 8.2. It does so because of two factors: (1) a preliminary sample is required to estimate M and σ_c^2 and (2) there is an implicit assumption that there will be a full canvassing of all members of the clusters selected. Hence this formula has the advantage of a greater geographical concentration of clusters, but there are disadvantages in terms of the costs of conducting an adequate preliminary sample and obtaining a 100 percent census within the clusters themselves. Also, because of the smaller number of clusters, there is a greater likelihood of needing to stratify the cluster sample. On balance, we believe it to be more consistent with the needs of the readers of this book, in terms of practical applications, cost factors, and other such considerations, to use the approaches presented in chapter 8 and this chapter in the application of cluster sampling.

2. American Association for Public Opinion Research: Best Practices (www.aapor .org/best_practical.htm).



PART THREE

PRESENTING AND ANALYZING SURVEY RESULTS



CHAPTER TEN

ANALYZING CROSS-TABULATED DATA

The distribution of a single variable is described by the frequency distribution, and this univariate relationship is further clarified and understood through the use of measures of central tendency and dispersion. Multivariate relationships (those with more than one variable) can be presented through the use of contingency tables, but unlike frequency distributions, descriptive statistics are not adequate to analyze such relationships. Inasmuch as the primary purpose of contingency tables (cross-tabulations) is to depict the relationship between two or more variables, tests of statistical significance and measures of association (analytical statistics) need to be applied to the data to verify the existence and strength of any apparent relationships between variables.

This chapter presents the most commonly applied tests of statistical significance and measures of association that relate particularly to cross-tabulated survey data at the nominal or ordinal level of measurement. These analytical methods are essential for researchers to understand because they have the ability to discriminate among voluminous amounts of data generated by the survey research process. They permit researchers to identify relationships among survey variables and address whether data from a sample can be used to represent facts about the general population from which the sample has been drawn.

Chapters 11 and 12 continue the discussion of analytical statistics, presenting several additional tests of significance and measures of association that are particularly useful in the analysis of interval level sample survey data. Chapter 13 discusses the presentation of the analyzed findings in a final report.

Cross-Tabulated Contingency Tables

The most elementary tabular display of data is the frequency distribution. A *frequency distribution* is a summary presentation of the frequency of response (f) of each category of a variable. An example of a frequency distribution is provided in table 10.1. The frequency distribution provides the response frequencies for each question in a survey.

Frequency distributions involve a description of one variable. Often, however, the research calls for a simultaneous analysis of more than one variable. For example, a researcher may be interested in knowing the relationship between the ethnic background of the survey respondents and their educational attainment in order to recommend certain culturally based proposals for increasing educational attainment in a study area. In this case, two variables are under study: ethnicity and education.

Since the researcher is interested in the influence one variable may have on another, the use of a contingency table is appropriate. Contingency tables add an explanatory dimension to the frequency distribution. Table 10.2 is an example of a contingency table drawn from the same data as table 10.1 but including two variables: years as a customer of the water district and area of customer residence.

If the data contained in table 10.2 had been presented in frequency distributions, the researcher would have found that the water district customers surveyed were relatively evenly split among the residential areas and that there were more north central residents surveyed than southeast and west residents, as indicated in the "Total" column and row. The contingency table provides the additional information that the particular residential areas differ in terms of the length of time their residents have been customers of the water district. These data are derived by a computer-generated cross-tabulation of two questions from the survey instrument and their associated variables: years as a customer of the Richmond Water District and area of residence. Table 10.2 reveals an apparent difference among these residential areas reflecting the finding that the west area residents have somewhat longer tenure (20.7 percent with thirty or more years and 7.8 percent with less than one year) than the other two residential areas.

f	%	
101		13.1
145		18.7
149		19.3
135		17.4
134		17.3
110	1	14.2
774	100.0	
	f 101 145 149 135 134 110 774	f % 101 145 145 149 135 134 110 1 774 100.0

TABLE 10.1.	NUMBER OF	YEARS AS A	CUSTOMER
OF THI	E RICHMOND	WATER DIST	RICT.

TABLE 10.2. NUMBER OF YEARS AS A CUSTOMER OF THE RICHMOND WATER DISTRICT, BY AREA OF RESIDENCE.

	North Central		South East		l	West		Total	
Years	f	%	f	%	f	%	f	%	
Less than 1	49	13.1	38	17.3	14	7.8	101	13.1	
1 and less than 5	77	20.6	31	14.1	37	20.7	145	18.7	
5 and less than 10	75	20.0	46	20.9	28	15.6	149	19.3	
10 and less than 20	62	16.5	35	16.9	38	21.2	135	17.4	
20 and less than 30	71	18.9	38	17.3	25	14.0	134	17.3	
30 or more	41	10.9	32	14.5	37	20.7	110	14.2	
Total	375	100.0	220	100.0	179	100.0	774	100.0	

Important principles apply specifically to contingency tables. Prior to enumerating these principles, it is critical to explain the concepts of independent and dependent variables.

The *independent variable* is the change agent, or the variable that attempts to explain changes in the dependent variable. It acts on, influences, or precedes the *dependent variable*, which is the variable that is being explained and is therefore dependent on the independent variable. In the case of the variables in table 10.2, the area of residence is the independent

variable that is said to have some influence on years as a customer. Reversing these characterizations is logically much less likely.

In determining which variable should be considered independent and which should be considered dependent, the researcher can use a certain rule of thumb if common sense does not clearly specify an independent variable for a given situation. An attempt should be made to identify which variable came first in time, and that variable can be specified as independent. In the case of two variables, such as "years of schooling" and "parents' socioeconomic class," that logic would dictate that parents' socioeconomic class be designated as independent because it too precedes the children's educational achievement.

When a temporal relationship is not clear, the researcher must use professional judgment about the study's intent and focus to choose which variable should be treated as the independent variable. A statistical measure is available to help in this determination. This measure is called lambda (λ) .¹

Contingency tables include a "Total" column in addition to the "Total" row that is shown in typical frequency distributions. The Total column depicts the sum of all categories of the column variable for each category of the row variable. Inasmuch as the stated purpose of a contingency table is to determine the relationship between two variables, the inclusion of a Total column is a critical and necessary component. In addition, the table should be prepared so that the independent variable is the column variable—the one along the top—and the dependent variable is the row variable. The title of the table should be expressed in terms of the dependent variable "by" the independent variable.

Percentages are calculated for the independent (column) variable only. In accordance with this principle, the percentage is calculated vertically, summing to 100 percent at the bottom of each column.² The variables can be compared by holding a category of the dependent variable (row) constant and comparing the percentages across the row. For example, in Table 10.2, whereas 16.5 percent of north central area residents have been customers for ten but less than twenty years, 21.2 percent of residents in the west area have been customers that long.

At least two situations may arise that would prompt the researcher to reverse the independent and dependent variables. First, the number of categories of the independent variable may be so extensive that it is difficult to fit them on a single sheet of paper for final report purposes. In such a case, it may be necessary to present the independent variable vertically in order to reconfigure the table so that it can be contained on one page. Second, it is possible that in a series of tables, one variable will be consistently identified as either independent or dependent (e.g., income is generally an independent variable in opinion and behavioral surveys), but an isolated instance may arise that identifies the variable in the opposite way (e.g., income cross-tabulated with ethnicity). For this single situation, the researcher may decide not to reverse the axis to which the reader has become oriented. In cases where the independent variable is located on the rows rather than across the top, percentages should still be calculated for the independent variable (although this will occur horizontally), and comparisons between the variables will be accomplished instead by comparing percentages down the columns.

It is possible to add a third variable to a contingency table analysis by constructing contingency tables that cross-tabulate the dependent variable and the independent variable while holding each category of a third (control) variable constant. For example, table 10.2 could be presented in a more detailed format by preparing a series of contingency tables that display customer tenure at the Richmond Water District by residence and gender. Table 10.3 is an example of one of these gender-based distributions. This is ordinarily referred to as a three-way cross-tabulation; it is particularly useful when it is suspected that additional variables are involved in the relationship. Men seem to have been water district customers slightly longer than have women. This is found in that men have lower percentages in table 10.3 than the overall (table 10.1) for the shorter periods and higher percentages for the longer tenures, with the exception of the longest period.

	Area							
	North Central		Southeast		West		Total	
Years	f	%	f	%	f	%	f	%
Less than 1	26	13.0	20	16.7	5	5.5	51	12.4
1 and less than 5	40	20.0	16	13.3	15	16.7	71	17.3
5 and less than 10	30	15.0	27	22.5	15	16.7	72	17.6
10 and less than 20	44	22.0	20	16.7	24	26.7	88	21.5
20 and less than 30	40	20.0	22	18.3	15	16.7	77	18.8
30 or more	20	10.0	15	12.5	16	17.7	51	12.4
Total	200	100.0	120	100.0	90	100.0	410	100.0

TABLE 10.3. NUMBER OF YEARS AS A CUSTOMER OF THE RICHMOND WATER DISTRICT, BY AREA OF RESIDENCE: MALE RESPONDENTS.

USING SPSS TO GENERATE CROSS-TABULATIONS

Much of chapter 10 is about analyzing cross-tabulated variables. SPSS prepares cross-tabulations and also identifies the values and significance of various tests of significance and measures of association. To initiate the process, once again access Transportation.sav. Let's say that the researcher wants to analyze the relationship between frequency of riding the bus and overall satisfaction with the bus service:

- 1. Select Analyze—Descriptive Statistics—Crosstabs.
- 2. Place the independent variable (freq_q6) in the Column box.
- 3. Place the dependent variable (satis_overall-q1) in the Row box.

	Row(s):	Exact
change_service_12m	satis_overall_q1	Statistics
satis ontime g3b		Cells
satis_freq_q3c	Column(s):	Format
satis_transfer#_q3d	freq_q6	Eormat
satis_closehome_q3e	•	Bootstrap
satis teleinfo g3g	-lover1 of 1	
satis_transferreliable		
satis_drivecourtesy_q3i	Previous	
satis_driveknow_q3j		1
satis safetystops g3l	4	
satis_clean_q3m		
	Display layer variables in table lay	Vers
Display clustered bar charts	Coopies reger valiables in table la	1010
Sunnress tables		

4. Click on the Cells button. Make certain that Observed is checked in the Counts section, and then check Column in the Percentages section. Click on Continue.

Counts	-z-test
✓ Observed	Compare column proportions
Expected	Adjust p-values (Bonferroni method)
Hide small counts	
Less than 5	
Percentages	Residuals
Row	Unstandardized
Column	Standardized
Total	Adjusted standardized
Noninteger Weights	
Round cell counts	◎ Round case <u>w</u> eights
O Truncate cell counts	◎ Truncate case weig <u>h</u> ts
O No adjustments	

Analyzing Cross-Tabulated Data

5. Press OK, and the Output shows the following cross-tabulation. This cross-tabulation shows that with the exception of the handful of those who ride the bus only a few times per year, the percentage of riders who are very satisfied decreases as frequency of riding increases. Even when Very Satisfied and Somewhat Satisfied are added together, this holds, ranging from 79.1% somewhat or very satisfied among those who ride the bus four to seven times per week to 83.0% for one to three days, 81.3% at least once per month and 82.3% a few times per year. Correspondingly, dissatisfaction is highest among those who ride frequently (again not including the five people who ride fewer than a few times per year).

				Freque	ncy Riding	Bus		
			4–7 Days per Week	1–3 Days per Week	At Least Once per Month	A Few Times per Year	Less Than a Few Times per Year	Total
		Count	571	125	12	10	2	720
	Very satisfied	% within frequency riding bus	38.8%	45.1%	52.2%	58.8%	40.0%	40.2%
		Count	591	105	9	4	1	710
	Somewhat satisfied	% within frequency riding bus	40.2%	37.9%	39.1%	23.5%	20.0%	39.6%
0		Count	123	25	2	1	1	152
Satisfaction with Bus Service	Neither satisfied nor dissatisfied	% within frequency riding bus	8.4%	9.0%	8.7%	5.9%	20.0%	8.5%
		Count	123	19	0	1	0	143
	Somewhat dissatisfied	% within frequency riding bus	8.4%	6.9%	0.0%	5.9%	0.0%	8.0%
		Count	63	3	0	1	1	68
	Very dissatisfied	% within frequency riding bus	4.3%	1.1%	0.0%	5.9%	20.0%	3.8%
		Count	1471	277	23	17	5	1793
Total		% within frequency riding bus	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%

Overall Satisfaction with Bus Service * Frequency Riding Bus Cross-Tabulation

USING SPSS TO MERGE NOMINAL AND ORDINAL CATEGORIES

Recoding (initially discussed in chapter 5 to replace coded ordinal variables with the category midpoint) can also be used to merge those five very infrequent respondents and the seventeen few-times-per-year respondents with the adjacent category ("At least once per month"). This will clean up the findings and remove the very small numbers of respondents from the last three categories. This will eliminate the illusion that they are more important to the overall satisfaction findings than they are.

- 1. The recoding process once again is accessed through Transform—Recode into Different Variable
- 2. Give the recoded variable a new name (freqrec) and label (recoded frequency).
- 3. In Old and New Values, insert code 5 in the Value box in the Old Value section and code 3 in the Value box in the New Value section. This will merge codes 3 ("At least once per month") with code 5 ("Less than a few times per year").
- 4. Do the same for code 4, merging it into code 3. Nothing need be done with code 3 itself. Cases that have been coded 3 ("At least once per month" will remain in code 3.
- 5. This same recoding can be accomplished in one fewer step using the Range button in the Old Value section by inserting 4 through 5 under the Old Value and code 3 in the New Value section.
- 6. Inasmuch as no other code needs to be changed, check "All other values" in the Old Value section and "Copy old values" in the New Value section.
- 7. Again add the missing value instruction (System or User-Missing to System-Missing)
- 8. Press Continue and then OK.

A Recode into Different Variables: Old and New Values Old Value Ølydate Ølydate	New Value @ Value @ Value @ System-missing Copy old value(s) Og ⇒> New: MSSNG ⇒> SYSMIS 5 → 3 4 → 3 Change Benove
O All other values	Output variagles are strings Wildh: Convert numeric strings to numbers (5'->5)
Continue	Cancel Help

- 9. Look at the Variable View or Data View to make certain that the new variable was created, and then go to the row that contains the new variable in Variable View.
- 10. The recoding of an interval variable that was done in chapter 5 required no new values to be input because interval data values are in fact their codes. That is not the case for ordinal or nominal data. They require labels, and labels must be input for the new variable.

Analyzing Cross-Tabulated Data

- 11. Click on the far right side of the Values column for the freqrec row and proceed to label codes 1 and 2 using the same label for codes 1 and 2 as in the original variable and a slightly altered label for code 3 that merges the meanings of the original codes 3, 4, and 5 ("Approximately once per month or less").
- 12. Finally, if desired, the decimals can be changed from a default of 2 (if that has not previously been edited) to 0 in Edit—Options, as was also discussed in chapter 5.

talue La	bels	×
-Value La Val <u>u</u> e:	ibels	Spelling
Label:		
<u>C</u> h <u>R</u> e	dd 1.00 = "4-7 times per week" 2.00 = "1-3 times per week" 3.00 = "Approximately once per month or le	
	OK Cancel Help	

			Recoded Frequency				
		-	1–3 Times per Week	4–7 Times per Week	Approximately Once per Month or Less	Total	
		Count	571	125	24	720	
	Very satisfied	% within recoded frequency	38.8%	45.1%	53.3%	40.2%	
		Count	591	105	14	710	
	Somewhat satisfied	% within recoded frequency	40.2%	37.9%	31.1%	39.6%	
Overall		Count	123	25	4	152	
satisfaction with bus service	Neither satisfied nor dissatisfied	% within recoded frequency	8.4%	9.0%	8.9%	8.5%	
		Count	123	19	1	143	
	Somewhat dissatisfied	% within recoded frequency	8.4%	6.9%	2.2%	8.0%	
		Count	63	3	2	68	
	Very dissatisfied	% within recoded frequency	4.3%	1.1%	4.4%	3.8%	
		Count	1471	277	45	1793	
Total		% within recoded frequency	100.0%	100.0%	100.0%	100.0%	

Overall Satisfaction with Bus Service * Recoded Frequency Cross-Tabulation

Running a new cross-tabulation for freqrec and satis_overall-q1 produces the preceding table, which is more easily recognizable as overall satisfaction declining when frequency of riding increases than was the case with the original cross-tabulation.

The Chi-Square Test of Significance

From a cursory reading of table 10.4, one might make the following assumptions: Democrats tend to favor capital punishment more than either Republicans or Independents (56.5 percent versus 52.4 percent and 50.0 percent, respectively), and Independents tend to have no opinion on the issue of gun control more than either Democrats or Republicans (16.7 percent versus 10.9 percent and 11.9 percent, respectively).

These perceived differences may actually exist within the general population, or they may simply be the result of built-in sampling error in the random sampling process. Determination of the statistical validity of these perceived differences is made through statistical significance tests, of which the most frequently used in survey research is the chi-square test. Chi square is the only significance test available for data with both variables measured on the nominal scale. However, data measured on the ordinal and interval scales, organized into categories and presented in a contingency table, can also be tested using chi square.

The *chi-square test* of significance is essentially concerned with the differences between the frequencies that are obtained from the sample survey and those that could be expected to be obtained if there were no differences among the categories of the variables. As discussed in chapter 7, the assumption that no difference exists among the categories of the variables is known as the *null hypothesis*. The chi-square test seeks to identify whether

	Den	nocrat	Rep	ublican	Inde	pendent	Total	
Opinion	f	%	f	%	f	%	f	%
Favor	130	56.5	110	52.4	30	50.0	270	54.0
Do not favor	75	32.6	75	35.7	20	33.3	170	34.0
No opinion	25	10.9	25	11.9	10	16.7	60	12.0
Total	230	100.0	210	100.0	60	100.0	500	100.0

TABLE 10.4. OPINION CONCERNING CAPITAL PUNISHMENT BY POLITICAL PARTY.

the perceived findings are genuine and therefore generalizable to the full population or the result of sampling error, in which case they would not be generalizable. With reference to table 10.4, under the assumption of no difference, the researcher would expect that the overall percentage of those who favor capital punishment (54.0 percent) would also be the percentage of Democrats, Republicans, and Independents who favor capital punishment. The essence of this assumption is that if the percentages associated with the categories are the same as those associated with the entire distribution, it can be said that the two variables in the contingency table have no relationship to each other; under these circumstances, political party affiliation would have no bearing on opinion concerning capital punishment, and any differences obtained by the sample would have occurred by sampling error alone. The chi-square test seeks to identify whether any differences among the categories of the variables in the sample are genuine or merely the result of sampling error.

Calculation of the chi-square statistic (χ^2) involves measuring the difference between the expected frequencies that are consistent with the null hypothesis and those frequencies obtained through the survey process, in accordance with the following equation:

$$\chi^{2} = \Sigma \frac{\left(f_{0} - f_{e}\right)^{2}}{f_{e}}$$
(10.1)

where

 f_0 = the frequency obtained in each cell

 f_e = the frequency expected in each cell under the assumption of no difference

	Democrat		Republican		Independent			
	f _o	f _e	f _o	f _e	f _o	f _e	Row Totals	
Favor	130	(124.2)	110	(113.4)	30	(32.4)	270	
Do not favor	75	(78.2)	75	(71.4)	20	(20.4)	170	
No opinion	25	(27.6)	25	(25.2)	10	(7.2)	_60	
Column totals	230		210		60		500	

WORKSHEET 10.1. CHI-SQUARE MATRIX (n = 500).

The first step in the calculation of chi square is the establishment of a chisquare matrix worksheet consisting of obtained and expected frequencies. Worksheet 10.1 contains the matrix for the data in table 10.1. The numbers in each cell represent the obtained frequency (without parentheses) and the expected frequency (within parentheses) in which the overall percentage distribution of opinion is assumed, under conditions of no difference, to be replicated for each category of political affiliation. Thus, for each political affiliation, the expected frequencies reflect a distribution of 54.0 percent in favor, 34.0 percent not in favor, and 12.0 percent with no opinion.

An alternative method for calculating the expected frequencies is as follows. For each cell, multiply the row total corresponding to that cell by the column total and then divide the product by *n*. In the case of Democrats in favor of capital punishment, for example, the expected frequency of 124.2 can be calculated by multiplying 54.0 percent by the total number of Democrats $(0.54 \times 230 = 124.2)$ or by multiplying 270 by 230 and dividing by 500 (270 × 230/500 = 62,100/500 = 124.2).

Application of the chi-square formula yields the following calculation:

$$\begin{split} \chi^2 = \Sigma \frac{\left(f_0 - f_e\right)^2}{f_e} \\ = \frac{\left(130 - 124.2\right)^2}{124.2} + \frac{\left(11 - 113.4\right)^2}{113.4} + \frac{\left(30 - 32.4\right)^2}{32.4} + \frac{\left(75 - 78.2\right)^2}{78.2} + \frac{\left(75 - 71.4\right)^2}{71.4} \\ + \frac{\left(20 - 20.4\right)^2}{20.4} + \frac{\left(25 - 27.6\right)^2}{27.6} + \frac{\left(25 - 25.2\right)^2}{25.2} + \frac{\left(10 - 7.2\right)^2}{7.2} \\ = 0.271 + 0.102 + 0.178 + 0.131 + 0.180 + 0.008 + 0.241 + 0.002 + 1.120 \\ = 2.23 \end{split}$$

To interpret the calculated chi square, the researcher must refer to a table of critical chi-square values (see exhibit 10.1). This exhibit shows the required magnitude of the calculated chi square in order to achieve statistical significance—or the rejection of the null hypothesis. That is, if the calculated chi square (χ^2) equals or is greater than the critical chi square from exhibit 10.1 (χ^{2*}), the differences between obtained and expected frequencies within the cells are considered to be a reflection of a genuine difference between the categories of the variable. This difference indicates that a statistically significant relationship exists between the variables (i.e., political party does make a difference in opinion on capital punishment in the general population), not just the sample. If the calculated chi square is less than the critical chi square, then no relationship between the variables

has been identified, which in this case would allow the researcher no immediate alternative other than to operate on the basis that no genuine relationship exists between political party and opinion on capital punishment.

df	$\chi^{2^\star}_{.05}$	$\chi^{2^{*}}_{.01}$
1	3.841	6.635
2	5.991	9.210
3	7.815	11.345
4	9.488	13.277
5	11.070	15.086
6	12.592	16.812
7	14.067	18.475
8	15.507	20.090
9	16.919	21.666
10	18.307	23.209
11	19.675	24.725
12	21.026	26.217
13	22.362	27.688
14	23.685	29.141
15	24.996	30.578
16	26.296	32.000
17	27.587	33.409
18	28.869	34.805
19	30.144	36.191
20	31.410	37.566
21	32.671	38.932
22	33.924	40.289
23	35.172	41.638
24	36.415	42.980
25	37.652	44.314
26	38.885	45.642
27	40.113	46.963
28	41.337	48.278
29	42.557	49.588
30	43.773	50.892

EXHIBIT 10.1. CRITICAL VALUES OF THE CHI-SQUARE DISTRIBUTION.

The process of identifying the critical chi square for the contingency table (table 10.4) requires two pieces of information. First, the researcher must decide on the appropriate level of confidence (generally 95 percent or 99 percent). Second, degrees of freedom must be determined. The simple rule for degrees of freedom for chi square involves the following formula:

$$df = (r-1)(c-1) \tag{10.2}$$

where

df = degrees of freedom

r = number of categories of the dependent variable (row variable)

c = number of categories of the independent variable (column variable)

Degrees of freedom can be explained in the context of chi square as the number of cells that are free to vary. Once the values of this number of cells are known and all row and column totals are known, the values of all other cells become fixed. In the context of table 10.4, equation 10.2 indicates that df = 4. Thus, with the row and column totals identified and the frequencies of any four cells known, the remaining cell frequencies can be calculated.

The critical chi square for table 10.4, at the 95 percent level of confidence and with df = 4, can be determined from exhibit 10.1 to be 9.488. For the 99 percent level of confidence, the critical chi square is 13.277. Therefore, the calculated chi square of 2.23 is less than the critical chi square and is indicative of sampling error rather than of a statistically significant relationship between the variables. Because the researcher cannot establish a statistical relationship, he or she will proceed under the assumption that there is none.

In another example of the use of chi square, consider table 10.2, where it appears that there is a difference among various residential areas in their time as customers of the Richmond Water District. The calculated chi square for this table is 23.712, and at 10 *df*, this is sufficiently large (greater than the critical chi squares of 18.307 and 23.209) to indicate a statistically significant relationship between the variables at the 95 percent and 99 percent levels of confidence, respectively. In other words, length of time as a customer of the Richmond Water District had been found to differ by area of residence, and the apparent longer customer tenure of the West residents in the sample is generalizable to the full population in this water district.

Cramér's V and Phi (Φ)

Tests of statistical significance determine whether a relationship exists between variables, but they do not measure the strength of that relationship. Measures of association reflect the strength of the relationship between two or more variables. They are very useful as descriptive tools to indicate the strength of a relationship and as qualifying devices for determining whether certain findings merit reporting.

In the case of a very large survey sample with margins of error, for example, of ± 1 percent or less, there likely will be many findings of statistical significance. A study we once performed contained 55,000 surveys—reflecting a margin of error of ± 0.4 percent. Differences as small as 1 percent therefore were found to be statistically significant. To have reported all such findings of significance would have made the final report unreadable. Using measures of association as a qualifying device allowed the report to include only findings that had achieved a certain strength.

There are measures of association that can be derived for nominal data directly from the calculated chi-square statistic. The most versatile of these measures is Cramér's *V*, for which the formula is as follows:

$$V = \sqrt{\frac{\chi^2}{n(M-1)}} \tag{10.3}$$

where

 $\chi^2 =$ calculated chi square

n =sample size

M = minimum number of rows or columns

The possible values for Cramér's V range from 0 to 1, with 0 representing no association and 1 representing a perfect association.

Table 10.5 depicts the results of a survey of 550 respondents to a questionnaire about the value of physical therapy. The table seems to show that middle-aged people are more favorably inclined toward physical therapy than are the younger and the older groups.

A chi-square test found a significant calculated chi square of 51.02, indicating that middle-aged people are in fact more favorably inclined toward physical therapy. Cramér's V can add further information by telling

BT AGE OF RESPONDENT.								
	Unc	der 35	35	5–55	Ov	er 55	T	otal
Value	f	%	f	%	f	%	f	%
Always helpful	15	7.8	21	10.1	8	5.3	44	8.0
Sometimes helpful	68	35.4	135	64.9	85	56.7	288	52.4
Rarely helpful	98	51.0	50	24.0	46	30.7	194	35.3
Never helpful	11	5.8	2	1.0	11	7.3	24	4.3
Total	192	100.0	208	100.0	150	100.0	550	100.0

TABLE 10.5. VALUE OF PHYSICAL THERAPY, BY AGE OF RESPONDENT.

the researcher how strong the relationship between age and opinion about physical therapy is. Cramér's *V* is calculated in the following manner:

$$V = \sqrt{\frac{51.02}{550(3-1)}}$$
$$= \sqrt{\frac{51.02}{1,100}}$$
$$= \sqrt{.0464}$$
$$= .22$$

Exhibit 10.2 represents a scale for interpreting the meaning of Cramér's *V*. From exhibit 10.2, it can be found that the statistically significant relationship between age and opinion concerning physical therapy can be labeled as moderately strong. Note that most significant relationships are found to be moderate or relatively strong at best and that Cramér's *V* rarely achieves a value of .80 or above.

A special case of Cramér's *V* is phi (Φ). Phi is the measure of association based on the chi-square distribution when one or both of the variables contains only two categories. The formula for phi is as follows:

$$\Phi = \sqrt{\frac{\chi^2}{n}} \tag{10.4}$$

Note that the formula for phi is the same as the formula for Cramér's *V*, with at least one variable containing only two categories. Therefore, phi is also interpreted in accordance with the scale represented by exhibit 10.2.

Measure	Interpretation
.00 and under .10	Negligible association
.10 and under .20	Weak association
.20 and under .40	Moderate association
.40 and under .60	Relatively strong association
.60 and under .80	Strong association
.80 to 1.00	Very strong association

EXHIBIT 10.2. INTERPRETATION OF CALCULATED CRAMÉR'S V, PHI, AND LAMBDA MEASURES OF ASSOCIATION.

Additional Chi-Square Considerations

The use of chi square is subject to certain restrictions. As is true of all other sampling techniques and the results derived from them, the chi-square statistic is more reliable as the overall sample size increases. Consistent with this principle is the rule of thumb that each cell of the contingency table should contain an expected frequency of at least 5. If the expected frequency falls below 5 in any one cell, categories should be merged to eliminate the problem. This should be done in accordance with logic and reasonableness.

For example, a survey question that asked people to identify their dominant mode of travel contained the following categories: automobile, large bus, rail, taxi, minibus, dial-a-ride, and car pool. Let us suppose that the cells associated with the minibus and dial-a-ride categories contained expected frequencies of fewer than 5. It could be appropriate to combine these categories into a single category with an expected frequency of 5 or more because each of these forms of travel carries passengers on street-oriented systems in relatively small public vehicles. Alternatively, a researcher might decide to combine the minibus category with the bus category, since both have similar mass transit features, and combine the taxi and dial-a-ride categories, since each of these modes of travel is associated with placing a telephone call for service. In general, however, the researcher's goal is to maintain as many of the original question categories as possible; therefore, the process of combining categories should be undertaken only when necessary to achieve requisite expected frequencies.

Contingency tables consisting of two categories of independent variables and two categories of dependent variables (commonly referred to as 2×2 tables) present additional concerns. For example, if a 2×2 table contains cells with expected frequencies of fewer than 5, no merging of categories is possible. The researcher must employ a somewhat different test of statistical differences, known as *Fisher's exact test*. Also, results in a 2×2 table can be distorted if any one cell contains an expected frequency below 10. In such cases, it is recommended that the *Yates correction* be employed. This entails reducing the magnitude of the difference between observed frequencies and expected frequencies in each of the four cells by .5. Most computer software programs perform this correction automatically.

When either variable is on the ordinal or interval scale, the use of chi square can become problematic in interpreting the results. Consider table 10.6, which indicates the overall political orientation of teachers in the Santa Rosa School District on an ordinal continuum from "very liberal" to "very conservative," by gender.

The calculated chi square for table 10.6 is statistically significant ($\chi^2 = 18.40$), indicating a relationship between political orientation and gender. An initial glance at the table might lead the researcher to think that men are more conservative than women because 25 percent of the men consider themselves very conservative, whereas only 20 percent of the women do so. However, further review of the table shows 60 percent of the women and only 55 percent of the men in both conservative categories combined. The review also shows 45 percent of the men in the two liberal categories, while only 30 percent of the women are so categorized. Thus, it is not clear, from an initial review of the table, whether there is a consistent pattern identifying a particular gender with either a liberal or a conservative orientation.

A significant chi square can result from data that may possess major differences between obtained and expected frequencies in as few as two cells. In this example, these differences are most pronounced in the "moderate" cells. With nominal data, valid conclusions can still be drawn from such differences, but when the data are based on a continuum (as is the case in table 10.6), the objective is to detect an overall pattern. In table 10.6, there is no such clearly identifiable pattern. A significant chi square that results from two or more major discrepancies within the cells of ordinal or interval data can therefore lead the researcher to misinterpret the overall meaning of the data. Hence, additional tests of significance may be necessary to evaluate significant relationships among ordinal and interval variables.

	٨	Male		male	Total	
Political Orientation	f	%	f	%	f	%
Very liberal	20	20.0	20	20.0	40	20.0
Liberal	25	25.0	10	10.0	30	15.0
Moderate	0	0.0	10	10.0	15	7.5
Conservative	30	30.0	40	40.0	70	35.0
Very conservative	25	25.0	20	20.0	45	22.5
Total	100	100.0	100	100.0	200	100.0

TABLE 10.6. POLITICAL ORIENTATION OF TEACHERS IN THE SANTA ROSA SCHOOL DISTRICT, BY GENDER.

USING SPSS TO TEST THE SIGNIFICANCE OF CROSS-TABULATIONS—CHI-SQUARE TEST OF STATISTICAL SIGNIFICANCE

The Statistics tab in Analyze—Descriptive Statistics—Crosstabs provides the way for researchers to determine whether a finding is statistically significant or simply the result of sampling error. The cross-tabulation below (change _service_12mos_q2 by gender) seems to indicate that women think that bus service has improved over the past 12 months (33.2%) more than do men (29.6%) and, correspondingly, women think that service has worsened (15.8%) less than do men (20.4%).

			Gender of Respondent		
			Male	Female	Total
Bus service change		Count	227	332	559
in past 12 months	Better	% within gender of respondent	29.6%	33.2%	31.7%
		Count	383	509	892
	Same	% within gender of respondent	50.0%	51.0%	50.5%
		Count	156	158	314
	Worse	% within gender of respondent	20.4%	15.8%	17.8%
		Count	766	999	1765
Total		% within gender of respondent	100.0%	100.0%	100.0%

Bus Service Change Past 12 Months * Gender of Respondent Cross-Tabulation

The question that arises, however, is whether this apparent finding is statistically significant. By using the Statistics tab and then requesting Chi-square and Phi and Cramer's V, followed by Continue and OK, this can be determined. The significance of the Chi-square is .032 (Asymp. Sig. (2-sided)), indicating a significant finding, with the significance being .05 or less, and the Value of Cramer's V (.063), although significant, is negligible according to exhibit 10.2.

Crosstabs: Statistics	X
Chi-square	Correlations
Nominal	Ordinal
Contingency coefficient	📕 <u>G</u> amma
Phi and Cramer's V	📃 Somers' d
📃 Lambda	📃 Kendall's tau- <u>b</u>
Uncertainty coefficient	📕 Kendall's tau- <u>c</u>
Nominal by Interval	🛅 <u>K</u> appa
🛅 <u>E</u> ta	🔲 R <u>i</u> sk
	McNemar
Cochr <u>a</u> n's and Mantel-Hae	enszel statistics
Test common odds ratio e	equals: 1
Continue	Help

Chi-Square Tests

	Value	df	Asymp. Sig. (2-sided)
Pearson chi square	6.895ª	2	.032
Likelihood ratio	6.862	2	.032
Linear-by-linear association	6.053	1	.014
N of valid cases	1,765		

^a No cells (.0%) have expected count less than 5. The minimum expected count is 136.27.

Symmetric Measures

		Value	Approx. Sig.
Nominal by	Phi	.063	.032
nominal	Cramer's V	.063	.032
N of valid cases		1,765	

Analyzing Cross-Tabulated Data

It is important to look at the note (superscript "a") beneath the Chi-Square Tests output. As discussed in this chapter, chi-square tests require that all cells have expected frequencies equal to 5 or more. This note indicates whether this requirement has been met. In this case, that requirement has been met and the researcher can proceed to honor the results as being significant, with a negligible association.

Employment Status * Condex of Despendent			Gend Respo	ler of ndent	
Cross-Tabulatio	atus * Gender of F	Respondent	Male	Female	Total
Employment		Count	287	314	601
Status	Employed full time	% within gender of respondent	36.8%	31.1%	33.6%
		Count	142	248	390
	Employed part time	% within gender of respondent	18.2%	24.6%	21.8%
		Count	99	163	262
	Student—not working	% within gender of respondent	12.7%	16.1%	14.6%
		Count	7	70	77
	Homemaker	% within gender of respondent	0.9%	6.9%	4.3%
		Count	125	108	233
	Not currently employed	% within gender of respondent	16.0%	10.7%	13.0%
		Count	79	66	145
	Disabled— unable to work	% within gender of respondent	10.1%	6.5%	8.1%
		Count	36	39	75
	Retired	% within gender of respondent	4.6%	3.9%	4.2%
		Count	4	2	6
	Other	% within gender of respondent	0.5%	0.2%	0.3%
		Count	779	1010	1789
Total		% within gender of respondent	100.0%	100.0%	100.0%

	Value	df	Asymp. Sig. (2-sided)
Pearson chi square	71.764ª	7	.000
Likelihood ratio	79.310	7	.000
Linear-by-linear association	2.903	1	.088
N of valid cases	1,789		

Chi-Square Tests

^aTwo cells (12.5%) have expected count less than 5. The minimum expected count is 2.61.

If the note shows expected frequencies below 5, as it does in the example of employment (employment_q27) status and gender, the recoding process shown above to combine categories must be undertaken. In this case, it appears as if men are employed full time to a greater extent than are women. Men are also unemployed more than are women, who are more likely to be part-time employees and students. However, note a indicates that two categories fail the requirement. In order to identify which two categories fail, return to the Cells tab and check Expected Counts—Continue—OK.

The output now shows the expected counts and allows the researcher to identify the cells that violate the requirement: Male-Other Employment (expected count = 2.6) and Female-Other Employment (expected count = 3.4).

Counts	z-test
✓ Observed	Compare column proportions
Expected	Adjust p-values (Bonferroni method)
Hide small counts	
Less than 5	
Percentages	Residuals
Row	Unstandardized
Column	Standardized
Total	Adjusted standardized
Noninteger Weights	
Round cell counts	◎ Round case <u>w</u> eights
O Truncate cell counts	© Truncate case weig <u>h</u> ts
© No adjust <u>m</u> ents	

			Gen Respo	der of ondent	
			Male	Female	Total
		Count	287	314	601
		Expected count	261.7	339.3	601.0
	Employed full time	% within gender of respondent	36.8%	31.1%	33.6%
		Count	142	248	390
		Expected count	169.8	220.2	390.0
	Employed part time	% within gender of respondent	18.2%	24.6%	21.8%
		Count	99	163	262
		Expected count	114.1	147.9	262.0
	Student—not working	% within gender of respondent	12.7%	16.1%	14.6%
		Count	7	70	77
		Expected count	33.5	43.5	77.0
Employment	Homemaker	% within gender of respondent	0.9%	6.9%	4.3%
status		Count	125	108	233
		Expected count	101.5	131.5	233.0
	Not currently employed	% within gender of respondent	16.0%	10.7%	13.0%
		Count	79	66	145
	Disabled—	Expected count	63.1	81.9	145.0
	unable to work	% within gender of respondent	10.1%	6.5%	8.1%
		Count	36	39	75
		Expected count	32.7	42.3	75.0
	Retired	% within gender of respondent	4.6%	3.9%	4.2%
		Count	4	2	6
		Expected count	2.6	3.4	6.0
	Other	% within gender of respondent	0.5%	0.2%	0.3%
		Count	779	1,010	1,789
		Expected count	779.0	1,010.0	1,789.0
Total		% within gender of respondent	100.0%	100.0%	100.0%

Employment Status * Gender of Respondent Cross-Tabulation

Old Value	New Value
O <u>V</u> alue:	O Value:
	System-missing
O System-missing	Copy old value(s)
System- or user-missing	Old> New:
© Ra <u>n</u> ge:	10> 4
	MISSING> SYSMIS
inougn	Add ELSE -> Copy
Range, LOWEST through value:	Change
	Remove
Range, value through HIGHEST:	
◎ All <u>o</u> ther values	Output variables are strings Width
	Convert numeric strings to numbers ('5'->5)

Cells must be combined in a logical manner and then the chi square is to be rerun. The reader might choose, for example, to combine Other with Homemaker and to follow the instructions in this chapter for combining categories. The reader should try to do this himself or herself and should obtain the following chi square and Cramer's V output that indicate that the preliminary finding of significance still holds and that the association is now stronger with the elimination of this small category and is a weak-to-moderate association. Also note that all cells are of sufficient size regarding their expected counts.

	Value	df	Asymp. Sig. (2-sided)
Pearson chi square	64.258ª	6	.000
Likelihood ratio	68.929	6	.000
Linear-by-linear association	2.347	1	.126
N of valid cases	1,789		

Chi-Square Test

^aNo cells (.0%) have expected count less than 5. The minimum expected count is 32.66

		Value	Approx. Sig.
Nominal by Nominal	Phi	.190	.000
	Cramer's V	.190	.000
N of valid cases		1,789	

Symmetric Measures

$\overline{\text{Gamma}}(\gamma)$

The nature of chi square, a test built for nominal data, is such that significance is achieved if cells differ sufficiently from their null hypothesis values regardless of logical order. As such, when trends are being analyzed (e.g., a hypothesis that as incomes increase, people become more conservative politically), chi square may not be the best test of significance available. Gamma (γ) generally proves to be a better test. Gamma is not a significance test; rather, it is a measure of association (as is Cramér's *V*) that has a *Z* significance test that can be performed along with it.

When each of two variables is on the ordinal or interval scale, γ is an appropriate measure of association and one that measures the strength of trends in the data. The formula for gamma is

$$\gamma = \frac{\Sigma(f_i \cdot \Sigma f_s) - \Sigma(f_i \cdot \Sigma f_d)}{\Sigma(f_i \cdot \Sigma f_s) + \Sigma(f_i \cdot \Sigma f_d)}$$
(10.5)

where

 f_i = the frequency of any cell

 f_s = the frequency of a cell ordered in the same direction from the subject cell

 f_d = the frequency of a cell ordered in a different (or inverse) direction from the subject cell

Consider table 10.7, which results from cross-tabulating two survey questions: one eliciting information about the educational level of the college graduate respondents to the survey and the other about the respondent's socioeconomic status.

The researcher can determine $\Sigma(f_i \cdot \Sigma f_s)$ by taking the frequency in each cell and multiplying that number by the frequencies in each category cell that is both below that cell and to the right of it—in other words, cells for which both variables are either lower in rank or higher in rank than the cell under consideration. In the case of table 10.7, $\Sigma(f_i \cdot \Sigma f_s)$ can be calculated as follows:

$$\begin{split} \Sigma(f_i \cdot \Sigma f_s) &= 25(55+25+30+25) = 3,375 \\ &+ 10(25+25) = 500 \\ &+ 20(30+25) = 1,100 \\ &+ 55(25) = 1,375 \\ \mathrm{Total} &= 6,350 \end{split}$$

	Educational Level							
Socioeconomic	Doctorate		Master's		Bachelor's		Total	
Status	f	%	f	%	f	%	f	%
Upper	25	62.5	20	20.0	5	8.3	50	25.0
Middle	10	25.0	55	55.0	30	50.0	95	47.5
Lower	5	12.5	25	25.0	25	41.7	55	27.5
Total	40	100.0	100	100.0	60	100.0	200	100.0

TABLE 10.7. EDUCATIONAL LEVEL, BY SOCIOECONOMIC STATUS.

The determination of $\Sigma(f_i \cdot \Sigma f_d)$ requires taking the frequency in each cell and multiplying that number by the frequencies in each category cell that is both below and to the left of it—below and left being inverse directions, with one variable higher in rank and the other variable lower. Again from table 10.7:

$$\begin{split} \Sigma(f_i \cdot \Sigma f_d) &= 5(55{+}25{+}10{+}5) = 475 \\ &+ 30(5{+}25) = 900 \\ &+ 20(10+5) = 300 \\ &+ 55(5) = 275 \\ \mathrm{Total} &= \overline{1,950} \end{split}$$

Hence, gamma can be calculated as follows:

$$\gamma = \frac{6,350 - 1,950}{6,350 + 1,950} = \frac{4,400}{8,300} = +.53$$

The calculated gamma of +.53 is interpreted on a scale that ranges from -1.00 to +1.00. A positive gamma is read from the upper left corner of the cross-tabulation (table 10.7 in this case) to the right across the top (i.e., education declines) and from the upper left corner down the rows (i.e., status declines). The calculated gamma therefore indicates that as education declines, so does socioeconomic status.

A negative gamma is read again from the upper left and again to the right across the top, but it is read from the bottom row to the top row (in contrast to the positive gamma). As such, had the calculated gamma been negative, the interpretation would have been that a decline in education is associated with a higher socioeconomic status. The scale presented in exhibit 10.3 can serve as a guideline for interpreting the calculated gamma measure, which, in the case of table 10.7, can be thought of as moderate in nature.

Testing the Statistical Significance of Gamma

Any apparent statistical pattern or trend that the researcher observes in the data must be formally tested for statistical significance. The chi square serves as the test for the significance of Cramér's *V*, phi, and lambda, but the presence of ordinal or interval data used in the calculation of gamma permits the use of a more direct test of significance for the calculated gamma.

EXHIBIT 10.3. INTERPRETATION OF CALCULATED GAMMA.

Measure	Interpretation		
0	No association		
.01–.09	Negligible association		
.10–.29	Low association		
.30–.59	Moderate association		
.60–.74	Strong association		
.75–.99	Very strong association		
1.00	Perfect association		

The following equation is used to calculate the Z score necessary for determining the significance of gamma:

$$Z = \gamma \left(\sqrt{\frac{\Sigma(f_i \cdot \Sigma f_s) - \Sigma(f_i \cdot \Sigma f_d)}{n(1 - \gamma^2)}} \right)$$
(10.6)

When equation 10.6 is applied, the gamma derived from table 10.7 yields

$$Z = .53 \left(\sqrt{\frac{4,400}{200(1-(.53)^2)}} \right)$$
$$= .53 \left(\sqrt{\frac{4,400}{143.82}} \right)$$
$$= 2.93$$

Inasmuch as gamma can be either positive or negative, the significance test for gamma's Z score is a two-tail test, with critical Z scores of 1.96 (95 percent confidence) and 2.575 (99 percent confidence). As such, the calculated Z of 2.93 is indicative of significance at both the 95 percent and 99 percent levels of confidence.

It is noteworthy that the significance test for gamma can be used to test the significance of ordinal data in contingency tables when the results of the chi-square test are not definitive because of the insensitivity of chi square to patterns and trends, as discussed previously.

USING SPSS TO TEST THE SIGNIFICANCE OF CROSS-TABULATIONS—GAMMA

Gamma is found in SPSS in the same way as chi square and Cramer's V are (Analyze—Descriptive Statistics—Crosstabs—Statistics). Gamma can be run by itself or with the chi square inasmuch as sometimes the chi square is sufficiently informative even though it is designed primarily for nominal data. Gamma, however, is considerably advantageous when categories need to be collapsed for chi square in that there is no such requirement that cells contain at least an expected value of 5 for gamma.

Readers should access the Transportation.sav data file and assign the independent variable to the columns and dependent to the rows. If that is not easily identifiable or not part of the research hypothesis (e.g., if causality is not the issue—only association), then it does not matter which is assigned to the columns and which to the rows. The gamma association for two ordinal variables (satis_overall_q1) and satis_traveltime_q3n) is as follows, indicating a positive association: as the satisfaction with travel time declines (reading right across the top), so does the satisfaction with the overall system (top to bottom on the rows).

Symmetric Measures

		Asymp. Std.			Approx.	
		Value	Error ^a	Approx. T ^b	Sig.	
Ordinal by ordinal	Gamma	.513	.024	19.491	.000	
N of valid cases		1782				

^aNot assuming the null hypothesis.

^bUsing the asymptotic standard error assuming the null hypothesis.

According to exhibit 10.3, a gamma value of +.513 is a moderate association and, with an Approx. Sig. of .000, this is a statistically significant gamma.
					Satisfaction: Travel	Time		
			Very Satisfied	Somewhat Satisfied	Neither Satisfied nor Dissatisfied	Somewhat Dissatisfied	Very Dissatisfied	Total
		Count	362	227	62	30	26	707
		Expected count	216.2	233.7	128.1	79.3	49.6	707.0
	Very satisfied	% within satisfac- tion: travel time	66.4%	38.5%	19.2%	15.0%	20.8%	39.7%
		Count	134	287	168	100	22	711
	Somewhat	Expected count	217.4	235.0	128.9	79.8	49.9	711.0
	satisfied	% within satisfac- tion: travel time	24.6%	48.7%	52.0%	50.0%	17.6%	39.9%
Overall		Count	18	36	45	35	18	152
Satisfaction	Neither	Expected count	46.5	50.2	27.6	17.1	10.7	152.0
with Bus Service	satisfied nor dissatisfied	% within satisfac- tion: travel time	3.3%	6.1%	13.9%	17.5%	14.4%	8.5%
		Count	20	35	34	27	29	145
		Expected count	44.3	47.9	26.3	16.3	10.2	145.0
	Somewhat dissatisfied	% within satisfac- tion: travel time	3.7%	5.9%	10.5%	13.5%	23.2%	8.1%
		Count	11	4	14	8	30	67
	Verv	Expected count	20.5	22.1	12.1	7.5	4.7	67.0
	dissatisfied	% within satisfac- tion: travel time	2.0%	0.7%	4.3%	4.0%	24.0%	3.8%
		Count	545	589	323	200	125	1,782
		Expected count	545.0	589.0	323.0	200.0	125.0	1,782.0
Total		% within satisfac- tion: travel time	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%

Overall Satisfaction with Bus Service * Satisfaction: Travel Time Cross tabulation

Notes

1. (This note is best understood after reading the entirety of chapter 10.) Another measure of association associated with nominal scale data is lambda (λ). The formula for lambda is as follows:

$$\lambda = \frac{\Sigma F_{iv} - R_{dv}}{n - R_{dv}} \tag{10.7}$$

where

 F_{iv} = largest cell frequency for each category of the independent variable R_{dv} = largest row total among the categories of the dependent variable n = sample size

Table 10.8 depicts the results of a survey of 510 residents of a metropolitan area in Rhode Island. Respondents were asked to indicate their most frequent method of commuting to work. Results of this question were cross-tabulated by subarea. Applying equation 10.7 to these data yields the calculation for lambda, as follows:

$$\Sigma F_{iv} = 68 + 70 + 103 + 100 = 341$$

$$R_{dv} = 273$$

$$n = 510$$

$$\lambda = \frac{341 - 273}{510 - 273}$$

$$= \frac{68}{237}$$

$$= .29$$

The calculated lambda can also be interpreted according to exhibit 10.2 as indicative of a moderate association between metropolitan subareas and mode of travel to work. As with Cramér's V and phi, lambda values can range from 0 (no association) to 1 (perfect association).

	Inner City		Mic	d-City	Sub	urban	Exi	urban	Total	
_	f	%	f	%	f	%	f	%	f	%
Auto	20	17.0	50	31.3	103	83.1	100	92.6	273	53.5
Bus	68	57.6	70	43.7	16	12.9	4	3.7	158	31.0
Bicycle/walk	30	25.4	40	25.0	5	4.0	4	3.7	79	15.5
	118	100.0	160	100.0	124	100.0	108	100.0	510	100.0

TABLE 10.8. MOST FREQUENTLY USED MODE OF TRANSPORTATION TO WORK, BY SUBAREA.

Beyond its usefulness as a measure of association, lambda also measures the extent to which one variable serves to explain the variation in the other variable and can, therefore, be used as a clue (not definitive) in determining the variable that is more likely to be the independent variable when that is otherwise not obvious.

USING SPSS TO DETERMINE LAMBDA

Lambda is also found in Analyze—Descriptive Statistics—Crosstabs—Statistics. As indicated, lambda is particularly useful in providing some indication about which variable affects the other more. That is, when it is difficult to determine an independent variable, lambda can be called on to help—not necessarily be the one and only answer—but to help by providing one possible indicator.

Again in the Transportation.sav file, it might be difficult to identify theoretically whether the length of time that someone has been a bus rider (length_rider_q8) affects his or her overall satisfaction (e.g., the longer someone has ridden, the more disenchanted and less satisfied he or she is) more than whether overall satisfaction affects ridership length (e.g., the more satisfied someone is with the bus, the longer he or she has chosen to ride the bus).

Lambda shows that although the relationship between overall satisfaction and length of time that the bus rider has used the bus for his or her transportation needs is not quite statistically significant, researchers have a clue that seems to indicate that the effect that length of ridership has on satisfaction (.052 with satisfaction dependent) is stronger than the effect that satisfaction has on length of ridership (.000). The researcher would be cautioned, however, to undertake more tests than just this one—especially when not significant. Such tests are beyond the scope of this book.

			Value	Asymp. Std. Error ^a	Approx. T ^ь	Approx. Sig.
Nominal by nominal	Lambda	Symmetric	.028	.014	1.926	.054
		How long been bus rider dependent	.000	.000	·c	. ^c
		Overall satisfaction with bus service dependent	.052	.027	1.926	.054
	Goodman and Kruskal tau	How long been bus rider dependent	.006	.002		.002 ^d
		Overall satisfaction with bus service dependent	.005	.002		.001 ^d

Directional Measures

^aNot assuming the null hypothesis.

^bUsing the asymptotic standard error assuming the null hypothesis.

^cCannot be computed because the asymptotic standard error equals zero.

^dBased on chi-square approximation.

^eGoodman and Kruskal's tau is another measure of association that identifies the degree to which one variable predicts the other. In that sense, it is not unlike Lambda.

2. The frequency percentages should always be shown to sum to 100 percent. If, due to rounding error, the percentages total slightly less or slightly more than 100 percent, the researcher can address the problem in one of two ways. First, she or he may show a total percentage of 100 percent even if the individual percentages result in a slightly different sum; a footnote should be added to the table stating that percentages may not add to 100 percent because of rounding error.

Second, a category can be identified with a percentage that can be adjusted to achieve a total of 100 percent. It is recommended that the category chosen be the one that is associated with the percentage closest to the point at which rounding takes place. For example, in table 10.1, the category "Less than 1" year is associated with a percentage of exactly 13.049. Properly rounded, this percentage should read 13.0. In so doing, however, the total percentage for the entire adjusted frequency distribution equals 99.9 percent. The closeness of 13.049 to the rounding point of 13.05 warrants its selection as the category to be adjusted.

We recommend the second approach because the slight advantage in accuracy of using 13.0 percent is far less important than the disadvantage of the appearance of inaccuracy resulting from a column of percentages that does not add up to the total percentage indicated. The primary purpose of tabular construction is to communicate data visually with simplicity and clarity, and the second approach more readily achieves this objective.



CHAPTER ELEVEN

TESTING THE DIFFERENCE BETWEEN MEANS

Chapter Ten discussed analytical statistics for data that lend themselves to presentation in the form of contingency tables. Contingency table data typically are not interval in level of measurement because the very large number of individual values associated with interval data would render the table unwieldy and unreadable. This chapter continues the discussion of significance tests that are most appropriately applied when the dependent variable is interval, on ordinal scale treated as interval (see chapter 7), or on ordinal containing categories of grouped interval data (chapters 3 and 5).

These significance tests are more powerful statistical tools, with widerranging applicability than the tests discussed in the previous chapter.

Independent Samples t Test

When the dependent variable is on the interval scale and an independent variable consists of only two categories on any scale, there is a test of statistical significance that is more rigorous than the chi-square test of significance or gamma's Z. This test involves the comparison of the arithmetic means of the two categories of the independent variable, and it is known as the *independent samples t test*.

Table 11.1 reflects two distinct sample groups of government employees who sat for a promotional examination. A review of these data would

Test	Water U	Jtilities	Final Manag	ncial Jement	Total		
Scores	f	%	f	%	f	%	
90–100	30	15.0	20	10.0	50	12.5	
80–89	65	32.5	35	17.5	100	25.0	
70–79	60	30.0	65	32.5	125	31.3	
60–69	30	15.0	30	15.0	60	15.0	
50–59	10	5.0	30	15.0	40	10.0	
40–49	5	2.5	_20	10.0	_25	6.2	
	200	100.0	200	100.0	400	100.0	
	$\bar{x}_1 = 77.6$		$\bar{x}_{2} = 70.8$				
	s ₁ = 12		$s_2 = 14.5$				

TABLE 11.1. PROMOTIONAL EXAMINATION SCORE, BY DEPARTMENT AFFILIATION.

lead the researcher to suspect that water utilities employees tend to perform better than financial management employees (note the one-tail question; see chapter 7), with the mean score for water utilities employees almost seven points higher than for financial management employees. This suspicion should prompt the researcher to test whether this apparent difference is genuine or the result of sampling error.

In the case at hand, the dependent variable, being on the interval scale, will permit the researcher to employ the independent samples *t* test. The equation for the test is as follows:

$$t = \frac{\overline{x_1} - \overline{x_2}}{\sqrt{\frac{s_1^2}{n_1 - 1} + \frac{s_2^2}{n_2 - 1}}}$$
(11.1)

Application of the equation to the data in table 11.1 yields

$$t = \frac{77.6 - 70.8}{\sqrt{\frac{(12)^2}{199} + \frac{(14.5)^2}{199}}}$$
$$t = 5.1$$

Degrees of freedom are n - 1 for each group, so the total degrees of freedom are

$$df = n_1 + n_2 - 2, (11.2)$$

which, when applied to this case, equals 398.

Exhibit 7.2 can be consulted, and it is seen that t = 5.1 exceeds the critical t for one-tail questions at both the 95 percent (critical t = 1.645) and 99 percent (critical t = 2.33) confidence levels. Note that when df exceeds 120, the critical t and critical Z (see chapter 7) are considered to effectively coincide. The independent samples t test can be used to test the difference between any two arithmetic means, whether or not these means derive from a table, such as table 11.1. For example, suppose that 6,000 individuals across the United States are asked how many miles they commute to work. The 2,000 respondents from west of the Mississippi River indicate a mean of 6.25 miles (standard deviation = 2.12 miles). The 4,000 respondents from east of the Mississippi reveal a mean commute of 6.03 miles (standard deviation = 1.31 miles). Can the researcher conclude that a significant difference (two-tail question) exists based on geographic considerations?

Applying equation 11.1 provides the following calculations:

$$t = \frac{6.25 - 6.03}{\sqrt{\frac{(2.12)^2}{1,999} + \frac{(1.31)^2}{3,999}}} = \frac{0.22}{\sqrt{0.0022 + 0.0004}} = \frac{0.22}{0.05} = 4.4$$

A calculated t of 4.4 exceeds both the 95 percent confidence level critical two-tail t of 1.96 and the 99 percent confidence level critical t of 2.575. Hence, the researcher can conclude with 99 percent confidence that the two sections of the country differ significantly in their commuting mileage.

USING SPSS TO CONDUCT THE INDEPENDENT SAMPLES *t* TEST

It might be a legitimate question in the Transportation.sav file to find out if the number of transfers differs between white riders and Latino riders. Whether any differences are cultural or economic, the first step is to identify if a difference exists. Inasmuch as the number of transfers is scale and that the question is posed between two ethnic group categories, an independent samples *t* test is appropriate. The process is as follows:

- 1. Analyze—Compare Means—Independent-Samples T Test.
- The Test Variables is the scale variable for which means are to be compared. (transfers#_q15)
- 3. The Grouping Variable is the one that has two categories being compared (ethnicity_q34)

Independent-Samples T Te: figure effectiveinfo_ema figure effectiveinfo_onb figure effectiveinfo_broc figure effectiveinfo_tele figure effectiveinfo_tele figure passby_q12 figure passby_q14 figu	t Test Variable(s):	Options Bootstrap
othermodes_wal	Define Groups	

- 4. The two question marks in the Grouping Variable box are to be filled in with the code values for white and Latino in the ethnicity_q34 variable, which requires clicking on Define Groups and then inserting the code values for White (1) and Latino (2)—either order works in a two-tailed question such as this (difference between the two groups without direction designated).
- 5. Click on Continue and Options. Make certain that the Confidence Interval is 95% and that the cases are excluded analysis by analysis.
- 6. Click on Continue—OK, and the first output table (Group Statistics) will show the mean number of transfers by white riders at 1.57 and the mean for Latino riders at 2.03. It is the second table that tells the researcher if this difference is significant.

The manual equation that we have used in this chapter for determining the independent samples *t* has an underlying assumption that the variances (standard deviation squared) of the two groups are not statistically dissimilar. SPSS provides a *t* calculation that establishes whether this assumption holds by including Levene's Test for Equality of Variances. In this case, the significance of Levene is .087, which is not significant. The absence of significance means that the two variances are not dissimilar—the researcher can continue to assume that the variances are equal.

With equal variances assumed (Levene significance greater than .05), the *t* value is found on the first row of the Independent Samples Test output, which, in the case of ethnicity and number of transfers, indicates that this is a statistically significant difference, (Sig. (2-tailed) = .000), and that whites and Latinos do (at 95% confidence) have a different mean number of transfers. Were the variances not determined by Levene to be equal, then the second row of the output would be used, and, as is the case under most circumstances, the conclusion would be the same—a statistically significant difference between white and Latino riders regarding the number of transfers they make while riding the bus.

Group Statistics

	Ethnicity	N	Mean	Std. Deviation	Std. Error Mean
Number of transfers— subject trip	White/Caucasian Hispanic/Latino	421 969	1.57 2.03	1.253 1.294	.061 .042

Independent Samples Test

		Levene's Test for Equality of Variances			t	Test for E	quality o	of Means	5	
		F Sig.				Sig. (2-	Mean Diffe-	Std. Error Diffe-	95% Co Interva Diffe	onfidence al of the erence
			t	t df tailed) rence renc		rence	Lower	Upper		
Number of	Equal variances assumed	2.931	.087	-6.088	1388	.000	455	.075	602	309
transfers— subject trip	Equal vari- ances not assumed			-6.166	822.826	.000	455	.074	600	310

One-tailed variation: Note that the SPSS independent samples t test is a twotailed test. In order to read it as a one-tailed test, make certain that the order of the variables in the Define Variables box conforms to the rules of this chapter and chapter 7 by placing the code of the category that is the subject of the research question in group 1 and the category code to which that subject category is being compared is placed in group 2. Once that is done, change the 95% confidence interval in Options to 90% (5% in each tail); then press Continue and OK.

As was discussed in chapter 7, make certain that the sign of the calculated t is correct, and then judge the finding to be significant if the two-tailed significance is .10 or less. This is indicative of .05 in each tail; however, inasmuch as the sign must

be correct, this remains a 95% test, with no tail on one side (because of the requirement that the sign be correct) and a 5% tail on the other.

Paired Samples t Test

The independent samples t test assumes that the variances for each sample are mutually exclusive. That is, they are independent samples. There are situations when two samples are not independent. A before-and-after test is a situation where the second sample (after) is dependent on the first (before). For example, if a specially designed efficiency program were to be introduced to the San Antonio Police Department (see table 5.6 in chapter 5) in an effort to improve the department's nonemergency response times, and the 9.3 minute response time were improved to 7.9 for a sample of 25 officers with a standard deviation of 3.4 (recall that table 5.6 indicates n = 225 and s = 3.6), the police chief would be interested in determining if this increase were statistically significant (one tail). These 25 officers were chosen at random from among the 225 who took the test originally and for whom individual performance data from that first test are no longer available. Equations 11.3 and 11.4 would provide this information.

First, calculate the overall standard deviation of the paired samples:

$$s = \sqrt{((n_1 - 1)s_1^2 + (n_2 - 1)s_2^2) \div (n_1 + n_2 - 2)}$$
(11.3)

Then calculate the *t* test, as follows:

$$t = (\overline{x}_1 - \overline{x}_2) \div s\sqrt{(1/n_1 + 1/n_2)}$$
(11.4)

In this case, the calculation would yield

$$s = \sqrt{(24(11.56) + 224(12.96)) \div (25 + 225 - 2))}$$
$$= \sqrt{(277.44 + 2903.04) \div 248}$$
$$= \sqrt{12.82}$$

$$s = 3.58$$

$$t = (7.9 - 9.3) \div 3.58 \left(\sqrt{1/25 + 1/225}\right)$$

$$= -1.4 \div 3.58 \left(\sqrt{.0444}\right)$$

$$= -1.4 \div 3.56 (.211)$$

$$= -1.4 \div .76$$

$$t = -1.84$$

The one-tailed (looking for significant increase) critical *t* (from exhibit 7.2, $-df = n_1 + n_2 - 2$) is 1.645 at 95 percent confidence; therefore, the efficiency program did improve police performance.

USING SPSS FOR THE PAIRED SAMPLES t TEST

The paired samples *t* test is accessed on SPSS via Analyze—Compare Means—Paired-Samples T Test. With this test being used in this chapter in the event of some form of intervention in order to assess changes in the tested population, data found in the other data set included with this text (Water.sav) are more amenable to this test. After accessing that data set, there are variables labeled percent#1comb, pctaftmess-comb#2, and pctnotUScomb#3. These three variables indicate respondents' opinions about the percentage of the region's water supply that should come from desalinated water—#1 being an initial impression, #2 being after messages about the benefits of desalinated water are shared, and #3 being after it becomes known that the source of this water will be from outside the United States. This is a perfect scenario for the paired samples *t* test. To see how the messages affect opinion, Descriptives shows that the mean percentage for the first question is 47.53 and the mean percentage for the second is 50.81. This would seem to indicate that the messages made a difference—enlarging the percentage by more than three percentage points.

	N	Minimum	Maximum	Mean	Std. Deviation
q7 and q7arec combined	361	0	100	47.53	28.021
q9 and q9arec combined	364	0	100	50.81	28.954
Valid <i>N</i> (listwise)	356				

Descriptive Statistics

The statistical significance of this change can be determined using the paired samples *t* test, which provides the following window when first opened.

Index gender Image: gender	Paired-Samples T Te	st	Paired	Variables:			
✓ respiration 1 Induct Fundable Eootstra Image: Second sec	A response	4	Pair	Variable1	Variable?	1	Options
Importance I	Cust Cust Cust Cust Cust Cust Cust Cust		1		Valiablez	•	Bootstrap
Seprecwater Seprecwater	 defwatpure importance usedrecwat whereused 					•	
	exprecwater positive pegative	•					

Inasmuch as the research question is a one-tailed question about whether #2 improves upon #1, place the #2 percentage in the Variable1 box and the #1 percentage in the Variable2 box. Under Options, select 90% as the one-tailed confidence interval and make certain that cases are excluded analysis by analysis. The output is as follows:

Paired-Samples Statistics

		Mean	N	Std. Deviation	Std. Error Mean
D · 1	q9 and q9arec combined	51.29	356	28.882	1.531
Pair I	q7 and q7arec combined	47.80	356	28.109	1.490

Note that there is a small difference in the mean percentages from Compare Means because only 356 of the total cases have provided both of these percentages in contrast to 361 that provided percentage #1 and 364 that provided percentage #2 separately.

Paired-Samples Correlation

		Ν	Correlation	Sig.
Pair 1	q9 and q9arec combined & q7 and q7arec combined	356	.774	.000

Pair	ed-Sa	amp	les 1	Fest
		p		

			Paired Differences							
			Std	Std. Error	90% Co Interva Diffe	nfidence l of the rence			Sig (2	
		Mean	Deviation	Mean	Lower	Upper	t	df	tailed)	
Pair 1	q9 and q9arec combined— q7 and q7arec combined	3.486	19.188	1.017	1.809	5.163	3.428	355	.001	

Again, making certain the sign of t is correct (positive t = 3.428 indicating an increased percentage (with the second opinion being entered first as Variable 1), a two-tailed significance of .10 or less will indicate a statistically significant improvement in this one-tailed environment. The results therefore show that this is a significant improvement and that the messages had a positive impact. If the question were two-tailed (difference versus improvement), the order of the percentages as Variable1 and Variable2 would not make a difference, nor would the sign of t; however, the Options should reset the confidence interval to 95% and the two-tailed significance would need to be measured against .05 (which standard would still be met).

Difference of Proportions Test

There exists a variation on the independent samples t test that is applicable to percentage values for the dependent variable for any two categories of the independent variable. This test is very specific to only two independent variable categories. Therefore, for the vast majority of analyses of survey data that are nominal or ordinal, chi-square and gamma's Z are still the significance tests of choice.

Frequently survey data from two groups are analyzed in conjunction with proportions and percentages rather than arithmetic means. This is particularly so in cases where ordinal and nominal data are present—data that do not lend themselves to the calculation of means. For instance, a sample survey of 267 respondents in Missoula, Montana, finds that 45 percent of the respondents have trust in their public leaders. A similar survey of 320 respondents in Bozeman, Montana, finds that only 38 percent have such trust. A researcher decides to evaluate this apparent difference and seeks initially to ascertain statistical significance. The researcher poses this question: Is Missoula more trusting of its public leaders than is Bozeman?

The response to this research question can be derived by application of the difference of proportions test:

$$t = \frac{\overline{p}_1 - \overline{p}_2}{\sqrt{\left(\frac{(n_1 - 1)\overline{p}_1 + (n_2 - 1)\overline{p}_2}{n_1 + n_2 - 2}\right) \left(1 - \frac{(n_1 - 1)\overline{p}_1 + (n_2 - 1)\overline{p}_2}{n_1 + n_2 - 2}\right) \left(\frac{n_1 + n_2 - 2}{(n_1 - 1)(n_2 - 1)}\right)}$$
(11.5)

where

 \overline{P}_1 = sample proportion of first subgroup

 \overline{p}_2 = sample proportion of second subgroup

 $n_1 =$ sample size of first subgroup

 $n_{\rm o}$ = sample size of second subgroup

The calculated t is then compared to the same critical t values as in the independent samples t test to establish the presence or absence of statistical significance. This can be illustrated as follows:

$$\begin{split} t &= \frac{0.45 - 0.38}{\sqrt{\left(\frac{266(0.45) + 319(0.38)}{266 + 319}\right)\left(1 - \left(\frac{266(0.45) + 319(0.38)}{266 + 319}\right)\right)\left(\frac{266 + 319}{(266)(319)}\right)}} \\ &= \frac{0.07}{\sqrt{\left(\frac{119.7 + 121.22}{585}\right)\left(1 - \left(\frac{119.7 + 121.22}{585}\right)\right)\left(\frac{585}{84,854}\right)}} \\ &= \frac{0.07}{\sqrt{(0.41)(0.588)(0.0069)}} \\ &= \frac{0.07}{0.041} \\ &= 1.71 \end{split}$$

Because the calculated *t* of 1.71 exceeds the one-tail, 95 percent critical *t* of 1.645 (df = 585), the researcher can conclude with 95 percent

244

confidence that Missoula residents trust their public leaders more than the residents of Bozeman trust theirs.

Analysis of Variance

These independent and paired samples *t* tests compare two categories (subgroups) of the independent variable—for instance, men versus women or a before group to the same individuals after some change has been introduced. The world, however, is not always so easily categorized into only two groups, and even when it is, it is often in the researcher's interest to analyze the data in greater detail. In the case of ethnicity, the researcher's purposes might best be served by replacing two categories, such as "white" and "nonwhite," with "white," "African American," "Latino," and "Asian." When there are multiple categories of the independent variable, with a dependent variable on the interval scale, the researcher can attempt to determine statistical significance by a series of applications of these means tests or by a single application of an analysis of variance test.

Besides the cumbersome nature of a series of the means tests, there is an even more important shortcoming to that option. Repeated application of a test to the same data increases the likelihood of committing a Type I error, with each test adding its 5 percent error factor (at 95 percent confidence) to previous applications. Hence, there is a significant advantage to identifying and implementing one single test with its own 5 percent Type I error factor only. That single test is the *analysis of variance test*.

The analysis of variance test measures the amount of the total variability of the dependent variable that can be attributed to the differences among the categories of the independent variable. Analysis of variance compares that portion of the variance in the data that can be attributed to the independent variable categories to that portion of the variance that is attributable to all other potential factors. For example, suppose the researcher is interested in determining if a significant difference exists among ethnic groups with regard to income. Analysis of variance would measure the amount of all the variation in the data that can be explained by ethnic differences as opposed to such other factors as education, marital status, and age. The relative importance of this one independent variable is determined by the analysis of variance test and is tested for statistical significance.

The implementation of the analysis of variance test requires dividing the total variation associated with the dependent variable into two parts: (1) between-group variation (that portion of the variation explained by grouping the dependent variable in a particular fashion) and (2) within-group variation (that portion of the variation accounted for by all factors other than the groups). The test is not dissimilar from the means tests that measure distance between group means and divides this distance by a factor that is indicative of the variation within the groups themselves.

Consider the following example of hourly wage rates paid to a sample of public works employees in four cities: Fargo, St. Paul, Watertown, and South Bend:

Fargo	St. Paul	Watertown	South Bend
\$12.60	\$13.80	\$15.00	\$12.60
\$10.40	\$15.60	\$16.00	\$10.60
\$16.00	\$10.40	\$15.60	\$12.60
\$12.60	\$10.40	\$15.40	\$10.60
\$15.00	\$13.80	\$14.40	\$14.80
\$12.60	\$12.60	\$15.40	
\$17.00	\$15.60	\$15.60	\$14.80
		Μ	ean wage =
\$13.74	\$13.17	15.28	\$12.24

The researcher wishes to determine if the apparent difference among the wage rates paid in these cities can be attributed to sampling error or is in fact a statistically significant finding.

In order to proceed to answer this research question, the researcher would conduct an analysis of variance test and begin by setting up a worksheet as shown in worksheet 11.1. Note that the individual city means differ from the overall mean and from each other. The total variation related to the overall distribution of wage rates can be expressed by the following equations:

Total sum of squares (SS_{total}) = Between groups sum of squares $(SS_{between})$ + Within groups sum of squares (SS_{within})

This can be expressed in algebraic form as

$$\sum f_{x}(x-\bar{x})^{2} = \sum n_{i}(\bar{x}_{i}-x)^{2} + \sum f_{x_{i}}(x_{i}-\bar{x}_{i})^{2}$$
(11.6)

where

- x = individual wage rates
- x_i = individual wage rates by city
- \overline{x}_i = overall mean wage rate
- \overline{x}_i = mean wage rate by city
- n_i = sample size for each city

X Hourly Wage	Overall f	X₁ Fargo f₁	X ₂ St. Paul f ₂	X_{3} Watertown f_{3}	X_4 South Bend f_4
\$10.40	3	1	2	0	0
\$10.60	2	0	0	0	2
\$12.60	6	3	1	0	2
\$13.80	2	0	2	0	0
\$14.40	1	0	0	1	0
\$14.80	2	0	0	1	1
\$15.00	2	1	0	1	0
\$15.40	2	0	0	2	0
\$15.60	4	0	2	2	0
\$16.00	2	1	0	1	0
\$17.00	1	1	0	0	0
	n = 27	<i>n</i> ₁ = 7	n ₂ = 7	$n_{_{3}} = 8$	$n_{4} = 5$
	<i>x</i> = \$13.77	$\bar{x}_1 = $ \$13.74	$\bar{x}_2 = \$13.17$	$\bar{x}_3 = $ \$15.28	$\bar{x}_4 = $ \$12.24

WORKSHEET 11.1. PUBLIC WORKS EMPLOYEES' EARNINGS.

Calculation of Sum of Squares

Whenever equations contain expressions requiring the calculation of deviation, the arithmetic procedure becomes laborious. In order to minimize this problem in the case of analysis of variance, the researcher will find that such inconvenience can be minimized by calculating the SS_{total} and $SS_{between}$ and obtaining SS_{within} by subtraction ($SS_{within} = SS_{total} - SS_{between}$).

In the example at hand, SS_{TOTAL} can be calculated in the following manner:

a.

$$SS_{\text{total}} = \sum f_x (x - \bar{x})^2$$
(11.7)

$$3(10.40 - 13.77)^2 + 2(10.60 - 13.77)^2 + 6(12.60 - 13.77)^2 + 2(13.80 - 13.77)^2 + 1(14.40 - 13.77)^2 + 2(14.80 - 13.77)^2 + 2(15.00 - 13.77)^2 + 2(15.40 - 13.77)^2 + 4(15.60 - 13.77)^2 + 2(16.00 - 13.77)^2 + 1(17.00 - 13.77)^2 + 3S_{\text{TOTAL}} = 107.24$$

b.

$$SS_{\text{BETWEEN}}$$
(11.8)
= $\sum n_i (\bar{x}_i - \bar{x})^2$
= $7(13.74 - 13.77)^2 + 7(13.17 - 13.77)^2 + 8(15.28 - 13.77)^2 + 5(12.24 - 13.77)^2$
= 32.47
c. $SS_{\text{within}} = SS_{\text{total}} - SS_{\text{between}}$
= $107.24 - 32.47 = 74.77$ (11.9)

Calculation of Variances and F Ratio

The essence of analysis of variance is the calculation of an *F* ratio that comprises the ratio of two variances. More specifically, it is the variance associated with SS_{between} (S_b^2) divided by the variance associated the SS_{within} (S_w^2). When the sum of squares is divided by its appropriate degrees of freedom, the result is the variance, also known as the mean square (*MS*). Hence,

$$MS_{\rm between} = \frac{SS_{\rm between}}{df_{\rm between}} \tag{11.10}$$

and

$$MS_{\rm within} = \frac{SS_{\rm within}}{df_{\rm within}} \tag{11.11}$$

where

 $df_{\text{between}} =$ number of categories of the independent variable (k) - 1 = k - 1 $df_{\text{within}} =$ overall sample size minus the number of categories of the independent variable = n - k

In terms of the problem at hand, therefore,

$$MS_{\text{between}} = \frac{32.47}{3} = 10.82$$
$$MS_{\text{within}} = \frac{74.77}{23} = 3.25$$

The *F* ratio is calculated by dividing MS_{between} by MS_{within} :

$$F = MS_{\text{between}} \div MS_{\text{within}} \tag{11.12}$$

This process is best depicted in an ANOVA table. An ANOVA table (Analysis of Variance) presents these calculations in an orderly, easily readable format. Figure 11.1 is the ANOVA table for this public works example, and it shows in one place the sum of squares and degrees of freedom for between, within, and total. It then divides these sums of squares by degrees of freedom and depicts the mean squares for between and within. Finally, the ANOVA table divides MS_{between} by MS_{within} to generate the *F* ratio.

FIGURE 11.1. ANOVA TABLE FOR PUBLIC WORKS EMPLOYEES.

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F
Between groups	32.47	<i>k</i> – 1 = 3	10.82	3.33
Within groups	74.77	n - k = 23	3.25	
Total	107.24	26		

Statistical Significance

In order to determine if the F ratio is statistically significant and therefore indicative of a genuine difference in wage rates among the cities, refer to the critical values of the F distribution in exhibit 11.1 (95 percent confidence) and exhibit 11.2 (99 percent confidence). The reader must determine from these exhibits a critical value of F that is identified by two factors: the confidence level (95 percent or 99 percent) and the degrees of freedom for both $MS_{between}$ and MS_{within} . For this example, the critical F at 95 percent is 3.03, and at 99 percent, it is 4.77. The conclusion that the researcher can draw is that there is 95 percent confidence that the apparent difference in wage rates among these four cities is a genuine one and not sampling error. However, this conclusion cannot be drawn with 99 percent confidence.

Correlation Ratio E² and the Measure of Association—Eta

Eta is the appropriate measure of association when the independent variable is on the nominal scale and the dependent variable is on the interval scale. The equation for *Eta* is

$$Eta = \sqrt{\left(SS_{\text{between}} \div SS_{\text{total}}\right)} \tag{11.13}$$

The portion of the equation that is under the radical sign is known as the *correlation ratio* (E^2), and it represents the total variation explained by the grouping of the independent variable. In the example provided above, $E^2 = 32.47/107.24 = .303 \dots$ and Eta = .55. Interpretation of the correlation ratio is that the city in which the worker is employed explains approximately 30 percent of the variation in pay, with all other factors accounting for the other 70 percent. *Eta* is interpreted in the context of exhibit 10.2 (Cramér's V and phi). Hence, an *Eta* of .55 can be considered to be relatively strong.

Post Hoc Tests: A Multiple Comparison of Means

A significant F ratio informs the researcher that an overall difference among groups exists. If only two groups were being investigated, no further analysis would be required. However, when a significant F is obtained for three or more groups, this does not mean that all groups differ from all others—just that at least one is different from one other. In such cases, it is important to determine precisely which groups are significantly different from the others and which are not.

Repeated applications of independent or paired samples t tests for two groups would increase the chance that a Type I error might occur. Fortunately there exist a number of statistical significance tests, known as *post hoc tests*, that have been developed to make multiple comparisons of means after a significant *F* has been found. These tests can pinpoint where the significant means differences exist.

EXHIBIT 11.1. TABLE OF *F*-STATISTICS, P = .05.

df _{NUM} / df _{DEN}	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81 8	.79	8.76	8.74	8.73	8.71	8.70	8.69	8.68	8.67	8.67
4	7.71	6.94 5.79	6.59 5.41	6.39 5.19	6.26 5.05	6.16 4 95	6.09 4.88	6.04 (5.00 5 4 77 4	.96	5.94 4 70	5.91	5.89	5.87	5.86	5.84	5.83	5.82 4.58	5.81 4 57
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15 4	4.10 4	.06	4.03	4.00	3.98	3.96	3.94	3.92	3.91	3.90	3.88
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68 3	.64	3.60	3.57	3.55	3.53	3.51	3.49	3.48	3.47	3.46
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39 3	.35	3.31	3.28	3.26	3.24	3.22	3.20	3.19	3.17	3.16
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02 2	.98	2.94	2.91	2.89	2.86	2.85	2.83	2.81	2.80	2.79
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95 2	2.90 2	.85	2.82	2.79	2.76	2.74	2.72	2.70	2.69	2.67	2.66
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.802 712	.75	2.72	2.69	2.66	2.64	2.62	2.60	2.58	2.57	2.56
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70 2	2.65 2	.60	2.57	2.53	2.50	2.35	2.35	2.44	2.43	2.40	2.47
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59 2	.54	2.51	2.48	2.45	2.42	2.40	2.38	2.37	2.35	2.34
16 17	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54 2	.49 .	2.46	2.42	2.40	2.37	2.35	2.33	2.32	2.30	2.29
18	4.41	3.55	3.16	2.90	2.77	2.66	2.58	2.55 2	2.46 2	.43	2.37	2.34	2.35	2.29	2.27	2.25	2.27	2.20	2.24
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42 2	.38	2.34	2.31	2.28	2.26	2.23	2.21	2.20	2.18	2.17
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45 2	2.39 2	.35	2.31	2.28	2.25	2.23	2.20	2.18	2.17	2.15	2.14
22	4.30	3.44	3.03	2.82	2.60	2.55	2.40	2.40 2	2.34 Z 2.30 Z	.25	2.20	2.23	2.20	2.17	2.13	2.15	2.11	2.10	2.08
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27 2	.22	2.18	2.15	2.12	2.09	2.07	2.05	2.03	2.02	2.00
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24 2	.19	2.15	2.12	2.09	2.06	2.04	2.02	2.00	1.99	1.97
35	4.17	5.5∠ 3.27	2.92 2.87	2.69 2.64	2.33 2.49	2.42 2.37	2.33 2.29	2.22	∠.∠ı 2 2.16 2	.10	∠.13 2.08	∠.09 2.04	2.00 2.01	2.04 1.99	2.01 1.96	1.99	1.98	1.90	1.89
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12 2	.08	2.04	2.00	1.97	1.95	1.92	1.90	1.89	1.87	1.85
45 50	4.06	3.20	2.81	2.58	2.42	2.31	2.22	2.15	2.10 2	.05	2.01	1.97	1.94	1.92	1.89	1.87	1.86	1.84	1.82
60	4.00	3.15	2.76	2.50	2.40	2.25	2.20	2.10 2	2.07 2	.99	1.95	1.92	1.89	1.86	1.84	1.82	1.80	1.78	1.76
70	3.98	3.13	2.74	2.50	2.35	2.23	2.14	2.07	2.02 1	.97	1.93	1.89	1.86	1.84	1.81	1.79	1.77	1.75	1.74
80	3.96	3.11	2.72	2.49	2.33	2.21	2.13	2.06 2	2.00 1	.95	1.91	1.88	1.84	1.82	1.79	1.77	1.75	1.73	1.72
200	3.89	3.09	2.65	2.40	2.26	2.19	2.10	1.98	1.97 1	.88	1.89	1.80	1.62	1.79	1.72	1.69	1.67	1.66	1.69
500	3.86	3.01	2.62	2.39	2.23	2.12	2.03	1.96	1.90 1	.85	1.81	1.77	1.74	1.71	1.69	1.66	1.64	1.62	1.61
1000	3.85	3.00	2.61	2.38	2.22	2.11	2.02	1.95 1	1.89 1	.84	1.80	1.76	1.73	1.70	1.68	1.65	1.63	1.61	1.60
>1000	1.04	5.00	2.01	2.37	Z.Z I	2.10	2.01	1.94	1 00 1	~ ~	1 / 9	1.7.3	1.72	1.02	1.07	1.04	1.02	1.01	1.39
df /										.05									
${ m df}_{_{ m NUM}}/{ m df}_{_{ m DEN}}$	20	22	24	26	28	30	35	40	45	50) (60	70	80	100	200	500	1,000	>1,000
$\frac{df_{\rm NUM}}{df_{\rm DEN}} / \frac{1}{3}$	20 8.66	22 8.65	24 8.64	26 8.63	28 8.62	30 8.62	35 8.60	40) 8.59	45 8.59	50 8.5) (5 0	70	80 8.56	100 8.55	200 8.54	500 8.53	1,000 8.53	> 1,000 8.54
$\frac{df_{_{\sf NUM}}}{df_{_{\sf DEN}}}$	20 8.66 5.80 4.56	22 8.65 5.79	24 8.64 5.77 4.53	26 8.63 5.76 4.52	28 8.62 5.75 4 50	30 8.62 5.75	35 8.60 5.72	40) 8.59 3 5.72 4 46	45 8.59 5.71	50 8.5 5.7) (8 8 0 5 4 4	5 0 .57 .69	70 3.57 5.68	80 8.56 5.67	100 8.55 5.66 4.41	200 8.54 5.65 4 39	500 8.53 5.64 4.37	1,000 8.53 5.63 4.37	> 1,000 8.54 5.63 4.36
df _{NUM} / df _{DEN} 3 4 5 6	20 8.66 5.80 4.56 3.87	22 8.65 5.79 4.54 3.86	24 8.64 5.77 4.53 3.84	26 8.63 5.76 4.52 3.83	28 8.62 5.75 4.50 3.82	30 8.62 5.75 4.50 3.81	35 8.60 5.73 4.48 3.79	40 0 8.59 3 5.72 3 4.46 9 3.77	45 8.59 5.71 4.45 3.76	50 8.5 5.7 4.4 3.7	8 8 0 5 4 4 5 3	50 .57 .69 .43 .74	70 3.57 5.68 4.42 3.73	80 8.56 5.67 4.42 3.72	100 8.55 5.66 4.41 3.71	200 8.54 5.65 4.39 3.69	500 8.53 5.64 4.37 3.68	1,000 8.53 5.63 4.37 3.67	> 1,000 8.54 5.63 4.36 3.67
df _{NUM} / df _{DEN} 3 4 5 6 7	20 8.66 5.80 4.56 3.87 3.44	22 8.65 5.79 4.54 3.86 3.43	24 8.64 5.77 4.53 3.84 3.41	26 8.63 5.76 4.52 3.83 3.40	28 8.62 5.75 4.50 3.82 3.39	30 8.62 5.75 4.50 3.81 3.38	35 8.60 5.73 4.48 3.79 3.30	40 3 5.72 3 4.46 3 .377 5 3.34	45 8.59 5.71 4.45 3.76 3.33	50 8.5 5.7 4.4 3.7 3.3) (8 8 0 5 4 4 5 3 2 3	50 .69 .43 .74 .30	70 3.57 5.68 4.42 3.73 3.29	80 8.56 5.67 4.42 3.72 3.29	100 8.55 5.66 4.41 3.71 3.27	200 8.54 5.65 4.39 3.69 3.25	500 8.53 5.64 4.37 3.68 3.24	1,000 8.53 5.63 4.37 3.67 3.23	> 1,000 8.54 5.63 4.36 3.67 3.23
df _{NUM} / df _{DEN} 3 4 5 6 7 8 9	20 8.66 5.80 4.56 3.87 3.44 3.15 2.94	22 8.65 5.79 4.54 3.86 3.43 3.13 2.92	24 8.64 5.77 4.53 3.84 3.41 3.12 2.90	26 8.63 5.76 4.52 3.83 3.40 3.10 2.89	28 8.62 5.75 4.50 3.82 3.39 3.09 2.87	30 8.62 5.75 4.50 3.81 3.38 3.08 2.86	35 8.60 5.73 4.48 3.79 3.30 3.00 2.84	40 8.59 3.5.72 3.4.46 3.77 5.3.34 5.3.04 4.2.83	45 8.59 5.71 4.45 3.76 3.33 3.03 2.81	50 8.5 5.7 4.4 3.7 3.3 3.0 2 8	8 8 0 5 4 4 5 3 2 3 12 3 0 2	50 .57 .69 .74 .30 .01 .79	70 3.57 5.68 4.42 3.73 3.29 2.99 2.78	80 8.56 5.67 4.42 3.72 3.29 2.99 2.77	100 8.55 5.66 4.41 3.71 3.27 2.97 2.76	200 8.54 5.65 4.39 3.69 3.25 2.95 2.73	500 8.53 5.64 4.37 3.68 3.24 2.94 2.72	1,000 8.53 5.63 4.37 3.67 3.23 2.93 2.71	> 1,000 8.54 5.63 4.36 3.67 3.23 2.93 2.71
df _{NUM} / df _{DEN} 3 4 5 6 7 8 9 10	20 8.66 5.80 4.56 3.87 3.44 3.15 2.94 2.77	22 8.65 5.79 4.54 3.86 3.43 3.13 2.92 2.75	24 8.64 5.77 4.53 3.84 3.41 3.12 2.90 2.74	26 8.63 5.76 4.52 3.83 3.40 3.10 2.89 2.72	28 8.62 5.75 4.50 3.82 3.39 3.09 2.87 2.71	30 8.62 5.75 4.50 3.81 3.38 3.08 2.86 2.70	35 8.60 5.73 4.48 3.79 3.30 3.00 2.84 2.68	40 8.59 5.72 4.46 3.77 5.3.34 5.3.04 4.2.83 3.2.66	45 8.59 5.71 4.45 3.76 3.33 3.03 2.81 2.65	50 8.5 5.7 4.4 3.7 3.3 3.0 2.8 2.6	8 8 0 5 4 4 15 3 12 3 12 3 10 2 14 2	50 .57 .69 .43 .74 .30 .01 .79 .62	70 3.57 5.68 4.42 3.73 3.29 2.99 2.78 2.61	80 8.56 5.67 4.42 3.72 3.29 2.99 2.77 2.60	100 8.55 5.66 4.41 3.71 3.27 2.97 2.76 2.59	200 8.54 5.65 4.39 3.69 3.25 2.95 2.73 2.56	500 8.53 5.64 4.37 3.68 3.24 2.94 2.72 2.55	1,000 8.53 5.63 4.37 3.67 3.23 2.93 2.71 2.54	> 1,000 8.54 5.63 4.36 3.67 3.23 2.93 2.71 2.54
df _{NUM} / df _{DEN} 3 4 5 6 7 8 9 10 11	20 8.66 5.80 4.56 3.87 3.44 3.15 2.94 2.77 2.65	22 8.65 5.79 4.54 3.43 3.13 2.92 2.75 2.63	24 8.64 5.77 4.53 3.84 3.41 3.12 2.90 2.74 2.61	26 8.63 5.76 4.52 3.83 3.40 3.10 2.89 2.72 2.59	28 8.62 5.75 4.50 3.82 3.39 3.09 2.87 2.71 2.58	30 8.62 5.75 4.50 3.81 3.38 3.08 2.86 2.70 2.57	35 8.60 5.73 4.48 3.79 3.30 2.84 2.68 2.55	40 8.59 5.72 4.46 3.77 5.3.34 5.3.04 4.2.83 3.2.66 5.2.53	45 8.59 5.71 4.45 3.76 3.33 3.03 2.81 2.65 2.52	50 8.5 5.7 4.4 3.7 3.3 3.0 2.8 2.6 2.5	8 8 0 5 4 4 5 3 2 3 12 3 30 2 44 2 1 2	50 .57 .69 .43 .74 .30 .01 .79 .62 .49	70 3.57 5.68 4.42 3.73 3.29 2.99 2.78 2.61 2.48	80 8.56 5.67 4.42 3.72 3.29 2.99 2.77 2.60 2.47	100 8.55 5.66 4.41 3.71 3.27 2.97 2.76 2.59 2.46	200 8.54 5.65 4.39 3.25 2.95 2.73 2.56 2.43	500 8.53 5.64 4.37 3.68 3.24 2.94 2.72 2.55 2.42	1,000 8.53 5.63 4.37 3.67 3.23 2.93 2.71 2.54 2.41	> 1,000 8.54 5.63 4.36 3.67 3.23 2.93 2.71 2.54 2.41
df _{NUM} / df _{DEN} 3 4 5 6 7 8 9 10 11 12 13	20 8.66 5.80 4.56 3.87 3.44 2.77 2.65 2.54 2.46	22 8.65 5.79 4.54 3.86 3.43 3.13 2.92 2.75 2.63 2.52 2.24	24 8.64 5.77 4.53 3.84 3.41 3.12 2.90 2.74 2.61 2.51 2.42	26 8.63 5.76 4.52 3.83 3.40 3.10 2.89 2.72 2.59 2.49 2.41	28 8.62 5.75 4.50 3.82 3.39 3.09 2.87 2.71 2.58 2.48 2.48 2.49	30 8.62 5.75 4.50 3.81 3.38 3.08 2.86 2.70 2.57 2.47 2.38	35 8.60 5.73 4.48 3.30 3.00 2.84 2.68 2.54 2.54 2.44	40 8.59 5.72 4.46 3.77 5.3.34 5.3.04 4.2.83 3.2.66 5.2.53 4.2.43 5.2.43 5.2.43	45 8.59 5.71 4.45 3.76 3.33 2.81 2.65 2.52 2.52 2.41 2.41	50 8.5 5.7 4.4 3.7 3.3 3.0 2.8 2.6 2.5 2.4 2.3	8 8 0 5 4 4 5 3 12 3 10 2 11 2 11 2	50 .57 .69 .43 .74 .30 .01 .79 .62 .49 .38 .30	70 3.57 5.68 4.42 3.73 3.29 2.99 2.78 2.61 2.48 2.37 2.28	80 8.56 5.67 4.42 3.72 3.29 2.99 2.77 2.60 2.47 2.36 2.27	100 8.55 5.66 4.41 3.71 2.97 2.76 2.59 2.46 2.35 2.26	200 8.54 5.65 4.39 3.69 3.25 2.95 2.73 2.56 2.43 2.32 2.32	500 8.53 5.64 4.37 3.68 3.24 2.94 2.72 2.55 2.42 2.31 2 22	1,000 8.53 5.63 4.37 3.67 3.23 2.93 2.71 2.54 2.41 2.30 2.21	> 1,000 8.54 5.63 4.36 3.67 3.23 2.71 2.54 2.41 2.30 2.21
$\frac{df_{_{\sf NUM}}}{df_{_{\sf DEN}}} \\ \frac{df_{_{\sf DEN}}}{3} \\ \frac{3}{4} \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \\ 13 \\ 14 \\ 14 \\ 14 \\ 10 \\ 10 \\ 10 \\ 10 \\ 10$	20 8.66 5.80 4.56 3.87 3.44 3.15 2.94 2.77 2.65 2.54 2.46 2.39	22 8.65 5.79 4.54 3.86 3.43 3.13 2.92 2.75 2.63 2.52 2.44 2.37	24 8.64 5.77 4.53 3.84 3.41 2.90 2.74 2.61 2.51 2.42 2.35	26 8.63 5.76 4.52 3.83 3.40 2.89 2.72 2.59 2.49 2.41 2.33	28 8.62 5.75 4.50 3.82 3.39 2.87 2.71 2.58 2.48 2.39 2.32	30 8.62 5.75 4.50 3.81 3.38 3.08 2.86 2.70 2.57 2.47 2.38 2.31	35 8.60 5.73 4.48 3.30 2.84 2.68 2.55 2.44 2.30 2.24	40 8.59 3.5.72 3.4.46 3.77 5.3.34 5.3.04 4.2.83 3.2.66 5.2.53 4.2.43 5.2.34 3.2.27	45 8.59 5.71 4.45 3.76 3.33 3.03 2.81 2.52 2.52 2.41 2.33 2.25	50 8.5 5.7 4.4 3.7 3.3 3.0 2.8 2.6 2.5 2.4 2.3 2.2	8 8 8 8 0 5 4 4 5 3 2 3 2 3 2 3 0 2 1 2 1 2 1 2 1 2 2 3	50 .57 .69 .74 .30 .01 .79 .62 .49 .38 .30 .22	70 3.57 5.68 4.42 3.73 3.29 2.99 2.78 2.61 2.48 2.37 2.28 2.21	80 8.56 5.67 4.42 3.72 3.29 2.99 2.77 2.60 2.47 2.36 2.27 2.20	100 8.55 5.66 4.41 3.71 3.27 2.97 2.76 2.59 2.46 2.35 2.26 2.19	200 8.54 5.65 4.39 3.25 2.95 2.73 2.56 2.43 2.32 2.23 2.16	500 8.53 5.64 4.37 3.68 3.24 2.72 2.55 2.42 2.31 2.22 2.14	1,000 8.53 5.63 4.37 3.67 3.23 2.71 2.54 2.41 2.30 2.21 2.14	> 1,000 8.54 5.63 4.36 3.67 3.23 2.93 2.71 2.54 2.41 2.30 2.21 2.13
df _{NUM} / df _{DEN} 3 4 5 6 7 8 9 10 11 12 13 14 15	20 8.66 5.80 4.56 3.87 3.44 3.15 2.94 2.77 2.65 2.54 2.46 2.39 2.33	22 8.65 5.79 4.54 3.86 3.43 3.13 2.92 2.75 2.63 2.52 2.44 2.37 2.31	24 8.64 5.77 4.53 3.84 3.12 2.90 2.74 2.61 2.51 2.42 2.35 2.29	26 8.63 5.76 4.52 3.83 3.40 3.10 2.89 2.72 2.59 2.49 2.41 2.33 2.27	28 8.62 5.75 4.50 3.39 3.09 2.87 2.71 2.58 2.48 2.39 2.32 2.22	30 8.62 5.75 4.50 3.81 3.38 2.86 2.57 2.47 2.38 2.31 2.25	35 8.60 5.73 4.48 3.79 3.00 2.84 2.68 2.55 2.44 2.30 2.24 2.22 2.22	40 8.59 3.5.72 3.4.46 3.77 5.3.34 5.3.04 4.2.83 5.2.53 4.2.43 5.2.53 4.2.43 5.2.34 3.2.27 2.2.02	45 8.59 5.71 4.45 3.76 3.33 2.81 2.55 2.52 2.41 2.33 2.25 2.19	50 8.5 5.7 4.4 3.7 3.0 2.8 2.6 2.5 2.4 2.3 2.2 2.1	8 8 8 8 0 5 4 4 5 3 2 3 2 3 0 2 1 2 2 1 2 2 1 2 2 3 2 3 0 2 1 2 2 3 2 3 2 3 2 3 2 3 2 3 2 3 2 3 2 3 2 3 2 3 2 3 3 2 3 2 3 2 3 2 3 2 3 2 3 2 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 <td>50 .57 .69 .43 .74 .30 .01 .79 .62 .49 .38 .30 .22 .11</td> <td>70 3.57 5.68 4.42 3.73 3.29 2.99 2.78 2.61 2.48 2.37 2.28 2.21 2.28 2.21 2.15</td> <td>80 8.56 5.67 4.42 3.72 3.29 2.99 2.77 2.60 2.47 2.36 2.27 2.20 2.12</td> <td>100 8.55 5.66 4.41 3.71 3.27 2.97 2.76 2.59 2.46 2.35 2.26 2.19 2.12</td> <td>200 8.54 5.65 4.39 3.69 3.25 2.95 2.73 2.56 2.43 2.32 2.23 2.16 2.10</td> <td>500 8.53 5.64 4.37 3.68 3.24 2.94 2.72 2.55 2.42 2.31 2.22 2.14 2.02</td> <td>1,000 8.53 5.63 4.37 3.67 3.23 2.93 2.71 2.54 2.41 2.30 2.21 2.14 2.07 2.07</td> <td>>1,000 8.54 5.63 4.36 3.23 2.71 2.54 2.41 2.54 2.21 2.13 2.07</td>	50 .57 .69 .43 .74 .30 .01 .79 .62 .49 .38 .30 .22 .11	70 3.57 5.68 4.42 3.73 3.29 2.99 2.78 2.61 2.48 2.37 2.28 2.21 2.28 2.21 2.15	80 8.56 5.67 4.42 3.72 3.29 2.99 2.77 2.60 2.47 2.36 2.27 2.20 2.12	100 8.55 5.66 4.41 3.71 3.27 2.97 2.76 2.59 2.46 2.35 2.26 2.19 2.12	200 8.54 5.65 4.39 3.69 3.25 2.95 2.73 2.56 2.43 2.32 2.23 2.16 2.10	500 8.53 5.64 4.37 3.68 3.24 2.94 2.72 2.55 2.42 2.31 2.22 2.14 2.02	1,000 8.53 5.63 4.37 3.67 3.23 2.93 2.71 2.54 2.41 2.30 2.21 2.14 2.07 2.07	> 1,000 8.54 5.63 4.36 3.23 2.71 2.54 2.41 2.54 2.21 2.13 2.07
$\frac{df_{NUM}}{df_{DEN}} \\ \frac{df_{DEN}}{3} \\ \frac{3}{4} \\ \frac{5}{5} \\ \frac{6}{7} \\ \frac{7}{8} \\ \frac{9}{10} \\ \frac{11}{12} \\ \frac{13}{13} \\ \frac{14}{15} \\ \frac{15}{16} \\ \frac{17}{12} \\ \frac{17}{12$	20 8.66 5.80 4.56 3.87 3.44 3.15 2.94 2.77 2.65 2.54 2.46 2.39 2.33 2.28 2.23	22 8.65 5.79 4.54 3.86 3.43 3.13 2.92 2.75 2.63 2.52 2.44 2.37 2.31 2.23 2.21	24 8.64 5.77 4.53 3.84 3.41 3.12 2.90 2.74 2.61 2.51 2.42 2.35 2.29 2.24 2.19	26 8.63 5.76 4.52 3.83 3.40 3.10 2.89 2.72 2.59 2.49 2.41 2.33 2.27 2.22 2.17	28 8.62 5.75 4.50 3.82 3.39 2.87 2.71 2.58 2.48 2.39 2.32 2.26 2.21 2.16	30 8.62 5.75 4.50 3.81 3.38 2.86 2.70 2.57 2.47 2.38 2.31 2.25 2.19 2.15	35 8.60 5.73 4.48 3.30 2.84 2.55 2.44 2.36 2.24 2.25 2.24 2.25 2.24 2.25 2.24 2.25 2.24 2.25 2.24 2.25 2.24 2.25 2.24 2.25 2.24 2.25 2.24 2.25 2.24 2.25 2.24 2.25 2.24 2.25 2.25	40 8.59 5.72 4.46 5.3.34 5.3.34 4.2.83 3.2.66 5.2.34 4.2.43 5.2.34 4.2.43 5.2.34 4.2.43 5.2.27 2.2.00 7.2.15 2.15 2.15 2.215 2.	45 8.59 5.71 4.45 3.76 3.33 2.81 2.652 2.41 2.33 2.252 2.19 2.19 2.14 2.052	50 8.5 5.7 4.4 3.7 3.3 3.0 2.8 2.6 2.5 2.4 2.3 2.2 2.1 2.1 2.1 2.1	8 8 0 5 4 4 5 3 2 3 2 3 2 3 2 3 2 3 0 2 1 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2	50 .57 .69 .43 .74 .30 .01 .79 .62 .49 .38 .30 .22 .16 .11 .06	70 3.57 5.68 4.42 3.73 3.29 2.99 2.78 2.61 2.48 2.37 2.28 2.21 2.15 2.09 2.05	80 8.56 5.67 4.42 3.72 3.29 2.97 2.60 2.47 2.36 2.27 2.20 2.14 2.03	100 8.55 5.66 4.41 3.71 2.97 2.59 2.46 2.35 2.26 2.19 2.12 2.07 2.07	200 8.54 5.65 4.39 3.25 2.75 2.75 2.75 2.56 2.43 2.32 2.23 2.16 2.10 2.04 1.99	500 8.53 5.64 4.37 3.68 3.24 2.94 2.72 2.55 2.42 2.31 2.22 2.14 2.08 2.02 1.97	1,000 8.53 5.63 4.37 3.67 3.23 2.93 2.71 2.54 2.41 2.30 2.21 2.14 2.07 2.07 2.07 2.07	> 1,000 8.54 5.63 4.36 3.67 3.23 2.71 2.54 2.41 2.30 2.21 2.13 2.07 2.01 1.96
df _{NUM} / df _{DEN} 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18	20 8.66 5.80 4.56 3.87 3.44 3.15 2.94 2.77 2.65 2.54 2.46 2.39 2.33 2.28 2.23 2.19	22 8.65 5.79 4.54 3.86 3.43 3.13 2.92 2.75 2.63 2.52 2.44 2.37 2.31 2.25 2.21 2.17	24 8.64 5.77 4.53 3.84 3.41 3.12 2.90 2.74 2.61 2.51 2.42 2.35 2.29 2.24 2.19 2.15	26 8.63 5.76 4.52 3.83 3.40 2.89 2.72 2.59 2.49 2.41 2.33 2.27 2.22 2.17 2.13	28 8.62 5.75 4.50 3.82 3.39 3.09 2.87 2.71 2.58 2.48 2.39 2.32 2.26 2.21 2.16 2.12	30 8.62 5.75 4.50 3.81 3.38 3.08 2.70 2.57 2.47 2.38 2.31 2.25 2.15 2.11	35 8.60 5.73 4.48 3.36 3.30 2.88 2.68 2.55 2.44 2.33 2.24 2.21 2.11 2.11 2.00	40 8.59 5.72 4.46 5.334 4.283 3.266 5.334 4.2.43 3.2.66 5.2.34 4.2.43 5.2.34 4.2.43 5.2.34 3.2.27 2.2.00 7.2.15 2.2.10 3.2.06 3.2.06 3.2.27 3.2.24 3.2.27 3.2.24 3.2.27 3.2.24 3.2.27 3.2.24 3.2.27 3.2.24 3.2.27 3.2.24 3.2.27 3.2.24 3.2.27 3.2.24 3.2.27 3.2.24 3.2.27 3.2.24 3.2.27 3.2.24 3.2.27 3.2.24 3.2.27 3.2.24 3.2.27 3.2.24 3.2.27 2.2.34 3.2.27 2.2.34 3.2.27 2.2.34 3.2.27 2.2.34 3.2.27 2.2.34 3.2.27 2.2.34 3.2.27 2.2.34 3.2.27 2.2.34 3.2.27 2.2.34 3.2.27 2.2.00 3.2.06 3.2.24 3.2.27 2.2.00 3.2.26 3.2.24 3.2.27 2.2.00 3.2.26 3.2.26 3.2.27 2.2.00 3.2.26 3.2.27 2.2.00 3.2.26 3.2.27 2.2.00 3.2.26 3.2.27 2.2.00 3.2.26 3.2.27 2.2.15 2.2.16 3.2.06 3.2.26 3.2.27 2.2.15 2.2.16 3.2.06 3.2.26 3.2.27 2.2.15 2.2.16 3.2.06 3.2.06 3.2.26 3.2.27 2.2.15 2.2.16 3.2.06 3.2.06 3.2.26 3.2.27 3.2.26 3.	45 8.59 5.71 4.45 3.76 3.73 2.81 2.65 2.52 2.52 2.52 2.52 2.19 2.19 2.19 2.19 2.05	50 8.5 5.7 4.4 3.3 3.0 2.8 2.6 2.5 2.4 2.5 2.4 2.3 2.2 2.1 2.1 2.0 2.0	8 8 8 0 5 3 2 3 2 32 3 2 30 2 3 4 2 3 0 2 3 1 2 2 10 2 2 14 2 2 18 2 2 14 2 2 18 2 2 14 2 2	50 .57 .69 .74 .30 .01 .79 .62 .49 .38 .30 .22 .16 .11 .06 .02	70 3.57 5.68 4.42 3.73 3.29 2.78 2.61 2.48 2.37 2.28 2.21 2.15 2.09 2.05 2.00	80 8.56 5.67 4.42 3.72 3.29 2.77 2.60 2.47 2.36 2.27 2.20 2.14 2.08 2.03 1.99	100 8.55 5.66 4.41 3.71 2.77 2.76 2.59 2.46 2.35 2.26 2.19 2.12 2.07 2.02 1.98	200 8.54 5.65 4.39 3.69 3.25 2.73 2.56 2.43 2.32 2.23 2.16 2.10 2.04 1.99 1.95	500 8.53 5.64 4.37 3.68 3.24 2.72 2.55 2.42 2.31 2.22 2.14 2.02 1.97 1.93	1,000 8.53 5.63 4.37 3.23 2.93 2.71 2.54 2.41 2.30 2.21 2.14 2.07 2.07 2.07 1.92	> 1,000 8.54 5.63 4.36 3.23 2.71 2.54 2.41 2.30 2.21 2.13 2.07 2.01 1.96 1.92
$\frac{df_{\text{NUM}}}{df_{\text{DEN}}} \\ \hline \\ \hline \\ \\ \hline \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ $	20 8.66 5.80 4.56 3.87 3.44 2.77 2.65 2.54 2.46 2.39 2.33 2.23 2.23 2.19 2.16	22 8.65 5.79 4.54 3.86 3.43 3.13 2.92 2.75 2.63 2.52 2.44 2.37 2.37 2.25 2.21 2.17 2.13	24 8.64 5.77 4.53 3.84 3.12 2.90 2.74 2.61 2.51 2.42 2.35 2.29 2.24 2.19 2.15 2.19 2.15 2.11 2.15	26 8.63 5.76 4.52 3.83 3.40 3.10 2.89 2.49 2.49 2.41 2.33 2.27 2.23 2.27 2.217 2.13 2.207	28 8.62 5.75 4.50 3.39 2.87 2.71 2.58 2.48 2.39 2.32 2.26 2.21 2.16 2.12 2.16	30 8.62 5.75 4.50 3.81 3.38 3.08 2.86 2.70 2.57 2.57 2.38 2.31 2.25 2.11 2.25 2.11 2.25 2.11	35 8.60 5.73 4.48 3.36 3.36 2.88 2.68 2.55 2.44 2.36 2.24 2.21 2.11 2.11 2.11 2.00 2.00	40 8.59 5.72 4.46 5.72 3.34 4.283 3.04 4.2.83 3.2.66 5.2.34 4.2.43 5.2.34 4.2.43 5.2.34 3.2.27 2.2.00 7.2.15 2.2.10 3.2.06 5.2.344 5.2.34 5.2.	45 8.59 5.71 4.45 3.76 3.73 2.81 2.65 2.41 2.33 2.25 2.19 2.19 2.19 2.19 2.05 2.05	50 8.5 5.7 4.4 3.7 3.3 3.0 2.8 2.6 2.5 2.4 2.3 2.2 2.1 2.1 2.1 2.1 2.0 2.0 2.0 2.0 2.0 2.0 2.0 2.0 2.0 2.0	8 8 8 8 0 5 3 2 3 2 3 2 3 2 3 2 3 2 3 2 3 2 3 2 3 2 3 2 3 2 3 2 3 2 3 2 4 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 3 2 3 2 3 2 3 2 3 2 3 3 3 3 3 3 3 3 <td>50 .57 .69 .43 .74 .30 .01 .79 .62 .49 .38 .30 .22 .16 .11 .06 .02 .98</td> <td>70 3.57 5.68 4.42 3.73 3.29 2.78 2.61 2.48 2.27 2.28 2.21 2.15 2.09 2.05 2.00 1.97</td> <td>80 8.56 5.67 4.42 3.29 2.99 2.77 2.60 2.47 2.36 2.27 2.20 2.14 2.08 2.03 1.99 1.96</td> <td>100 8.55 5.66 4.41 3.71 3.27 2.97 2.76 2.39 2.46 2.35 2.26 2.19 2.12 2.07 2.02 1.98 1.94</td> <td>200 8.54 5.65 4.39 3.25 2.95 2.73 2.56 2.43 2.32 2.23 2.16 2.10 2.04 1.99 1.95 1.91</td> <td>500 8.53 5.64 4.37 3.68 3.24 2.94 2.72 2.55 2.42 2.31 2.22 2.14 2.08 2.02 1.97 1.93 1.89</td> <td>1,000 8.53 5.63 4.37 3.23 2.93 2.71 2.54 2.41 2.30 2.21 2.14 2.07 2.07 2.07 1.97 1.92 1.88</td> <td>>1,000 8.54 5.63 4.36 3.67 3.23 2.71 2.54 2.41 2.30 2.21 2.13 2.07 2.01 1.96 1.92 1.88</td>	50 .57 .69 .43 .74 .30 .01 .79 .62 .49 .38 .30 .22 .16 .11 .06 .02 .98	70 3.57 5.68 4.42 3.73 3.29 2.78 2.61 2.48 2.27 2.28 2.21 2.15 2.09 2.05 2.00 1.97	80 8.56 5.67 4.42 3.29 2.99 2.77 2.60 2.47 2.36 2.27 2.20 2.14 2.08 2.03 1.99 1.96	100 8.55 5.66 4.41 3.71 3.27 2.97 2.76 2.39 2.46 2.35 2.26 2.19 2.12 2.07 2.02 1.98 1.94	200 8.54 5.65 4.39 3.25 2.95 2.73 2.56 2.43 2.32 2.23 2.16 2.10 2.04 1.99 1.95 1.91	500 8.53 5.64 4.37 3.68 3.24 2.94 2.72 2.55 2.42 2.31 2.22 2.14 2.08 2.02 1.97 1.93 1.89	1,000 8.53 5.63 4.37 3.23 2.93 2.71 2.54 2.41 2.30 2.21 2.14 2.07 2.07 2.07 1.97 1.92 1.88	> 1,000 8.54 5.63 4.36 3.67 3.23 2.71 2.54 2.41 2.30 2.21 2.13 2.07 2.01 1.96 1.92 1.88
$\begin{array}{c} df_{\text{NUM}} / \\ df_{\text{DEN}} \\ \hline \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \\ 13 \\ 14 \\ 15 \\ 16 \\ 17 \\ 18 \\ 19 \\ 20 \\ 22 \end{array}$	20 8.66 5.80 4.56 3.87 3.44 2.77 2.65 2.54 2.46 2.39 2.33 2.23 2.23 2.19 2.16 2.22 2.07	22 8.65 5.79 4.54 3.43 3.13 2.92 2.75 2.63 2.52 2.44 2.37 2.31 2.22 2.21 2.17 2.13 2.10 2.05	24 8.64 5.77 4.53 3.84 3.41 3.12 2.90 2.74 2.61 2.51 2.42 2.35 2.29 2.24 2.19 2.15 2.11 2.01 2.15 2.11 2.03	26 8.63 5.76 4.52 3.83 3.40 2.89 2.72 2.59 2.49 2.41 2.33 2.27 2.23 2.27 2.217 2.13 2.10 2.07 2.01	28 8.62 5.75 4.50 3.39 2.87 2.71 2.58 2.48 2.39 2.32 2.26 2.21 2.16 2.12 2.08 2.22 2.08 2.22 2.08 2.05	30 8.62 5.75 4.50 3.81 3.38 3.08 2.86 2.70 2.57 2.57 2.38 2.31 2.25 2.11 2.07 2.11 2.07 2.11	35 8.60 5.73 4.48 3.30 2.84 2.68 2.24 2.25 2.24 2.22 2.21 2.11 2.11 2.11 2.00 2.00 2.00	40 8.59 3.72 3.34 5.3.34 5.3.34 5.3.34 5.2.53 4.2.83 3.2.66 5.2.53 4.2.43 5.2.53 4.2.43 5.2.24 3.2.27 7.2.15 2.2.10 3.2.06 5.2.03 1.99 5.2.94 5.2.94	45 8.59 5.71 4.45 3.76 3.33 3.03 2.65 2.52 2.41 2.23 2.219 2.14 2.209 2.05 2.01 2.09 2.05 2.01 2.19 2.01 2.09 2.05 2.01 2.19 2.01 2.09 2.05 2.01 2.01 2.01 2.09 2.05 2.01 2.01 2.01 2.01 2.01 2.01 2.01 2.01	50 8.5 5.7 4.4 3.7 3.3 3.0 0 2.8 2.6 2.5 2.4 2.3 2.2 2.1 1 2.1 2.1 2.1 2.1 2.1 2.1 2.1 2	0 0 8 8 8 8 5 3 2 3 2 3 0 2 1 2 2 2 1 2 2 3 2 3 2 3 3 3 3 3 3 3 3 3 3 3	50 57 69 43 74 30 62 49 62 49 62 49 62 49 38 30 62 49 38 30 62 69 62 69 62 69 69 69 69 69 69 69 69 69 69 69 69 69 69 69 69 69 69 69 69 69 69 69 69 69 69 69 69 69 62 62 62 62 62 62 62 62 62 62 62 62 62 62 62 62 62 62 62 62 62 62 62 62 62 62 62 62 62 62 62 62 62 62 62 62 62 62 62 62 62 62 62 62 62 62 62 62 62 62 62 62 62 62 62 62 62 62 62 62 62 62 62 62 62 62 62 62 62 62 62 62 62 63 65 66 60 62 62 62 62 63 65 66 66 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67 67	70 3.57 5.68 4.42 3.73 3.29 2.99 2.78 2.61 2.48 2.37 2.28 2.37 2.28 2.37 2.28 2.37 2.28 2.09 2.05 2.00 1.97 1.88	80 8.56 5.67 4.42 3.29 2.99 2.77 2.60 2.47 2.36 2.27 2.20 2.14 2.03 1.99 1.96 1.92	100 8.55 5.66 4.41 3.27 2.97 2.76 2.39 2.46 2.35 2.26 2.19 2.12 2.07 2.02 1.98 1.94 1.91 1.85	200 8.54 5.65 4.39 3.25 2.95 2.73 2.56 2.43 2.32 2.23 2.16 2.10 2.04 1.99 1.95 1.91 1.82	500 8.53 5.64 4.37 3.68 3.24 2.72 2.55 2.42 2.31 2.22 2.31 2.22 2.31 2.22 2.14 2.08 2.02 1.97 1.93 1.89 1.80	1,000 8.53 5.63 4.37 3.67 3.23 2.71 2.54 2.41 2.30 2.21 2.14 2.07 2.02 1.92 1.88 1.85 1.79	> 1,000 8.54 5.63 4.36 3.67 3.29 2.71 2.54 2.41 2.30 2.21 2.13 2.07 2.01 1.96 1.92 1.88 1.84 1.78
$\frac{df_{\text{NUM}}}{df_{\text{DEN}}} \\ \hline \\ \hline \\ \\ \hline \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ $	20 8.66 5.80 3.87 3.44 3.15 2.94 2.77 2.65 2.54 2.46 2.39 2.33 2.28 2.23 2.19 2.16 2.12 2.07 2.03	22 8.65 5.79 4.54 3.86 3.33 2.75 2.43 2.52 2.44 2.37 2.52 2.21 2.25 2.25	24 8.64 5.77 4.53 3.84 3.41 3.12 2.90 2.74 2.61 2.42 2.35 2.29 2.24 2.15 2.21 2.42 2.15 2.11 2.08 2.03 1.98	26 8.63 5.76 4.52 3.83 3.40 3.10 2.89 2.49 2.49 2.49 2.49 2.49 2.43 2.27 2.23 2.27 2.13 2.10 2.07 2.01 1.97	28 8.62 5.75 4.500 3.82 3.39 3.09 2.87 2.71 2.58 2.48 2.32 2.26 2.21 2.16 2.12 2.08 2.05 2.00 1.95	30 8.62 5.75 4.50 3.81 3.38 3.08 2.86 2.70 2.57 2.47 2.25 2.19 2.15 2.11 2.07 2.15 2.11 2.07 2.14 2.15 2.11 2.07 2.14 2.15 2.11 2.15 2.11 2.15 2.11 2.15 2.11 2.15 2.11 2.15 2.11 2.15 2.11 2.15 2.11 2.15 2.15	355 8.60 5.77 3.30 3.300 2.88 2.66 2.24 2.22 2.21 2.22 2.21 2.20 2.00 2.00 2.00	40 8.59 8.573 4.46 3.77 5.3.04 5.3.04 4.2.83 3.2.65 2.34 3.2.27 2.2.20 7.2.15 2.2.10 3.2.07 2.2.00 3.2.07 2.2.00 3.2.07 2.1.5 2.2.10 3.2.07 2.1.5 2.2.10 3.2.07 2.1.5 2.2.10 3.2.07 2.1.5 2.2.10 3.2.07 2.1.5 2.2.10 3.2.07 2.1.5 2.2.10 3.2.01 1.9.9 5.1.94 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1.9.9 1	45 8.59 5.71 4.45 3.76 3.33 3.03 2.81 2.252 2.41 2.23 2.23 2.23 2.219 2.14 2.09 2.05 2.01 2.19 2.019 1.98 2.019 1.92 2.05 2.114 2.09 2.05 2.114	50 8,5 5,7 4,4 3,7 3,3 3,0 0,2 8,8 2,6 2,5 2,4 4,2,3 2,2 2,1,1 2,10 2,0 0,2,0 2,0 2,0 2,0 2,0 2,0 2,0 2,0	0 6 8 8 0 2 1 2 2 3 2 3 2 3 2 3 2 3 2 3 2 3 2 3 2 3 2 3 2 3 2 3 2 3 2 3 2 3 2 3 3 3 3 4 2 4 2 2 2 4 2 2 2 4 2 2 2 4 2 2 2 4 2 4 2 4 2 4 2 4 2 4 2 4 2 4 2	50 57 69 .69 .74 .30 .01 .79 .62 .30 .62 .30 .30 .30 .30 .32 .33 .33 .33 .33 .36 .36 .36 .36 .37 .37 .38 .38 .30 .30 .31 .36 .32 .36 .36 .36 .37 .37 .38 .39 .39 .39 .39 .30 .30 .30 .30 .30 .31 .31 .36 .32 .36 .36 .36 .36 .36 .37 .37 .38 .39 .39 .39 .39 .39 .39 .39 .39 .39 .39 .39 .39 .39 .39 .38 .39 .39 .39 .38 .39 .39 .39 .38 .39 .39 .38 .39 .39 .38 .39 .39 .39 .38 .39 .39 .39 .38 .39 .39 .39 .39 .39 .39 .39 .39 .39 .39 .39 .39 .39 .39 .39 .39 .39 .39 .39 .39 .39 .39 .39 .39 .39 .39 .39 .39 .39 .39 .39 .39 .39 .39 .39 .39 .39 .39 .39 .39 .39 .39 .39 .39 .39 .39 .39 .39 .39 .39 .39 . 1	70 3.57 5.68 4.42 2.37 2.29 2.78 2.61 2.24 2.27 2.28 2.09 2.05 2.00 1.97 1.93 1.88 1.88	80 8.56 5.67 4.42 3.72 3.29 2.97 2.60 2.47 2.36 2.27 2.20 2.14 2.03 1.99 1.96 1.92 1.86 1.82	100 8.55 5.66 4.41 3.71 3.27 2.76 2.59 2.46 2.35 2.26 2.19 2.12 2.07 2.02 1.98 1.94 1.91 1.85 1.80	200 8.54 5.65 4.39 3.25 2.73 2.73 2.56 2.43 2.32 2.23 2.16 2.10 2.04 1.99 1.95 1.91 1.88 1.82 1.77	500 8.53 5.64 4.37 3.68 2.94 2.72 2.42 2.31 2.22 2.14 2.02 1.97 1.93 1.89 1.80 1.75	1,000 8.53 5.63 4.37 3.67 3.23 2.71 2.54 2.41 2.30 2.21 2.14 2.07 2.02 1.92 1.88 1.85 1.79 1.74	> 1,000 8.54 5.63 4.36 3.67 3.23 2.93 2.71 2.54 2.41 2.54 2.41 2.54 2.21 2.13 2.01 1.96 1.92 1.88 1.88 1.84 1.73
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2.27 2.20 2.14 2.03 1.92 1.96 1.92 1.96 1.92 1.82 1.82 1.82	100 8.55 5.66 4.41 3.27 2.97 2.27 2.27 2.27 2.27 2.27 2.27 2	200 8.54 5.65 3.69 3.25 2.73 2.56 2.43 2.32 2.23 2.16 2.10 2.04 1.99 1.95 1.91 1.88 2 1.77 1.73	500 8.53 5.64 4.37 3.68 3.24 2.94 2.72 2.55 2.42 2.31 2.22 2.14 2.08 2.02 2.14 2.08 2.02 1.97 1.93 1.89 1.80 1.75 1.71	1,000 8.53 5.63 4.37 3.67 3.23 2.71 2.54 2.41 2.30 2.21 2.54 2.07 2.02 1.97 1.92 1.88 1.85 1.79 1.74 1.70	> 1,000 8.54 5.63 4.36 3.23 2.93 2.71 2.54 2.41 2.30 2.21 2.13 2.01 1.96 1.92 1.88 1.84 1.78 1.69
$\begin{array}{c} df_{\text{NUM}} / \\ df_{\text{DEN}} \\ \hline \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \\ 13 \\ 14 \\ 15 \\ 16 \\ 17 \\ 18 \\ 19 \\ 20 \\ 22 \\ 24 \\ 26 \\ 28 \\ 30 \\ \end{array}$	20 8.66 5.80 4.56 3.87 3.44 3.15 2.94 2.65 2.54 2.46 2.23 2.23 2.24 2.23 2.23 2.24 2.23 2.21 2.16 2.12 2.03 1.99 1.96	22 8.65 5.79 4.54 3.86 3.43 3.13 2.52 2.63 2.52 2.44 2.37 2.31 2.25 2.44 2.37 2.31 2.21 2.13 2.10 2.20 0 1.97 1.93 1.91	24 8.64 5.77 3.84 3.41 3.12 2.90 2.74 2.61 2.51 2.29 2.24 2.19 2.29 2.24 2.19 2.29 2.24 2.19 2.29 2.24 2.19 2.11 2.08 2.03 1.98 1.95 1.91 1.95 1.94 1.95 1.94 1.94 2.11 2.12 2.14 2.15 2.17 1.24 2.15 2.17 2.14 2.15 2.17 1.24 2.15 2.17 2.14 2.15 2.17 2.14 2.15 2.17 2.14 2.15 2.17 2.14 2.15 2.17 2.14 2.15 2.17 2.14 2.15 2.17 2.14 2.15 2.17 2.14 2.15 2.17 2.14 2.15 2.17 2.14 2.15 2.17 2.14 2.15 2.14 2.15 2.14 2.15 2.17 2.14 2.15 2.14 2.15 2.14 2.15 2.14 2.15 2.14 2.15 2.14 2.15 2.14 2.15 2.14 2.15 2.14 2.15 2.14 2.15 2.11 2.11 2.11 2.11 2.11 2.11 2.11	26 8.63 3.5.76 4.52 3.83 3.40 2.89 2.49 2.49 2.49 2.49 2.22 2.17 2.22 2.17 2.22 2.17 2.13 1.10 2.07 2.01 1.97 1.93 1.90	28 8.62 5.75 4.50 3.82 2.39 2.27 2.21 2.16 2.22 2.26 2.21 2.16 2.12 2.28 2.20 5.200 1.95 1.91 1.88	30 8.622 5.75 4.50 3.81 3.08 2.86 2.707 2.55 2.47 2.247 2.247 2.25 2.19 2.15 2.207 2.07 2.04 1.90 1.87 1.94 1.90	355 8.66 5.77 3.33 3.00 2.66 2.55 2.44 2.32 2.22 2.22 2.22 2.21 2.12 2.00 2.00 2.0	40 8.59 8.572 8.4.46 5.3.34 4.46 5.3.34 8.2.66 5.2.34 8.2.63 5.2.34 8.2.63 5.2.34 8.2.63 5.2.34 8.2.63 5.2.34 8.2.63 5.2.06 5.2.04 1.99 1.89 1.89 1.85 1.82 1.77 1.85 1.72 1.85 1.72	45 8.59 5.71 4.45 3.76 3.33 2.81 2.65 2.52 2.41 4.23 2.52 2.41 2.19 2.14 2.05 5.2.52 2.14 2.19 2.14 2.05 5.2.52 2.14 1.92 2.14 1.92 2.14 1.92 2.15 2.15 2.11 2.15 2.15 2.15 2.15 2.1	50 50 5 5 7 4 4 4 3 3 3 0 2 8 5 7 4 4 4 3 3 0 2 8 5 7 4 4 4 4 3 3 0 2 8 5 7 1 4 4 4 3 3 0 2 8 2 6 6 2 5 5 7 1 4 4 4 3 3 0 2 8 2 6 6 2 5 2 4 4 2 3 3 3 0 2 2 2 1 1 2 1 1 2 1 1 2 1 1 1 1 1 1 1 1 1 1	0 (1) 8 8 0 5 3 2 3 2 3 2 3 2 3 2 3 2 3 2 3 2 3 2 3 2 3 2 3 2 4 4 2 2 4 4 2 2 4 4 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 1 1 1 3 1 3 1 4 2 1 1 1 1 1 1 1 1 1 1 <td< td=""><td>50 57 69 57 43 30 77 43 30 30 31 30 31 30 31 30 31 31 31 31 31 31 31 31</td><td>70 3.557 5.68 3.73 3.299 2.99 2.278 2.278 2.278 2.278 2.278 2.278 2.278 2.278 2.278 2.278 2.278 2.278 2.205 2.000 1.97 1.93 1.88 1.883 1.79 1.75</td><td>80 8.56 5.67 4.42 3.72 2.99 2.77 2.60 2.27 2.20 2.27 2.20 2.21 4 2.03 2.03 1.99 1.92 1.92 1.92 1.92 1.92 1.92 1.92</td><td>100 8.55 5.66 4.41 3.71 2.76 2.97 2.76 2.97 2.26 2.12 2.07 2.02 2.07 2.02 2.07 2.02 1.98 1.94 1.94 1.94 1.94 1.76 1.73</td><td>200 8.54 5.65 3.69 3.25 2.73 2.26 2.23 2.24 2.23 2.210 2.04 1.99 1.91 1.88 1.82 1.77 1.73 1.69</td><td>500 8.53 5.64 4.37 3.68 3.24 2.72 2.25 2.42 2.22 2.21 4.208 2.02 2.22 1.97 1.89 1.89 1.89 1.80 1.75 1.71 1.67</td><td>1,000 8.53 5.63 3.23 2.71 2.54 2.30 2.21 2.54 2.30 2.21 2.30 2.07 2.02 1.97 1.92 2.02 1.97 1.88 1.85 1.79 1.70 1.66</td><td>>1,000 8.54 5.63 4.36 3.23 2.93 2.71 2.54 2.41 2.30 2.21 2.13 2.07 2.01 1.96 1.92 1.88 1.78 1.78 1.69 1.66 1.62</td></td<>	50 57 69 57 43 30 77 43 30 30 31 30 31 30 31 30 31 31 31 31 31 31 31 31	70 3.557 5.68 3.73 3.299 2.99 2.278 2.278 2.278 2.278 2.278 2.278 2.278 2.278 2.278 2.278 2.278 2.278 2.205 2.000 1.97 1.93 1.88 1.883 1.79 1.75	80 8.56 5.67 4.42 3.72 2.99 2.77 2.60 2.27 2.20 2.27 2.20 2.21 4 2.03 2.03 1.99 1.92 1.92 1.92 1.92 1.92 1.92 1.92	100 8.55 5.66 4.41 3.71 2.76 2.97 2.76 2.97 2.26 2.12 2.07 2.02 2.07 2.02 2.07 2.02 1.98 1.94 1.94 1.94 1.94 1.76 1.73	200 8.54 5.65 3.69 3.25 2.73 2.26 2.23 2.24 2.23 2.210 2.04 1.99 1.91 1.88 1.82 1.77 1.73 1.69	500 8.53 5.64 4.37 3.68 3.24 2.72 2.25 2.42 2.22 2.21 4.208 2.02 2.22 1.97 1.89 1.89 1.89 1.80 1.75 1.71 1.67	1,000 8.53 5.63 3.23 2.71 2.54 2.30 2.21 2.54 2.30 2.21 2.30 2.07 2.02 1.97 1.92 2.02 1.97 1.88 1.85 1.79 1.70 1.66	> 1,000 8.54 5.63 4.36 3.23 2.93 2.71 2.54 2.41 2.30 2.21 2.13 2.07 2.01 1.96 1.92 1.88 1.78 1.78 1.69 1.66 1.62
$\begin{array}{c} df_{\text{NUM}} / \\ df_{\text{DEN}} \\ \hline \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \\ 13 \\ 14 \\ 15 \\ 16 \\ 17 \\ 18 \\ 19 \\ 20 \\ 22 \\ 24 \\ 26 \\ 28 \\ 30 \\ 35 \\ \end{array}$	20 8.66 5.80 4.56 3.87 3.44 3.15 2.94 2.65 2.54 2.46 2.23 2.23 2.23 2.24 2.23 2.23 2.23 2.23	22 8.65 5.79 4.54 3.83 3.13 2.92 2.75 2.63 2.63 2.63 2.25 2.21 2.13 2.10 2.05 2.11 2.13 2.10 2.00 1.97 1.93 1.91 1.85	24 8.64 5.77 3.84 3.41 2.90 2.74 2.61 2.51 2.29 2.24 2.19 2.25 2.29 2.24 2.19 2.11 2.08 2.03 1.95 1.91 1.89 1.83	26 8.63 3.40 2.89 2.49 2.49 2.49 2.49 2.49 2.22 2.17 2.22 2.17 2.13 2.10 2.07 2.10 1.97 1.93 1.90 1.87	28 8.62 5.75 4.50 3.82 2.39 2.27 2.21 2.16 2.22 2.26 2.21 2.16 2.12 2.28 2.26 2.21 2.16 2.12 2.08 2.05 2.00 1.95 1.91 1.88 1.85	30 8.622 5.75 4.50 3.81 3.38 2.86 2.70 2.57 2.47 2.38 2.22 2.15 2.21 2.07 2.247 2.215 2.215 2.07 2.04 2.15 2.07 2.04 1.90 1.90 1.87 4.10 2.15 2.15 2.15 2.15 2.15 2.15 2.15 2.15	355 8.66 5.77 3.33 3.00 2.66 2.55 2.44 2.33 2.22 2.22 2.21 2.12 2.00 2.00 2.00 2.00	40 8.599 3.5.72 3.4.46 3.77 5.3.34 4.2.83 3.044 4.1.99 5.003 1.1.99 5.1.94 1.855 4.1.85	45 8.59 5.71 4.45 3.766 3.33 3.03 2.815 2.52 2.433 2.25 2.19 2.19 2.14 2.09 5.205 2.19 2.14 1.92 2.00 1.98 4.192 2.00 1.98 4.192 2.00 1.98 4.192 2.00 1.98 4.192 2.00 1.98 4.192 2.00 1.98 4.192 2.00 1.98 4.192 2.00 1.984 2.00 2.00 1.984 2.00 2.00 2.00 2.00 2.00 2.00 2.00 2.0	500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 500 50 5	0 (1) 8 8 0 5 3 2 3 2 3 2 3 2 3 2 3 2 3 2 3 2 3 2 3 2 3 2 3 2 4 4 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 <td< td=""><td>50 57 69 57 43 30 77 43 30 30 31 30 31 30 31 30 31 31 31 31 31 31 31 31</td><td>70 3.57 5.68 4.42 3.73 3.29 2.78 2.29 2.278 2.29 2.278 2.29 2.275 2.209 2.205 2.00</td><td>80 8.56 5.67 4.42 3.72 2.99 2.77 2.60 2.27 2.20 2.27 2.20 2.21 2.21 2.20 2.21 2.14 2.08 2.03 1.99 1.92 1.82 1.92 1.82 1.78 1.74 1.92 1.82 1.78 1.74 1.92</td><td>100 8.55 5.66 4.41 3.71 2.76 2.97 2.76 2.97 2.26 2.12 2.07 2.02 2.07 2.02 2.07 2.02 2.12 2.07 2.02 1.98 1.94 1.94 1.94 1.94 1.94 1.76 1.76 1.74 1.94 1.94 1.94 1.94 1.94 1.94 1.94 1.9</td><td>200 8.54 5.65 2.73 2.95 2.73 2.26 2.243 2.22 2.23 2.10 2.04 1.99 1.95 1.82 1.69 1.66</td><td>500 8.53 5.64 4.37 3.68 3.24 2.72 2.55 2.42 2.23 1.222 2.14 2.20 2.02 1.97 1.89 1.86 1.80 1.89 1.86 1.81 1.71 1.67 1.67</td><td>1,000 8.53 5.63 4.37 3.67 2.93 2.71 2.24 2.21 2.21 2.21 2.21 2.22 1.87 1.92 1.85 1.79 1.70 1.66 1.63 1.57</td><td>>1,000 8.54 5.63 4.36 3.23 2.93 2.71 2.51 2.21 2.13 2.07 2.01 1.96 1.92 1.88 1.78 1.78 1.69 1.66 1.62 1.56</td></td<>	50 57 69 57 43 30 77 43 30 30 31 30 31 30 31 30 31 31 31 31 31 31 31 31	70 3.57 5.68 4.42 3.73 3.29 2.78 2.29 2.278 2.29 2.278 2.29 2.275 2.209 2.205 2.00	80 8.56 5.67 4.42 3.72 2.99 2.77 2.60 2.27 2.20 2.27 2.20 2.21 2.21 2.20 2.21 2.14 2.08 2.03 1.99 1.92 1.82 1.92 1.82 1.78 1.74 1.92 1.82 1.78 1.74 1.92	100 8.55 5.66 4.41 3.71 2.76 2.97 2.76 2.97 2.26 2.12 2.07 2.02 2.07 2.02 2.07 2.02 2.12 2.07 2.02 1.98 1.94 1.94 1.94 1.94 1.94 1.76 1.76 1.74 1.94 1.94 1.94 1.94 1.94 1.94 1.94 1.9	200 8.54 5.65 2.73 2.95 2.73 2.26 2.243 2.22 2.23 2.10 2.04 1.99 1.95 1.82 1.69 1.66	500 8.53 5.64 4.37 3.68 3.24 2.72 2.55 2.42 2.23 1.222 2.14 2.20 2.02 1.97 1.89 1.86 1.80 1.89 1.86 1.81 1.71 1.67 1.67	1,000 8.53 5.63 4.37 3.67 2.93 2.71 2.24 2.21 2.21 2.21 2.21 2.22 1.87 1.92 1.85 1.79 1.70 1.66 1.63 1.57	> 1,000 8.54 5.63 4.36 3.23 2.93 2.71 2.51 2.21 2.13 2.07 2.01 1.96 1.92 1.88 1.78 1.78 1.69 1.66 1.62 1.56
$\begin{array}{c} df_{\text{NUM}} / \\ df_{\text{DEN}} \\ \hline \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \\ 13 \\ 14 \\ 15 \\ 16 \\ 17 \\ 18 \\ 19 \\ 20 \\ 22 \\ 24 \\ 26 \\ 28 \\ 30 \\ 35 \\ 40 \end{array}$	20 8.66 5.80 4.56 3.87 3.44 3.15 2.94 2.33 2.27 2.65 2.54 2.39 2.46 2.39 2.42 2.23 2.28 2.23 2.19 2.12 2.07 2.03 1.99 1.96 1.93 1.88 1.88 1.88 1.88 1.88 1.88 1.93 1.94 1.95 1.94 1.94 1.95 1.95 1.94 1.95 1.94 1.95 1.95 1.95 1.95 1.95 1.95 1.95 1.95	22 8.65 5.79 4.54 3.83 3.13 2.92 2.75 2.63 2.52 2.44 2.37 2.13 2.10 2.05 2.21 2.13 2.10 2.00 1.97 1.93 1.91 1.85 1.81 5.40 5.40 5.40 5.40 5.40 5.40 5.40 5.40	24 8.64 5.77 4.53 3.84 2.51 2.290 2.74 2.61 2.51 2.42 2.35 2.29 2.24 2.15 2.29 2.24 2.19 2.15 2.29 2.24 2.19 2.15 2.03 1.98 1.99 1.83 1.99 1.83 1.79	26 8.63 3.76 4.52 3.83 3.40 3.10 2.72 2.59 2.41 2.33 2.27 2.22 2.21 2.13 2.10 2.01 1.97 1.93 1.90 1.82 1.72	28 8.62 5.75 4.50 3.39 3.09 2.87 2.26 2.21 2.26 2.22 2.26 2.22 2.26 2.22 2.26 2.21 2.20 8.23 2.20 1.91 1.88 8.185 1.80 1.76 1.76 2.00 1.91 1.88 1.80 2.91 2.00 1.91 1.88 1.80 2.91 2.00 1.91 1.88 1.80 2.91 2.00 2.92 2.00 2.92 2.00 2.92 2.00 2.92 2.00 2.92 2.00 2.92 2.00 2.92 2.00 2.92 2.00 2.92 2.00 2.92 2.00 2.92 2.92	30 8.62 5.75 4.50 3.811 3.38 3.08 2.66 2.77 2.47 2.31 2.25 2.111 2.22 2.111 2.151 2.141 2.19 2.141 2.19 2.19 2.19 2.19 2.19 2.19 2.19 2.1	355 8.66 5.73 4.44 3.77 3.33 3.00 2.84 2.55 2.44 2.33 2.24 2.21 2.01 2.00 2.00 2.00 2.00 2.00 2.00	40 8.599 3.572 3.4.46 3.77 3.344 4.2.83 3.044 4.2.83 5.2.93 4.2.83 5.2.93 5.2.93 5.2.93 5.1.94 5.1	45 8.59 5.71 4.45 3.766 3.33 3.03 2.81 2.52 2.52 2.52 2.52 2.52 2.52 2.52 2.5	50 50 50 5 5 7 4 4 4 3 7 3 3 3 3 3 3 3 3 3 3	8 8 8 8 8 8 0 5 3 2 3 3 2 3 2 3 2 3 2 3 2 3 2 2 4 4 2 2 3 2 11 2 2 12 1 2 18 2 2 19 1 1 16 1 1 17 1 1 16 1 1 17 1 1 16 1 1 17 1 1 16 1 1 17 1 1 16 1 1 17 1 1 16 1 1 17 1 1 16 1 1 17 1 1 18 1 1<	50 57 6 69 69 74 30 01 79 62 49 30 01 79 62 49 33 30 30 31 30 32 32 33 33 33 33 33 34 35 36 36 37 37 38 38 38 38 38 38 38 38	70 3.57 5.68 4.42 3.73 3.29 2.78 2.29 2.278 2.29 2.275 2.20 2.05 2.00 2.05 2.00 2.05 2.00 2.05 2.00 1.97 1.93 1.83 1.79 1.75 1.66 1.62 2.62	80 8.56 5.67 4.42 2.99 2.77 2.36 2.47 2.26 2.27 2.20 2.14 2.03 1.99 1.96 1.82 1.78 1.92 1.82 1.78 1.74 1.71 1.65 1.61	100 8.55 5.66 4.41 3.71 2.77 2.79 2.26 2.35 2.26 2.19 2.22 2.20 2.12 2.07 2.02 1.98 1.94 1.91 1.85 1.70 1.76 1.73 1.70	200 8.54 3.69 3.25 2.95 2.73 2.25 2.23 2.23 2.23 2.24 2.23 2.24 2.23 2.24 2.10 2.04 1.99 1.95 1.91 1.88 1.82 1.77 1.69 1.66 1.55	500 8.53 5.64 4.37 3.68 3.24 2.72 2.55 2.42 2.31 2.22 2.14 2.08 2.02 1.97 1.89 1.89 1.89 1.89 1.89 1.89 1.89 1.89	1,000 8.53 5.63 4.37 3.67 2.93 2.71 2.24 2.21 2.21 2.21 2.21 2.22 1.97 1.92 1.97 1.92 1.79 1.74 1.85 1.79 1.74 1.63 1.57 1.52	> 1,000 8.54 5.63 4.36 3.67 3.23 2.93 2.71 2.54 2.41 2.30 2.21 2.13 2.07 2.01 1.96 1.92 1.88 1.78 1.78 1.78 1.69 1.66 1.62 1.56 1.51
df _{NUM} / df _{DEN} 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 22 24 26 28 30 35 40 45 50	20 8.66 5.80 4.56 3.87 3.44 3.15 2.27 2.65 2.54 2.39 2.12 2.07 2.23 2.19 2.12 2.07 2.03 1.99 1.96 1.93 1.88 1.84 1.81 1.88 1.84	22 8.65 5.79 4.54 3.83 3.43 3.13 2.92 2.75 2.63 2.37 2.31 2.25 2.44 2.21 2.17 2.13 2.05 2.00 2.05 2.00 2.197 1.93 1.91 1.85 1.81 1.78	24 8.64 5.77 4.53 3.84 2.61 2.74 2.61 2.74 2.25 2.29 2.24 2.15 2.15 2.15 2.29 2.24 2.15 2.15 2.19 1.208 2.03 1.98 1.99 1.83 1.99 1.79 1.79 1.79 1.79 1.79 1.79 1.79	26 8.63 3.76 4.52 3.83 3.40 3.10 2.72 2.59 2.41 2.33 2.27 2.22 2.17 2.13 2.10 2.07 2.01 1.97 1.93 1.90 1.82 1.77 1.74 2.72	28 8.62 5.75 4.50 3.89 2.87 2.71 2.58 2.48 2.32 2.26 2.21 2.12 2.12 2.12 2.12 2.12 2.1	30 8.62 5.75 4.50 3.811 3.38 3.08 2.86 2.77 2.47 2.38 2.57 2.47 2.151 2.151 2.151 2.111 2.04 1.98 1.99 1.87 1.84 4.90 1.87 1.74 1.79	35 8.66(5.77 3.333 3.00(2.8* 2.55; 2.44 2.35; 2.44 2.25; 2.24; 2.22; 2.12; 2.20; 2.00(2.00) 1.99(1.99) 1.8; 1.84 1.84 1.84 1.84 1.77 1.17; 1.66(1.74) 1.77; 1.66(1.74) 1.77; 1.66(1.74) 1.77; 1.66(1.74) 1.77; 1.66(1.74) 1.77; 1.66(1.74) 1.77; 1.66(1.74) 1.77; 1.66(1.74) 1.74;	40 8.599 8.572 8.446 9.377 3.345 5.334 4.243 4.243 4.243 3.265 2.253 4.243 3.243 4.243 3.242 2.243 3.205 2.210 3.205 2.103 3.205 2.1	45 8.59 5.71 4.45 3.76 3.33 2.81 2.52 2.52 2.52 2.52 2.52 2.52 2.52 2.5	50 50 5 5 7 4 4 3 3 0 2 8 5 5 7 4 4 3 3 0 2 8 2 4 4 3 7 3 3 0 2 8 2 5 5 7 7 3 3 3 0 2 8 2 2 4 4 4 3 7 3 3 0 2 8 2 2 4 4 4 4 3 7 3 3 0 2 8 2 4 4 4 4 4 4 4 4 4 4	8 8 8 8 8 0 5 3 3 2 3 2 3 2 2 3 2 2 3 2 3 2 2 3 2 2 3 2 3 2 2 3 2 2 3 4 2 2 2 10 2 1 2 12 1 1 1 13 1 1 1 14 2 2 2 18 2 2 1 19 1 1 1 13 1 1 1 14 2 2 2 19 1 1 1 10 1 1 1 10 1 1 1 10 1 1 1 10 1 1 1 10	50 57 6 69 74 30 79 62 49 38 30 62 49 38 30 62 22 16 111 066 02 98 884 800 777 74 68 664 665 669 69 69 69 69 69 69 6	70 3.57 3.68 4.42 2.37 2.29 2.28 2.22 2.22 2.20 2.20 2.20 2.20 2.20	80 8.56 5.67 4.42 3.72 2.99 2.77 2.36 2.27 2.26 2.27 2.20 2.14 2.03 1.99 1.96 1.82 2.03 1.99 1.92 1.86 1.82 1.74 1.71 1.54	100 8.55 5.66 4.41 3.71 2.97 2.76 2.35 2.26 2.29 2.22 2.07 2.02 1.98 1.94 1.91 1.85 1.80 1.73 1.70 1.55 1.55	200 8.54 3.69 3.69 2.73 2.25 2.25 2.23 2.23 2.23 2.24 2.23 2.24 2.23 2.24 1.99 1.95 1.91 1.88 1.82 1.77 1.69 1.69 1.55 1.51	500 8.53 5.64 4.37 3.68 3.24 2.72 2.31 2.22 2.31 2.22 2.31 2.22 2.31 2.22 1.47 1.93 1.89 1.80 1.75 1.67 1.67 1.53 1.49	1,000 8.53 5.63 4.37 3.23 2.71 2.54 2.241 2.30 2.21 2.241 2.30 2.21 2.241 2.30 2.21 1.97 1.92 1.97 1.92 1.97 1.92 1.97 1.92 1.97 1.92 1.97 1.92 1.97 1.92 1.97 1.92 1.97 1.92 1.97 1.92 1.97 1.92 1.97 1.92 1.93 1.97 1.92 1.93 1.97 1.92 1.93 1.92 1.93 1.93 1.94 1.94 1.95 1.94 1.95 1.95 1.95 1.95 1.95 1.95 1.95 1.95	> 1,000 8.54 5.63 4.36 3.67 3.23 2.71 2.54 2.41 2.30 2.21 2.13 2.07 2.01 1.96 1.92 1.84 1.78 1.73 1.66 1.62 1.56 1.51 1.44
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EXHIBIT 11.2. TABLE OF *F*-STATISTICS, P = .01.

ar _{NUM} / df	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
2 DEN	2412	20.92	20.46	20 71	20 24	27.01	27.67	27.40	27.25	27.22	27.12	27.05	26.09	26.02	26.97
3	21 20	30.82 18.00	29.46	26.71	28.24	27.91	27.67	27.49	27.55	27.25	27.15	27.05	20.98	26.92	20.87
5	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16	10.05	9.96	9.89	9.82	9.77	9.72
6	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.79	7.72	7.66	7.61	7.56
/	12.25	9.55	8.45 7.59	7.85	7.46	6 37	6.99	6.84 6.03	6.72 5.91	6.6Z	6.54 5.73	6.47 5.67	6.41 5.61	6.36 5.56	5.57
9	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	5.18	5.11	5.05	5.01	4.96
10	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.77	4.71	4.65	4.60	4.56
11 12	9.65	7.21	6.22 5.95	5.67 5.41	5.32	5.07	4.89	4.74	4.63 4 39	4.54	4.46	4.40	4.34	4.29	4.25
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10	4.02	3.96	3.91	3.86	3.82
14	8.86	6.51	5.56	5.04	4.70	4.46	4.28	4.14	4.03	3.94	3.86	3.80	3.75	3.70	3.66
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	3.73	3.67	3.61	3.56	3.52
10	8.40	6.11	5.19	4.77	4.44	4.20	3.93	3.79	3.68	3.59	3.52	3.46	3.40	3.35	3.31
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51	3.43	3.37	3.32	3.27	3.23
19	8.19	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	3.36	3.30	3.24	3.19	3.15
20 22	8.10	5.85	4.94	4.43	4.10	3.8/	3.70	3.56	3.46	3.3/	3.29	3.23	3.18	3.13	3.09
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	3.09	3.03	2.98	2.93	2.89
26	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09	3.02	2.96	2.90	2.86	2.82
28	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03	2.96	2.90	2.84	2.79	2.75
35	7.42	5.27	4.40	3.91	3.59	3.37	3.20	3.07	2.96	2.88	2.80	2.74	2.69	2.64	2.60
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	2.73	2.66	2.61	2.56	2.52
45	7.23	5.11	4.25	3.77	3.45	3.23	3.07	2.94	2.83	2.74	2.67	2.61	2.55	2.51	2.46
50 60	7.08	5.06 4.98	4.20	3.65	3.34	3.19	2.95	2.89	2.79	2.70	2.65	2.50	2.31	2.46	2.42
70	7.01	4.92	4.07	3.60	3.29	3.07	2.91	2.78	2.67	2.59	2.51	2.45	2.40	2.35	2.31
80	6.96	4.88	4.04	3.56	3.26	3.04	2.87	2.74	2.64	2.55	2.48	2.42	2.36	2.31	2.27
200	6.90 6.76	4.82	3.98	3.51	3.21	2.99	2.82	2.69	2.59	2.50	2.43	2.37	2.31	2.27	2.22
500	6.69	4.65	3.82	3.36	3.05	2.84	2.68	2.55	2.44	2.36	2.28	2.22	2.17	2.12	2.07
1000	6.66	4.63	3.80	3.34	3.04	2.82	2.66	2.53	2.43	2.34	2.27	2.20	2.15	2.10	2.06
>1000	1.04	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32	2.25	2.19	2.13	2.08	2.04
df _{NUM} /	16	17	18	10	20	22	24	26	28	30	35	40	45	50	60
	10		10	19	-20		24	20	20			40	45		
3	26.83	26.79	26.75	26./2	26.69	26.64	26.60	26.56	26.53	26.50	26.45	26.41	26.38	26.35	26.32
5			0.61	0.50	11.02	0.51	0 47	0 / 2	9.40	9.38	9.33	9.29	9.26	9.24	9.20
	9.68	9.64	9.61	9.30	9.55	9.51	9.47	2.45	2.40					7.27	
6	9.68 7.52	9.64 7.48	9.61 7.45	7.42	9.55 7.40	7.35	7.31	7.28	7.25	7.23	7.18	7.14	7.11	7.09	7.06
6 7	9.68 7.52 6.28	9.64 7.48 6.24	9.61 7.45 6.21	9.38 7.42 6.18	9.55 7.40 6.16	9.51 7.35 6.11	7.31 6.07	7.28 6.04	7.25	7.23 5.99	7.18 5.94	7.14 5.91	7.11 5.88	7.09 5.86	7.06 5.82
6 7 8 9	9.68 7.52 6.28 5.48 4.92	9.64 7.48 6.24 5.44 4.89	9.61 7.45 6.21 5.41 4.86	9.38 7.42 6.18 5.38 4.83	9.55 7.40 6.16 5.36 4.81	9.51 7.35 6.11 5.32 4.77	9.47 7.31 6.07 5.28 4.73	7.28 6.04 5.25 4.70	7.25 6.02 5.22 4.67	7.23 5.99 5.20 4.65	7.18 5.94 5.15 4.60	7.14 5.91 5.12 4.57	7.11 5.88 5.09 4.54	7.09 5.86 5.07 4.52	7.06 5.82 5.03 4.48
6 7 8 9 10	9.68 7.52 6.28 5.48 4.92 4.52	9.64 7.48 6.24 5.44 4.89 4.49	9.61 7.45 6.21 5.41 4.86 4.46	9.38 7.42 6.18 5.38 4.83 4.43	9.55 7.40 6.16 5.36 4.81 4.41	9.51 7.35 6.11 5.32 4.77 4.36	9.47 7.31 6.07 5.28 4.73 4.33	7.28 6.04 5.25 4.70 4.30	7.25 6.02 5.22 4.67 4.27	7.23 5.99 5.20 4.65 4.25	7.18 5.94 5.15 4.60 4.20	7.14 5.91 5.12 4.57 4.17	7.11 5.88 5.09 4.54 4.14	7.09 5.86 5.07 4.52 4.12	7.06 5.82 5.03 4.48 4.08
6 7 8 9 10 11	9.68 7.52 6.28 5.48 4.92 4.52 4.21	9.64 7.48 6.24 5.44 4.89 4.49 4.18	9.61 7.45 6.21 5.41 4.86 4.46 4.15	9.38 7.42 6.18 5.38 4.83 4.43 4.12	9.55 7.40 6.16 5.36 4.81 4.41 4.10	9.51 7.35 6.11 5.32 4.77 4.36 4.06	9.47 7.31 6.07 5.28 4.73 4.33 4.02	7.28 6.04 5.25 4.70 4.30 3.99	7.25 6.02 5.22 4.67 4.27 3.96	7.23 5.99 5.20 4.65 4.25 3.94	7.18 5.94 5.15 4.60 4.20 3.89	7.14 5.91 5.12 4.57 4.17 3.86	7.11 5.88 5.09 4.54 4.14 3.83	7.09 5.86 5.07 4.52 4.12 3.81	7.06 5.82 5.03 4.48 4.08 3.78
6 7 8 9 10 11 12 13	9.68 7.52 6.28 5.48 4.92 4.52 4.21 3.97 3.78	9.64 7.48 6.24 5.44 4.89 4.49 4.18 3.94 3.75	9.61 7.45 6.21 5.41 4.86 4.46 4.15 3.91 3.72	9.38 7.42 6.18 5.38 4.83 4.43 4.12 3.88 3.69	9.55 7.40 6.16 5.36 4.81 4.41 4.10 3.86 3.66	9.51 7.35 6.11 5.32 4.77 4.36 4.06 3.82 3.62	9.47 7.31 6.07 5.28 4.73 4.33 4.02 3.78 3.59	7.28 6.04 5.25 4.70 4.30 3.99 3.75 3.56	7.25 6.02 5.22 4.67 4.27 3.96 3.72 3.53	7.23 5.99 5.20 4.65 4.25 3.94 3.70 3.51	7.18 5.94 5.15 4.60 4.20 3.89 3.65 3.46	7.14 5.91 5.12 4.57 4.17 3.86 3.62 3.43	7.11 5.88 5.09 4.54 4.14 3.83 3.59 3.40	7.09 5.86 5.07 4.52 4.12 3.81 3.57 3.38	7.06 5.82 5.03 4.48 4.08 3.78 3.54 3.34
6 7 8 9 10 11 12 13 14	9.68 7.52 6.28 5.48 4.92 4.52 4.21 3.97 3.78 3.62	9.64 7.48 6.24 5.44 4.89 4.49 4.18 3.94 3.75 3.59	9.61 7.45 6.21 5.41 4.86 4.46 4.15 3.91 3.72 3.56	9.38 7.42 6.18 5.38 4.83 4.43 4.43 4.12 3.88 3.69 3.53	9.55 7.40 6.16 5.36 4.81 4.41 4.10 3.86 3.66 3.51	9.51 7.35 6.11 5.32 4.77 4.36 4.06 3.82 3.62 3.46	9.47 7.31 6.07 5.28 4.73 4.33 4.02 3.78 3.59 3.43	7.28 6.04 5.25 4.70 4.30 3.99 3.75 3.56 3.40	7.25 6.02 5.22 4.67 4.27 3.96 3.72 3.53 3.37	7.23 5.99 5.20 4.65 4.25 3.94 3.70 3.51 3.35	7.18 5.94 5.15 4.60 4.20 3.89 3.65 3.46 3.30	7.14 5.91 5.12 4.57 4.17 3.86 3.62 3.43 3.27	7.11 5.88 5.09 4.54 4.14 3.83 3.59 3.40 3.24	7.09 5.86 5.07 4.52 4.12 3.81 3.57 3.38 3.22	7.06 5.82 5.03 4.48 4.08 3.78 3.54 3.34 3.18
6 7 8 9 10 11 12 13 14 15	9.68 7.52 6.28 5.48 4.92 4.52 4.21 3.97 3.78 3.62 3.49	9.64 7.48 6.24 5.44 4.89 4.49 4.18 3.94 3.75 3.59 3.45	9.61 7.45 6.21 5.41 4.86 4.46 4.15 3.91 3.72 3.56 3.42	9.38 7.42 6.18 5.38 4.83 4.43 4.12 3.88 3.69 3.53 3.40	9.55 7.40 6.16 5.36 4.81 4.41 4.10 3.86 3.66 3.51 3.37	9.51 7.35 6.11 5.32 4.77 4.36 4.06 3.82 3.62 3.46 3.33	9.47 7.31 6.07 5.28 4.73 4.33 4.02 3.78 3.59 3.43 3.29	7.28 6.04 5.25 4.70 4.30 3.99 3.75 3.56 3.40 3.26	7.25 6.02 5.22 4.67 4.27 3.96 3.72 3.53 3.37 3.24	7.23 5.99 5.20 4.65 4.25 3.94 3.70 3.51 3.35 3.21	7.18 5.94 5.15 4.60 4.20 3.89 3.65 3.46 3.30 3.17	7.14 5.91 5.12 4.57 4.17 3.86 3.62 3.43 3.27 3.13	7.11 5.88 5.09 4.54 4.14 3.83 3.59 3.40 3.24 3.10	7.09 5.86 5.07 4.52 4.12 3.81 3.57 3.38 3.22 3.08	7.06 5.82 5.03 4.48 4.08 3.78 3.54 3.34 3.18 3.05
6 7 8 9 10 11 12 13 14 15 16 17	9.68 7.52 6.28 5.48 4.92 4.52 4.21 3.97 3.78 3.62 3.49 3.37 3.27	9.64 7.48 6.24 5.44 4.89 4.49 4.18 3.94 3.75 3.59 3.45 3.34 3.24	9.61 7.45 6.21 5.41 4.86 4.46 4.15 3.91 3.72 3.56 3.42 3.31 3.21	9.36 7.42 6.18 5.38 4.83 4.43 4.12 3.88 3.69 3.53 3.40 3.28 3.19	9.55 7.40 6.16 5.36 4.81 4.41 4.10 3.86 3.66 3.51 3.37 3.26 3.16	9.51 7.35 6.11 5.32 4.77 4.36 4.06 3.82 3.62 3.46 3.33 3.22 3.12	9.47 7.31 6.07 5.28 4.73 4.33 4.02 3.78 3.59 3.43 3.29 3.18 3.08	7.28 6.04 5.25 4.70 4.30 3.99 3.75 3.56 3.40 3.26 3.15 3.05	7.25 6.02 5.22 4.67 4.27 3.96 3.72 3.53 3.37 3.24 3.12 3.03	7.23 5.99 5.20 4.65 4.25 3.94 3.70 3.51 3.35 3.21 3.10 3.00	7.18 5.94 5.15 4.60 4.20 3.89 3.65 3.46 3.30 3.17 3.05 2.96	7.14 5.91 5.12 4.57 4.17 3.86 3.62 3.43 3.27 3.13 3.02 2.92	7.11 5.88 5.09 4.54 4.14 3.83 3.59 3.40 3.24 3.10 2.99 2.89	7.09 5.86 5.07 4.52 4.12 3.81 3.57 3.38 3.22 3.08 2.97 2.87	7.06 5.82 5.03 4.48 4.08 3.78 3.54 3.34 3.18 3.05 2.93 2.83
6 7 8 9 10 11 12 13 14 15 16 17 18	9.68 7.52 6.28 5.48 4.92 4.52 4.21 3.97 3.78 3.62 3.49 3.37 3.27 3.19	9.64 7.48 6.24 5.44 4.89 4.18 3.94 3.94 3.55 3.55 3.55 3.34 3.24 3.24 3.16	9.61 7.45 6.21 5.41 4.86 4.46 4.15 3.91 3.72 3.56 3.42 3.31 3.21 3.13	9.36 7.42 6.18 5.38 4.83 4.43 4.12 3.88 3.69 3.53 3.40 3.28 3.19 3.10	9.55 7.40 6.16 5.36 4.81 4.41 4.10 3.86 3.66 3.51 3.37 3.26 3.16 3.08	9.51 7.35 6.11 5.32 4.77 4.36 4.06 3.82 3.62 3.46 3.33 3.22 3.12 3.03	9.47 7.31 6.07 5.28 4.73 4.33 4.02 3.78 3.59 3.43 3.29 3.18 3.08 3.00	7.28 6.04 5.25 4.70 3.99 3.75 3.56 3.40 3.26 3.15 3.05 2.97	7.25 6.02 5.22 4.67 4.27 3.96 3.72 3.53 3.37 3.24 3.12 3.03 2.94	7.23 5.99 5.20 4.65 4.25 3.94 3.70 3.51 3.35 3.21 3.10 3.00 2.92	7.18 5.94 5.15 4.60 4.20 3.89 3.65 3.46 3.30 3.17 3.05 2.96 2.87	7.14 5.91 5.12 4.57 4.17 3.86 3.62 3.43 3.27 3.13 3.02 2.92 2.84	7.11 5.88 5.09 4.54 4.14 3.83 3.59 3.40 3.24 3.24 3.10 2.99 2.89 2.81	7.09 5.86 5.07 4.52 4.12 3.81 3.57 3.38 3.22 3.08 2.97 2.87 2.78	7.06 5.82 5.03 4.48 4.08 3.78 3.54 3.34 3.05 2.93 2.83 2.75
6 7 8 9 10 11 12 13 14 15 16 17 18 19	9.68 7.52 6.28 5.48 4.92 4.21 3.97 3.78 3.62 3.49 3.37 3.27 3.19 3.12	9.64 7.48 6.24 5.44 4.89 4.49 4.18 3.94 3.75 3.59 3.45 3.34 3.24 3.16 3.08	9.61 7.45 6.21 5.41 4.86 4.46 4.15 3.91 3.72 3.56 3.42 3.31 3.21 3.13 3.21	9.36 7.42 6.18 5.38 4.83 4.43 4.12 3.88 3.69 3.53 3.40 3.28 3.19 3.10 3.03	9.55 7.40 6.16 5.36 4.81 4.41 4.10 3.86 3.51 3.37 3.26 3.16 3.08 3.08	9.51 7.35 6.11 5.32 4.77 4.36 4.06 3.82 3.62 3.46 3.33 3.22 3.12 3.03 2.96	9.47 7.31 6.07 5.28 4.73 4.33 4.02 3.78 3.59 3.43 3.29 3.18 3.08 3.00 2.92	7.28 6.04 5.25 4.70 4.30 3.99 3.75 3.56 3.40 3.26 3.15 3.05 2.97 2.89	7.40 6.02 5.22 4.67 4.27 3.96 3.72 3.53 3.37 3.24 3.12 3.03 2.94 2.87	7.23 5.99 5.20 4.65 4.25 3.94 3.70 3.51 3.35 3.21 3.10 3.00 2.92 2.84	7.18 5.94 5.15 4.60 4.20 3.89 3.65 3.46 3.30 3.17 3.05 2.96 2.87 2.80	7.14 5.91 5.12 4.57 4.17 3.86 3.62 3.43 3.27 3.13 3.02 2.92 2.84 2.76	7.11 5.88 5.09 4.54 4.14 3.83 3.59 3.40 3.24 3.10 2.99 2.89 2.81 2.73	7.09 5.86 5.07 4.52 4.12 3.81 3.57 3.38 3.28 3.08 2.97 2.87 2.78 2.71	7.06 5.82 5.03 4.48 4.08 3.78 3.34 3.34 3.18 3.05 2.93 2.83 2.75 2.67
6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 22	9.68 7.52 6.28 5.48 4.92 4.21 3.97 3.78 3.62 3.49 3.37 3.27 3.19 3.12 3.05 2.94	9.64 7.48 6.24 5.44 4.89 4.49 4.18 3.94 3.75 3.59 3.45 3.34 3.24 3.16 3.08 3.02 2.91	9.61 7.45 6.21 5.41 4.86 4.46 4.15 3.91 3.72 3.56 3.42 3.31 3.21 3.13 3.05 2.99 2.88	9.36 7.42 6.18 5.38 4.83 4.43 4.12 3.88 3.69 3.59 3.40 3.28 3.40 3.28 3.19 3.10 3.03 2.96	9.55 7.40 6.16 5.36 4.81 4.41 4.10 3.86 3.66 3.51 3.37 3.26 3.16 3.08 3.00 2.94 2.83	9.51 7.35 6.11 5.32 4.77 4.36 4.06 3.82 3.62 3.62 3.62 3.62 3.33 3.22 3.12 3.03 2.96 2.90 2.78	9.47 7.31 6.07 5.28 4.73 4.33 4.02 3.78 3.59 3.49 3.59 3.29 3.18 3.00 2.92 2.86 2.25	7.28 6.04 5.25 4.70 4.30 3.99 3.75 3.56 3.46 3.26 3.15 3.05 2.97 2.89 2.83 2.72	7.45 6.02 5.22 4.67 4.27 3.96 3.72 3.53 3.37 3.24 3.12 3.03 2.94 2.87 2.80 2.69	7.23 5.99 5.20 4.65 4.25 3.94 3.51 3.35 3.21 3.10 3.00 2.92 2.84 2.78	7.18 5.94 5.15 4.60 3.89 3.65 3.46 3.30 3.17 3.05 2.96 2.87 2.80 2.73 2.62	7.14 5.91 5.12 4.57 3.86 3.62 3.43 3.27 3.13 3.02 2.92 2.84 2.76 2.69 2.58	7.11 5.88 5.09 4.54 4.14 3.83 3.59 3.40 3.24 3.10 2.99 2.89 2.81 2.73 2.67	7.09 5.86 5.07 4.52 4.12 3.81 3.57 3.38 3.22 3.08 2.97 2.87 2.78 2.71 2.64 2.53	7.06 5.82 5.03 4.48 3.78 3.54 3.34 3.18 3.05 2.93 2.83 2.75 2.67 2.61 2.50
6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 22 24	9.68 7.52 6.28 5.48 4.92 4.52 4.21 3.78 3.78 3.62 3.49 3.37 3.19 3.12 3.05 2.94 2.85	9.64 7.48 6.24 5.44 4.89 4.49 4.18 3.75 3.59 3.45 3.34 3.24 3.16 3.08 3.02 2.91 2.82	9.61 7.45 6.21 5.41 4.86 4.46 4.46 3.91 3.72 3.56 3.42 3.31 3.21 3.13 3.05 2.99 2.88 2.79	7.42 6.18 5.38 4.83 4.43 4.43 3.88 3.69 3.53 3.40 3.29 3.10 3.03 2.96 2.85 2.76	9.55 7.40 6.16 5.36 4.81 4.41 4.41 3.86 3.66 3.51 3.37 3.26 3.16 3.08 3.00 2.94 2.83 2.74	9.51 7.35 6.11 5.32 4.77 4.36 4.06 3.82 3.62 3.46 3.33 3.22 3.12 3.03 2.96 2.90 2.70	9.47 7.31 6.07 5.28 4.73 4.33 4.03 3.78 3.59 3.43 3.29 3.18 3.08 3.00 2.92 2.86 2.75 2.66	7.28 6.04 5.25 4.70 4.30 3.99 3.75 3.56 3.40 3.26 3.15 3.05 2.97 2.89 2.83 2.72 2.63	7.25 6.02 5.22 4.67 3.96 3.72 3.53 3.37 3.24 3.12 3.03 2.94 2.80 2.60	7.23 5.99 5.20 4.65 3.94 3.70 3.51 3.35 3.21 3.10 3.00 2.92 2.84 2.78 2.67 2.58	7.18 5.94 5.15 4.60 3.89 3.65 3.46 3.30 3.17 3.05 2.96 2.87 2.80 2.73 2.62 2.53	7.14 5.91 5.12 4.57 4.17 3.86 3.62 3.43 3.27 3.13 3.02 2.92 2.84 2.76 2.69 2.58 2.49	7.11 5.88 5.09 4.54 4.14 3.83 3.59 3.40 3.24 3.10 2.99 2.89 2.89 2.81 2.73 2.67 2.55 2.246	7.09 5.86 5.07 4.52 4.12 3.81 3.57 3.38 3.22 3.08 2.97 2.87 2.78 2.71 2.64 2.53 2.44	7.06 5.82 5.03 4.48 3.78 3.54 3.34 3.18 3.05 2.93 2.83 2.75 2.67 2.61 2.50 2.40
6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 22 24 26	9.68 7.52 6.28 5.48 4.92 4.52 4.21 3.97 3.78 3.62 3.49 3.37 3.19 3.12 3.05 2.94 2.85 2.78	9.64 7.48 6.24 4.89 4.49 4.49 3.75 3.59 3.45 3.34 3.24 3.08 3.02 2.91 2.82 2.82	9.61 7.45 6.21 5.41 4.86 4.46 4.45 3.91 3.72 3.56 3.42 3.31 3.21 3.13 3.05 2.99 2.88 2.79 2.72	7.42 6.18 5.38 4.83 4.43 4.12 3.88 3.69 3.53 3.40 3.28 3.19 3.10 3.03 2.96 2.85 2.76 2.69	9.55 7.40 6.16 5.36 4.81 4.41 4.40 3.86 3.66 3.16 3.37 3.26 3.16 3.08 3.00 2.94 2.83 2.74 2.66	9.51 7.35 6.11 5.32 4.77 4.36 4.06 3.82 3.62 3.46 3.33 3.22 3.12 3.03 2.96 2.90 2.70 2.70 2.62	9.47 7.31 6.07 5.28 4.73 4.02 3.78 3.59 3.43 3.59 3.43 3.08 3.00 2.92 2.86 2.75 2.66 2.58	7.28 6.04 5.25 4.70 4.30 3.99 3.75 3.56 3.40 3.26 3.15 3.05 2.97 2.89 2.83 2.72 2.63 2.55	7.25 6.02 5.22 4.67 4.27 3.96 3.72 3.53 3.37 3.24 3.12 3.03 2.94 2.87 2.80 2.60 2.53	7.23 5.99 5.20 4.65 3.94 3.70 3.51 3.35 3.21 3.10 3.00 2.92 2.84 2.78 2.67 2.58 2.50	7.18 5.94 5.15 4.60 4.20 3.89 3.65 3.46 3.30 3.17 3.05 2.96 2.87 2.80 2.73 2.62 2.53 2.45	7.14 5.91 5.12 4.57 4.17 3.86 3.62 3.43 3.27 3.13 3.02 2.92 2.84 2.76 2.69 2.58 2.49 2.58 2.49	7.11 5.88 5.09 4.54 4.14 3.83 3.59 3.40 3.24 3.10 2.99 2.89 2.81 2.73 2.65 2.55 2.46 2.39	7.09 5.86 5.07 4.52 4.12 3.81 3.57 3.38 3.22 3.08 2.97 2.87 2.78 2.71 2.64 2.53 2.44 2.36	7.06 5.82 5.03 4.48 4.08 3.78 3.54 3.34 3.18 3.05 2.93 2.83 2.75 2.67 2.61 2.50 2.40 2.33
6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 22 24 26 28 30	9.68 7.52 6.28 5.48 4.92 4.52 4.52 4.51 3.97 3.78 3.62 3.49 3.37 3.12 3.05 2.94 2.85 2.78 2.78 2.72	9.64 7.48 6.24 5.44 4.89 4.18 3.94 3.75 3.59 3.45 3.34 3.24 3.16 3.08 3.02 2.91 2.82 2.75 2.68 2.63	9.61 7.45 6.21 5.41 4.86 4.46 4.15 3.91 3.72 3.56 3.42 3.31 3.21 3.13 3.05 2.99 2.88 2.79 2.72 2.65 2.60	7.42 6.18 5.38 4.43 4.43 4.12 3.88 3.69 3.53 3.40 3.28 3.19 3.10 3.03 2.96 2.85 2.76 2.69 2.63	9.55 7.40 6.16 5.36 4.81 4.41 3.86 3.51 3.37 3.26 3.00 2.94 2.83 2.74 2.66 2.60 2.55	9.51 7.355 6.11 5.32 4.77 4.36 4.06 3.82 3.62 3.46 3.33 3.22 3.12 3.03 2.96 2.90 2.78 2.90 2.62 2.51	7.31 6.07 5.28 4.73 4.33 4.02 3.78 3.59 3.43 3.29 3.18 3.08 3.00 2.92 2.86 2.75 2.66 2.58 2.52	7.28 6.04 5.25 4.70 3.99 3.75 3.56 3.40 3.26 3.15 3.05 2.97 2.89 2.83 2.79 2.63 2.55 2.49	7.25 6.02 5.22 4.67 3.96 3.72 3.53 3.37 3.24 3.12 3.03 2.94 2.80 2.60 2.53 2.40	7.23 5.99 5.20 4.65 4.25 3.94 3.70 3.51 3.35 3.21 3.10 3.00 2.92 2.84 2.78 2.67 2.58 2.50 2.44	7.18 5.94 5.15 4.60 4.20 3.89 3.65 3.46 3.30 3.17 3.05 2.96 2.87 2.80 2.73 2.62 2.53 2.45 2.53 2.45 2.39	7.14 5.91 5.12 4.57 4.17 3.86 3.62 3.43 3.27 3.13 3.02 2.92 2.84 2.76 2.69 2.84 2.49 2.49 2.49 2.49 2.42 2.30	7.11 5.88 5.09 4.54 4.14 3.83 3.59 3.40 2.99 2.81 2.73 2.67 2.55 2.46 2.39 2.81 2.73 2.67 2.55 2.46 2.39 2.32	7.09 5.86 5.07 4.52 4.12 3.81 3.57 3.38 3.22 3.08 2.97 2.87 2.78 2.78 2.71 2.64 2.53 2.44 2.53 2.44 2.36 2.30	7.06 5.82 5.03 4.48 4.08 3.78 3.54 3.34 3.18 3.05 2.93 2.83 2.75 2.67 2.61 2.50 2.40 2.33 2.23 2.240 2.33
6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 22 24 26 28 30 35	9.68 7.52 6.28 5.48 4.92 4.52 4.51 3.97 3.78 3.62 3.49 3.37 3.19 3.12 3.05 2.94 2.85 2.78 2.78 2.76 2.56	9.64 7.48 6.24 5.44 4.89 4.18 3.94 3.75 3.59 3.45 3.34 3.16 3.08 3.02 2.91 2.82 2.75 2.68 2.63 2.53	9.61 7.45 6.21 5.41 4.86 4.46 4.15 3.91 3.72 3.56 3.42 3.31 3.21 3.13 3.05 2.99 2.88 2.89 2.72 2.65 2.60 2.50	7.42 6.18 5.38 4.43 4.43 4.12 3.88 3.69 3.53 3.40 3.28 3.19 3.10 3.03 2.96 2.85 2.85 2.69 2.63 2.57 2.47	9.55 7.40 6.16 5.36 4.81 4.41 3.86 3.66 3.61 3.37 3.26 3.00 2.94 2.83 2.84 2.66 2.60 2.55 2.44	9.51 7.355 6.11 5.32 4.77 4.36 4.06 3.82 3.46 3.33 3.22 3.12 3.03 3.296 2.90 2.78 2.90 2.62 2.56 2.51 2.40	7.31 6.07 5.28 4.73 4.33 4.02 3.78 3.59 3.43 3.29 3.18 3.08 3.00 2.92 2.86 2.52 2.66 2.58 2.52 2.52 2.36	7.28 6.04 5.25 4.70 3.99 3.75 3.56 3.40 3.26 3.15 3.05 2.97 2.89 2.83 2.72 2.63 2.55 2.49 2.44 2.33	7.25 6.02 5.22 4.67 3.96 3.72 3.53 3.37 3.24 3.12 3.03 2.94 2.80 2.60 2.53 2.46 2.53 2.46 2.41 2.31	7.23 5.99 5.20 4.65 4.25 3.94 3.70 3.51 3.10 3.00 2.92 2.84 2.78 2.67 2.58 2.50 2.44 2.39 2.28	7.18 5.94 5.15 4.60 4.20 3.89 3.65 3.46 3.30 3.17 3.05 2.96 2.87 2.80 2.73 2.62 2.53 2.45 2.39 2.34 2.24	7.14 5.91 5.12 4.57 4.17 3.86 3.62 3.43 3.27 3.13 3.02 2.92 2.84 2.76 2.69 2.84 2.49 2.49 2.49 2.42 2.35 2.30 2.19	7.11 5.88 5.09 4.54 4.14 3.83 3.59 3.40 3.24 3.10 2.99 2.81 2.73 2.67 2.55 2.46 2.39 2.32 2.32 2.27 2.16	7.09 5.86 5.07 4.52 4.12 3.81 3.57 3.38 3.22 3.08 2.97 2.87 2.78 2.71 2.64 2.53 2.44 2.36 2.30 2.25 2.14	7.06 5.82 5.03 4.48 4.08 3.54 3.54 3.34 3.18 3.05 2.93 2.83 2.75 2.67 2.61 2.50 2.40 2.33 2.26 2.21 2.10
6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 22 24 26 28 30 35 40	9.68 7.52 6.28 5.48 4.92 4.51 3.97 3.78 3.62 3.49 3.37 3.19 3.12 3.05 2.94 2.85 2.72 2.66 2.56 2.48	9.64 7.48 6.24 5.44 4.89 4.49 4.18 3.94 3.75 3.359 3.45 3.34 3.24 3.24 3.02 2.91 2.82 2.75 2.68 2.63 2.53 2.45	9.61 7.45 6.21 5.41 4.86 4.46 4.15 3.91 3.72 3.56 3.42 3.31 3.21 3.13 3.05 2.99 2.88 2.79 2.88 2.79 2.265 2.60 2.50 2.42	7.42 6.18 5.38 4.83 4.43 4.12 3.88 3.69 3.53 3.40 3.28 3.19 3.10 3.03 2.96 2.85 2.76 2.63 2.57 2.47 2.39	9.55 7.40 6.16 5.36 4.81 4.41 4.10 3.86 3.66 3.51 3.37 3.26 3.16 3.08 3.00 2.94 2.83 2.74 2.66 2.60 2.55 2.44 2.37	9.51 7.355 6.11 5.32 4.77 4.36 4.06 3.822 3.46 3.33 3.22 3.12 3.03 2.96 2.90 2.78 2.90 2.78 2.90 2.78 2.56 2.51 2.40 2.33	7.31 6.07 5.28 4.73 4.33 4.02 3.78 3.43 3.59 3.43 3.29 3.18 3.00 2.92 2.86 2.75 2.66 2.58 2.52 2.52 2.47 2.36 2.52 2.29	7.28 6.04 5.25 4.70 4.30 3.99 3.75 3.56 3.40 3.26 3.15 3.05 2.97 2.83 2.72 2.63 2.55 2.49 2.43 2.55 2.49 2.44 2.33 2.26	7.25 6.02 5.22 4.67 4.27 3.96 3.72 3.53 3.37 3.24 3.12 3.03 2.94 2.80 2.60 2.60 2.63 2.46 2.41 2.23	7.23 5.99 5.20 4.65 4.25 3.94 3.70 3.51 3.10 3.00 2.92 2.84 2.67 2.58 2.50 2.44 2.39 2.28 2.20	7.18 5.94 5.15 4.60 4.20 3.89 3.65 3.46 3.30 3.17 3.05 2.96 2.87 2.80 2.73 2.62 2.53 2.45 2.39 2.34 2.23 2.15	7.14 5.91 5.12 4.57 4.17 3.86 3.62 3.43 3.02 2.92 2.84 2.76 2.69 2.58 2.49 2.42 2.35 2.30 2.49 2.42 2.35 2.30 2.11	7.11 5.88 5.09 4.54 4.14 3.83 3.40 3.24 3.10 2.99 2.89 2.81 2.73 2.67 2.55 2.46 2.39 2.32 2.27 2.126 2.18	7.09 5.86 5.07 4.52 4.12 3.81 3.57 2.87 2.78 2.78 2.78 2.78 2.78 2.53 2.44 2.53 2.44 2.30 2.25 2.14 2.06	7.06 5.82 5.03 4.48 4.08 3.78 3.54 3.34 3.18 3.05 2.93 2.83 2.75 2.67 2.61 2.50 2.40 2.33 2.26 2.21 2.10 2.02
6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 22 24 22 24 22 24 22 24 30 35 40 45 50	9.68 7.52 6.28 5.48 4.92 4.21 3.97 3.78 3.62 3.49 3.37 3.19 3.12 3.05 2.94 2.85 2.78 2.66 2.56 2.48 2.43	9.64 7.48 6.24 5.44 4.89 4.49 4.18 3.75 3.59 3.45 3.34 3.24 3.16 3.08 3.02 2.91 2.82 2.75 2.68 2.63 2.45 2.45 2.45	9.61 7.45 6.21 5.41 4.86 4.46 4.15 3.91 3.72 3.56 3.42 3.31 3.21 3.13 3.05 2.99 2.88 2.79 2.88 2.79 2.265 2.60 2.50 2.42 2.265	7.42 6.18 5.38 4.83 4.43 4.12 3.88 3.69 3.53 3.40 3.29 6 2.85 2.76 2.63 2.57 2.47 2.39 2.39 2.39	9.55 7.40 6.16 5.36 4.81 4.41 4.10 3.86 3.66 3.51 3.37 3.26 3.16 3.08 3.00 2.94 2.83 2.74 2.60 2.55 2.44 2.55 2.44 2.37 2.37 2.37	9.31 7.35 6.11 5.32 4.77 4.36 4.06 3.82 3.42 3.42 3.12 3.03 2.96 2.50 2.51 2.40 2.51 2.51 2.51 2.51 2.51 2.53 2.51	7.31 6.07 5.28 4.33 4.33 4.02 3.78 3.43 3.59 3.43 3.29 3.43 3.29 3.43 3.00 2.92 2.86 2.52 2.68 2.52 2.47 2.36 2.29 2.229 2.229	7.28 6.04 5.25 4.70 4.30 3.99 3.75 3.56 3.15 3.05 2.97 2.89 2.83 2.75 2.63 2.55 2.49 2.44 2.33 2.26 2.24 2.44 2.33	7.25 6.02 5.22 4.67 3.96 3.72 3.53 3.37 3.24 3.12 3.03 2.94 2.80 2.69 2.60 2.60 2.63 2.46 2.41 2.23 2.17	7.23 5.99 5.20 4.65 4.25 3.94 3.70 3.51 3.35 3.21 3.10 3.00 2.92 2.84 2.67 2.58 2.67 2.58 2.67 2.44 2.39 2.28 2.24 2.28 2.20 2.14	7.18 5.94 5.15 4.60 4.20 3.89 3.65 3.46 3.30 5.2.96 2.87 2.80 2.73 2.62 2.53 2.45 2.39 2.34 2.23 2.34 2.23 2.15 2.09	7.14 5.91 5.12 4.57 4.17 3.86 3.62 3.43 3.27 2.34 2.62 2.84 2.76 2.69 2.58 2.49 2.58 2.49 2.58 2.30 2.19 2.11 2.05	7.11 5.88 5.09 4.54 4.14 3.83 3.59 3.40 3.20 2.89 2.89 2.81 2.73 2.65 2.32 2.27 2.16 2.32 2.27 2.16 2.08 2.32 2.27 2.16 2.08	7.09 5.86 5.07 4.52 4.12 3.81 3.57 2.87 2.78 2.78 2.78 2.78 2.78 2.78 2.7	7.06 5.82 5.03 4.48 4.08 3.78 3.54 3.34 3.18 3.05 2.93 2.67 2.61 2.50 2.40 2.350 2.40 2.26 2.21 2.10 2.02 1.96
6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 22 24 28 30 35 40 45 50 60	9.68 7.52 6.28 5.48 4.92 4.21 3.97 3.78 3.62 3.49 3.37 3.19 3.12 3.05 2.94 2.85 2.78 2.78 2.72 2.66 2.56 2.56 2.56 2.43 2.38 2.31	9.64 7.48 6.24 5.44 4.89 4.49 4.18 3.94 3.59 3.45 3.34 3.24 3.16 3.08 3.02 2.91 2.82 2.75 2.68 2.63 2.45 2.39 2.45 2.23 2.45 2.23 2.25 2.25	9.61 7.45 6.21 5.41 4.86 4.46 4.15 3.91 3.72 3.56 3.42 3.31 3.13 3.05 2.99 2.85 2.60 2.50 2.40 2.50 2.50 2.50 2.50 2.50 2.50 2.50 2.5	7.42 6.18 5.38 4.83 4.43 4.43 4.42 3.88 3.69 3.53 3.40 3.03 2.96 2.69 2.69 2.69 2.69 2.69 2.69 2.57 2.47 2.34 2.57 2.34 2.29 2.34	9.55 7.40 6.16 5.36 4.81 4.41 4.10 3.86 3.66 3.51 3.37 3.26 3.16 3.00 2.94 2.83 3.00 2.94 2.83 2.74 2.66 2.60 2.55 2.44 2.31 2.27 2.31 2.27	9.51 7.355 6.11 5.32 4.77 4.36 4.06 3.82 3.62 3.62 3.362 3.32 2.96 2.90 2.70 2.62 2.50 2.51 2.40 2.31 2.40 2.251 2.40 2.251 2.40 2.251 2.40 2.251 2.40 2.251 2.40 2.27 2.22 2.51	7.31 6.07 5.28 4.73 4.33 4.02 3.78 3.59 3.43 3.59 3.18 3.08 3.00 2.92 2.86 2.55 2.66 2.58 2.66 2.58 2.66 2.58 2.247 2.36 2.23 2.247 2.32 2.23 2.18 2.12	7.28 6.04 5.25 4.70 4.30 3.99 3.75 3.56 3.40 3.26 3.15 3.05 2.97 2.89 2.83 2.83 2.55 2.44 2.33 2.55 2.44 2.20 2.15 2.08	7.25 6.02 5.22 4.67 4.27 3.96 3.72 3.53 3.37 3.24 3.12 3.03 2.94 2.80 2.60 2.53 2.46 2.41 2.31 2.21 2.21 2.21 2.21 2.21 2.21 2.2	7.23 5.99 5.20 4.65 4.25 3.94 3.70 3.51 3.10 3.00 2.92 2.84 2.67 2.58 2.60 2.44 2.39 2.28 2.60 2.14 2.10 2.01	7.18 5.94 5.15 4.60 4.20 3.89 3.65 3.46 3.305 2.96 2.87 2.80 2.87 2.80 2.73 2.62 2.53 2.45 2.29 2.34 2.23 2.15 2.09 2.05 1 98	7.14 5.91 5.12 4.57 4.17 3.86 3.62 3.43 3.27 3.13 3.02 2.84 2.69 2.68 2.69 2.58 2.49 2.58 2.30 2.19 2.11 2.05 2.01 1 94	7.11 5.88 5.09 4.54 4.14 3.83 3.59 3.40 3.24 3.10 2.99 2.89 2.81 2.73 2.65 2.46 2.39 2.32 2.27 2.16 2.08 2.202 1.97 1.97	7.09 5.86 5.07 4.52 4.12 3.81 3.57 3.38 3.22 3.08 2.97 2.87 2.87 2.87 2.87 2.87 2.87 2.87 2.8	7.06 5.82 5.03 4.48 4.08 3.78 3.54 3.34 3.18 3.05 2.93 2.83 2.93 2.83 2.75 2.67 2.61 2.50 2.40 2.33 2.26 2.21 2.10 2.102 1.96 1.91 1.84
6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 22 24 26 28 30 35 40 45 50 60 70	9.68 7.52 6.28 5.48 4.92 4.21 3.97 3.78 3.69 3.49 3.37 3.19 3.12 3.05 2.94 2.85 2.78 2.66 2.56 2.56 2.43 2.38 2.38 2.38 2.38 2.38	9.64 7.48 6.24 5.44 4.89 4.49 4.18 3.75 3.59 3.45 3.34 3.16 3.08 3.02 2.91 2.82 2.75 2.63 2.53 2.45 2.63 2.53 2.45 2.39 2.35 2.23	9.61 7.455 6.21 5.41 4.86 4.46 4.15 3.91 3.72 3.56 3.42 3.31 3.13 3.05 2.99 2.88 2.79 2.72 2.60 2.50 2.40 2.50 2.46 2.32 2.25 2.25	7.42 6.18 5.38 4.83 4.43 4.43 3.69 3.53 3.40 3.28 3.19 3.10 3.03 2.96 2.65 2.76 2.69 2.63 2.57 2.47 2.34 2.29 2.34 2.22 2.12	9.55 7.40 6.16 5.36 4.81 4.41 4.10 3.86 3.51 3.36 3.16 3.00 2.94 2.83 3.00 2.94 2.83 2.74 2.66 2.66 2.66 2.66 2.65 2.44 2.31 2.27 2.21 5	9.51 7.355 6.11 5.32 4.77 4.36 4.06 3.82 3.62 3.46 3.32 3.12 3.03 2.96 2.90 2.70 2.62 2.50 2.51 2.40 2.351 2.27 2.25 2.51 2.21	9.47 7.31 6.07 5.28 4.73 4.33 4.02 3.78 3.59 3.43 3.29 3.18 3.08 3.00 2.92 2.86 2.58 2.52 2.47 2.36 2.23 2.47 2.36 2.23 2.18 2.12 2.07	7.28 6.04 5.25 4.70 4.30 3.99 3.75 3.56 3.40 3.26 3.15 3.05 2.97 2.89 2.83 2.55 2.49 2.63 2.55 2.44 2.33 2.20 2.15 2.20 2.15 2.20 3.20	7.25 6.02 5.22 4.67 4.27 3.96 3.72 3.53 3.37 3.24 3.03 2.60 2.69 2.60 2.53 2.46 2.41 2.31 2.217 2.12 2.05	7.23 5.99 5.20 4.65 4.25 3.94 3.70 3.51 3.35 3.21 3.10 3.00 2.92 2.84 2.67 2.58 2.50 2.44 2.39 2.28 2.20 2.14 2.10 2.14 2.10 2.14 2.10	7.18 5.94 5.15 4.60 4.20 3.89 3.65 3.46 3.30 2.87 2.80 2.87 2.80 2.73 2.62 2.53 2.45 2.29 2.34 2.23 2.15 2.29 2.34 2.23 2.15 2.09 2.05 1.93	7.14 5.91 5.12 4.57 4.17 3.86 3.62 3.43 3.02 2.92 2.84 2.76 2.69 2.84 2.76 2.69 2.84 2.49 2.55 2.30 2.19 2.11 2.05 2.01 1.94 1.84	7.11 5.88 5.09 4.54 4.14 3.83 3.59 3.40 3.240 2.89 2.89 2.89 2.89 2.89 2.85 2.46 2.39 2.32 2.27 2.16 2.02 2.197 1.90 1.85	7.09 5.86 5.07 4.52 4.12 3.81 3.57 3.38 3.22 3.08 2.97 2.87 2.87 2.87 2.87 2.87 2.87 2.87 2.8	7.06 5.82 5.03 4.48 4.08 3.78 3.54 3.34 3.18 3.05 2.93 2.83 2.75 2.67 2.61 2.50 2.40 2.33 2.21 2.10 2.02 2.21 2.10 2.02 1.96 1.91 1.84 1.78
6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 22 24 26 28 30 35 40 45 50 60 70 80	9.68 7.52 6.28 5.48 4.92 4.52 4.21 3.97 3.78 3.69 3.37 3.78 3.37 3.79 3.19 3.12 3.05 2.94 2.85 2.78 2.78 2.56 2.56 2.43 2.38 2.31 2.27 2.23	9.64 7.48 6.24 5.44 4.89 4.49 4.18 3.75 3.59 3.45 3.34 3.02 2.91 2.82 2.75 2.68 2.63 2.53 2.53 2.45 2.23 2.23 2.23 2.23 2.23 2.23 2.23 2.2	9.61 7.45 6.21 5.41 4.86 4.46 4.15 3.72 3.56 3.42 3.31 3.21 3.13 3.05 2.99 2.88 2.79 2.72 2.65 2.60 2.50 2.42 2.25 2.25 2.20 2.17	9.30 7.42 6.18 5.38 4.83 4.43 4.42 3.88 3.69 3.53 3.40 3.28 3.19 3.10 3.03 2.96 2.85 2.76 2.69 2.63 2.57 2.47 2.39 2.34 2.29 2.22 2.28 2.14 2.29	9.55 7.40 6.16 5.36 4.81 4.41 4.10 3.86 3.51 3.36 3.16 3.08 3.00 2.94 2.83 2.74 2.66 2.60 2.55 2.44 2.37 2.21 2.27 2.20 2.15 2.12	9.51 7.355 6.11 5.32 4.77 4.36 4.06 3.82 3.62 3.46 3.33 2.96 2.90 2.78 2.90 2.62 2.56 2.51 2.40 2.33 2.27 2.22 2.15 2.21 2.07	7.31 6.07 5.28 4.73 4.33 4.02 3.78 3.59 3.43 3.29 3.18 3.08 3.00 2.92 2.86 2.52 2.66 2.58 2.52 2.66 2.52 2.47 2.36 2.29 2.23 2.18 2.12 2.07 2.03 2.18 2.12 2.07 2.03 2.18 2.12 2.07 2.03 2.18 2.12 2.57 2.52 2.57 2.52 2.52 2.52 2.52 2.52 2.52 2.52 2.52 2.52 2.52 2.52 2.55	7.28 6.04 5.25 4.70 4.30 3.99 3.75 3.56 3.40 3.26 3.15 3.05 2.97 2.89 2.83 2.55 2.49 2.63 2.55 2.44 2.33 2.55 2.44 2.33 2.26 2.44 2.33 2.26 2.44 2.30 2.15 2.08 2.20 2.15 2.08	2.45 6.02 5.22 4.67 4.27 3.96 3.72 3.53 3.37 3.24 3.12 3.03 2.97 2.60 2.63 2.46 2.53 2.46 2.53 2.46 2.53 2.46 2.53 2.46 2.53 2.46 2.53 2.46 2.51 2.52 2.51 2.52 2.51 2.52 2.51 2.52 2.52 2.52 2.52 2.52 2.52 2.52 2.52 2.52 2.52 2.52 2.52 2.52 2.52 2.53 2.53 3.24 2.53 2.54 2.53 2.54 2.55 2.54 2.55 2.54 2.55 2.54 2.55 2.54 2.55 2.55 2.54 2.55 2.57	7.23 5.99 5.20 4.65 4.25 3.94 3.70 3.51 3.10 3.00 2.92 2.84 2.78 2.50 2.44 2.39 2.28 2.50 2.44 2.39 2.28 2.20 2.14 2.10 2.03 1.98 2.20	7.18 5.94 5.15 4.60 4.20 3.89 3.65 3.46 3.30 5.296 2.87 2.80 2.87 2.80 2.73 2.62 2.53 2.45 2.39 2.34 2.23 2.15 2.39 2.34 2.23 2.15 2.39 2.34 2.23 2.15 2.39 2.34 2.23 2.15 2.39 2.34 2.53 2.45 2.39 2.34 2.53 2.45 2.39 2.34 2.53 2.45 2.39 2.34 2.53 2.45 2.39 2.45 2.53 2.45 2.39 2.45 2.45 2.53 2.45 2.53 2.45 2.53 2.45 2.53 2.45 2.53 2.45 2.53 2.45 2.53 2.45 2.53 2.45 2.53 2.45 2.53 2.45 2.53 2.45 2.53 2.45 2.53 2.45 2.53 2.45 2.53 2.53 2.54 2.53 2.45 2.53 2.53 2.55 2.59 2.53 2.55 2.59 2.54 2.53 2.55 2.55 2.59 2.55 2.59 2.55 2.55 2.55	7.14 5.91 5.12 4.57 4.17 3.86 3.62 3.43 3.02 2.92 2.84 2.76 2.69 2.58 2.292 2.58 2.30 2.19 2.19 2.11 2.05 2.01 1.94 1.89 1.85 2.01	7.11 5.88 5.09 4.54 4.14 3.83 3.59 3.40 3.24 3.10 2.99 2.89 2.89 2.89 2.89 2.89 2.81 2.73 2.67 2.55 2.46 2.39 2.32 2.27 2.16 2.02 1.97 1.90 1.85 5.02 1.85 5.02 2.12 2.12 2.12 2.12 2.12 2.12 2.12 2	7.09 5.86 5.07 4.52 4.12 3.81 3.57 3.38 3.22 3.08 3.22 3.08 2.97 2.87 2.71 2.64 2.36 2.30 2.44 2.36 2.30 2.25 2.14 2.06 2.00 1.95 1.88 1.83 1.75 1.75 1.85 1.85 1.85 1.85 1.85 1.85 1.85 1.97 1.55 1	7.06 5.82 5.03 4.48 4.08 3.78 3.54 3.34 3.18 3.05 2.93 2.83 2.23 2.67 2.61 2.50 2.40 2.33 2.261 2.51 2.50 2.40 2.33 2.261 2.51 2.10 2.02 2.11 2.10 2.02 1.96 1.91 1.81 1.78 1.78 1.78 1.78
6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 22 24 26 28 30 35 40 45 50 60 70 80 100 200	9.68 7.52 6.28 5.48 4.92 4.21 3.97 3.78 3.62 3.49 3.327 3.19 3.12 3.05 2.94 2.85 2.78 2.72 2.66 2.56 2.48 2.48 2.43 2.38 2.31 2.23 2.23 2.23 2.23 2.29	9.64 7.48 6.24 5.44 4.89 4.49 4.18 3.75 3.59 3.45 3.34 3.02 2.91 2.82 2.75 2.68 2.63 2.53 2.45 2.35 2.45 2.28 2.23 2.25 2.28 2.20 2.15 2.05	9.61 7.45 6.21 5.41 4.86 4.46 4.15 3.91 3.72 3.56 3.42 3.31 3.21 3.13 3.05 2.99 2.88 2.79 2.72 2.65 2.60 2.50 2.42 2.25 2.20 2.25 2.20 2.17 2.12 2.02	7.42 6.18 5.38 4.83 4.43 4.43 4.42 3.88 3.69 3.53 3.40 3.28 3.19 3.10 3.03 2.96 2.85 2.76 2.69 2.63 2.57 2.47 2.39 2.24 2.29 2.22 2.14 2.29 2.214 2.09	9.55 7.40 6.16 5.36 4.81 4.41 4.10 3.86 3.51 3.37 3.26 3.16 3.08 3.00 2.94 2.83 2.74 2.66 2.60 2.55 2.44 2.37 2.27 2.20 2.15 2.12 2.07 1.97	9.51 7.355 6.11 5.32 4.77 4.36 4.06 3.82 3.62 3.46 3.362 3.46 3.33 2.96 2.90 2.78 2.90 2.62 2.56 2.51 2.40 2.33 2.22 2.55 2.11 2.07 2.07 2.07 2.02	7.31 6.07 5.28 4.73 4.33 4.02 3.78 3.59 3.43 3.29 3.18 3.08 3.00 2.92 2.86 2.52 2.66 2.58 2.52 2.66 2.58 2.52 2.46 2.52 2.64 2.52 2.47 2.36 2.29 2.23 2.18 2.12 2.03 1.98 1.98	7.28 6.04 5.25 4.70 4.30 3.99 3.75 3.56 3.40 3.26 3.15 3.05 2.89 2.83 2.55 2.49 2.63 3.255 2.49 2.63 2.55 2.49 2.64 2.33 2.55 2.49 2.63 2.26 2.20 2.15 2.08 2.00 2.15 2.00 2.15 2.00 2.15 2.00 2.15 2.00 2.15 2.00 2.15 2.00 2.15 2.00 2.15 2.00 2.15 2.00 2.15 2.00 2.15 2.00 2.15 2.55 2.55 2.55 2.55 2.55 2.55 2.55	7.25 6.02 5.22 4.67 4.27 3.96 3.72 3.53 3.37 3.24 3.12 3.03 2.94 2.87 2.80 2.60 2.53 2.46 2.53 2.46 2.53 2.41 2.31 2.23 2.12 2.05 2.01 1.97 1.82	7.23 5.99 5.20 4.65 4.25 3.94 3.70 3.51 3.10 3.00 2.92 2.84 2.78 2.67 2.58 2.50 2.44 2.39 2.28 2.20 2.14 2.10 2.03 1.98 1.94 1.89 1.79	7.18 5.94 5.15 4.60 4.20 3.89 3.65 3.46 3.305 2.96 2.87 2.80 2.73 2.62 2.53 2.45 2.39 2.34 2.23 2.15 2.05 1.98 1.98 1.89 1.84 1 74	7.14 5.91 5.12 4.57 4.17 3.86 3.62 3.43 3.02 2.92 2.84 2.76 2.69 2.58 2.49 2.42 2.30 2.19 2.11 2.05 2.01 1.94 1.85 1.80 1.60	7.11 5.88 5.09 4.54 4.14 3.83 3.59 3.40 3.24 3.10 2.99 2.89 2.89 2.89 2.81 2.73 2.67 2.55 2.46 2.39 2.32 2.27 2.16 2.08 2.02 2.27 2.16 2.08 2.02 1.97 1.90 1.85 1.82 1.82 1.66	7.09 5.86 5.07 4.52 4.12 3.81 3.57 2.71 2.64 2.53 2.71 2.64 2.30 2.25 2.14 2.06 2.30 2.21 2.64 2.30 2.21 2.64 2.30 2.25 2.14 2.06 2.00 1.95 1.88 1.83 1.79 1.74	7.06 5.82 5.03 4.48 4.08 3.78 3.54 3.34 3.15 2.93 2.83 2.75 2.67 2.61 2.50 2.40 2.33 2.267 2.61 2.50 2.40 2.33 2.267 2.61 2.50 2.40 2.33 2.267 2.61 2.50 2.40 2.33 2.267 2.61 2.50 2.40 2.33 2.21 2.10 2.02 1.96 1.91 1.95 1.91 1.95 1.95 1.95 1.95 1.95
$\begin{array}{c} 6\\ 7\\ 8\\ 9\\ 10\\ 11\\ 12\\ 13\\ 14\\ 15\\ 16\\ 17\\ 18\\ 19\\ 20\\ 22\\ 24\\ 26\\ 28\\ 30\\ 35\\ 40\\ 45\\ 50\\ 60\\ 70\\ 80\\ 100\\ 200\\ 500\\ \end{array}$	9.68 7.52 6.28 5.48 4.92 4.21 3.97 3.78 3.62 3.49 3.37 3.19 3.12 3.05 2.94 2.85 2.78 2.266 2.48 2.43 2.27 2.66 2.56 2.48 2.43 2.31 2.27 2.23 2.19 2.09 2.04	9.64 7.48 6.24 5.44 4.89 4.49 4.18 3.99 3.75 3.59 3.45 3.34 3.24 3.02 2.91 2.82 2.75 2.68 2.63 2.53 2.45 2.35 2.28 2.23 2.25 2.28 2.23 2.20 2.15 2.206 2.00	9.61 7.45 6.21 5.46 4.46 4.15 3.91 3.72 3.56 3.42 3.31 3.21 3.13 3.05 2.99 2.88 2.79 2.88 2.65 2.60 2.50 2.42 2.36 2.42 2.36 2.42 2.32 2.25 2.20 2.17 2.12 2.03 1.97	7.42 6.18 5.38 4.83 4.43 4.12 3.88 3.69 3.53 3.40 3.28 3.10 3.296 2.85 2.76 2.63 2.69 2.63 2.57 2.47 2.39 2.247 2.39 2.22 2.18 2.14 2.29 2.22 2.18 2.14 2.29 2.22 2.18 2.14 2.29 2.20 1.94 2.20 1.94 2.20 2.20 1.94 2.20 2.20 2.20 2.20 2.20 2.20 2.20 2.2	9.55 7.40 6.16 5.36 4.81 4.41 4.10 3.866 3.51 3.37 3.26 3.16 3.08 3.00 2.94 2.83 2.74 2.66 2.60 2.55 2.44 2.66 2.60 2.55 2.44 2.37 2.31 2.27 2.20 2.15 2.227 2.20 2.15 2.227 2.197 1.97	9.51 7.355 6.11 5.32 4.77 4.36 4.06 3.82 3.62 3.46 3.33 3.22 3.12 3.03 2.90 2.78 2.70 2.62 2.56 2.51 2.40 2.33 2.27 2.22 2.15 2.11 2.07 2.02 1.93 1.87	7.31 6.07 5.28 4.73 4.33 4.02 3.78 3.59 3.43 3.29 3.18 3.08 3.00 2.92 2.86 2.52 2.47 2.36 2.52 2.47 2.36 2.52 2.48 2.52 2.47 2.36 2.29 2.23 2.18 2.12 2.03 1.98 1.89 1.89	7.28 6.04 5.25 4.70 4.30 3.75 3.56 3.40 3.26 3.15 3.05 2.97 2.89 2.83 2.55 2.49 2.63 2.25 2.49 2.63 2.25 2.49 2.63 2.25 2.49 2.64 2.20 2.15 2.08 2.00 2.15 2.00 2.15 2.00 2.15 2.00 2.15 2.00 2.15 2.00 2.15 2.00 2.15 2.00 2.15 2.00 2.15 2.00 2.15 2.00 2.15 2.00 2.15 2.00 2.15 2.00 2.15 2.00 2.15 2.00 2.15 2.00 2.15 2.15 2.15 2.15 2.15 2.15 2.15 2.15	7.25 6.02 5.22 4.67 4.27 3.96 3.72 3.53 3.37 3.24 2.80 2.69 2.60 2.53 2.46 2.41 2.31 2.23 2.12 2.05 2.01 1.97 1.92 1.82	7.23 5.99 5.20 4.65 4.25 3.94 3.70 3.51 3.35 3.21 3.10 2.92 2.84 2.67 2.58 2.50 2.44 2.39 2.28 2.20 2.14 2.20 2.14 2.03 1.98 1.94 1.99 1.74	7.18 5.94 5.15 4.60 4.20 3.89 3.65 3.46 2.30 2.96 2.87 2.80 2.73 2.62 2.53 2.45 2.39 2.34 2.23 2.15 2.09 2.05 1.98 1.93 1.89 1.84 1.74 1.64	7.14 5.91 5.12 4.57 4.17 3.86 3.62 3.43 3.27 3.13 3.02 2.92 2.84 2.76 2.69 2.58 2.49 2.58 2.30 2.49 2.42 2.35 2.30 2.11 2.05 2.01 1.94 1.89 1.85 1.80 1.69 1.63	7.11 5.88 5.09 4.54 4.14 3.83 3.59 3.40 2.99 2.81 2.73 2.67 2.55 2.46 2.39 2.32 2.27 2.16 2.08 2.02 1.97 1.90 1.85 1.82 1.76 1.66 1.66	7.09 5.86 5.07 4.52 4.12 3.81 3.57 2.87 2.87 2.87 2.87 2.87 2.87 2.71 2.64 2.30 2.24 2.30 2.24 2.30 2.24 2.30 2.214 2.06 2.00 2.20 2.14 2.06 2.00 1.95 1.88 1.83 1.79 1.74 1.63 1.57	7.06 5.82 5.03 4.48 4.08 3.78 3.54 3.34 3.34 3.05 2.93 2.83 2.75 2.67 2.61 2.50 2.40 2.23 2.267 2.61 2.50 2.40 2.23 2.267 2.21 2.02 1.961 1.84 1.78 1.84 1.75 1.69 1.58
6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 22 24 26 28 30 35 40 45 50 60 70 80 100 200 200 21 24 24 26 28 30 35 40 40 10 11 10 11 10 11 10 11 10 11 10 11 10 11 10 11 10 11 11	9.68 7.52 6.28 5.48 4.92 4.21 3.97 3.78 3.62 3.49 3.37 3.19 3.12 3.05 2.94 2.85 2.78 2.72 2.66 2.56 2.48 2.43 2.27 2.26 2.56 2.248 2.31 2.27 2.219 2.09 2.04 2.02	9.64 7.48 6.24 5.44 4.89 4.49 4.18 3.99 3.75 3.59 3.45 3.34 3.24 3.16 3.02 2.91 2.82 2.75 2.68 2.63 2.53 2.45 2.23 2.23 2.23 2.23 2.23 2.20 2.15 2.06 2.00 2.15 2.00 2.15 2.00 2.15 2.00 2.15 2.00 2.15 2.00 2.15 2.00 2.15 2.00 2.15 2.00 2.15 2.00 2.15 2.00 2.15 2.00 2.15 2.00 2.15 2.20 2.25 2.25	9.61 7.45 6.21 5.41 4.86 4.46 4.15 3.91 3.72 3.56 3.42 3.31 3.21 3.13 3.05 2.99 2.88 2.79 2.88 2.79 2.88 2.79 2.65 2.60 2.50 2.42 2.36 2.42 2.36 2.42 2.32 2.25 2.20 2.17 2.12 2.03 1.97 2.195	7.42 6.18 5.38 4.83 4.43 4.12 3.88 3.69 3.53 3.40 3.28 3.10 3.296 2.85 2.76 2.69 2.63 2.57 2.47 2.39 2.34 2.29 2.22 2.18 2.29 2.22 2.18 2.24 2.29 2.22 2.18 2.24 2.29 2.22 2.18 2.24 2.29 2.22 2.18 2.24 2.29 2.22 2.18 2.24 2.29 2.22 2.18 2.24 2.29 2.22 2.18 2.24 2.29 2.22 2.18 2.24 2.29 2.20 2.20 2.20 2.20 2.20 2.20 2.20	9.55 7.40 6.16 5.36 4.81 4.41 4.10 3.866 3.51 3.37 3.26 3.16 3.08 3.00 2.94 2.83 2.74 2.66 2.60 2.55 2.44 2.66 2.60 2.55 2.44 2.27 2.20 2.15 2.12 2.07 1.97 1.97 1.97 1.92 2.197	9.51 7.355 6.11 5.322 4.77 4.36 4.06 3.822 3.62 3.46 3.333 3.22 3.03 2.90 2.78 2.90 2.62 2.56 2.51 2.40 2.33 2.27 2.15 2.11 2.07 2.02 1.93 1.87 1.85	7.311 6.07 5.28 4.73 4.33 4.02 3.78 3.59 3.43 3.09 3.18 3.08 3.00 2.92 2.86 2.75 2.66 2.58 2.52 2.47 2.36 2.29 2.23 2.18 2.12 2.03 1.98 1.89 1.83 1.81	7.28 6.04 5.25 4.70 4.30 3.75 3.56 3.40 3.26 3.15 3.05 2.87 2.83 2.83 2.83 2.62 2.63 2.64 2.20 2.44 2.33 2.26 2.20 2.15 2.08 2.03 2.00 1.95 1.85 1.77 1.77	7.25 6.02 5.22 4.67 4.27 3.96 3.72 3.53 3.37 3.24 2.80 2.69 2.60 2.53 2.46 2.41 2.31 2.23 2.12 2.05 2.01 1.97 1.92 1.82 1.74	7.23 5.99 5.20 4.65 4.25 3.94 3.70 3.51 3.35 3.21 3.30 2.92 2.84 2.87 2.58 2.67 2.58 2.67 2.58 2.67 2.58 2.20 2.14 2.39 2.20 2.14 2.19 2.14 2.19 2.14 2.19 2.14 2.19 2.14 2.10 2.14 2.10 2.14 2.14 2.14 2.14 2.14 2.14 2.14 2.14	7.18 5.94 5.15 4.60 4.20 3.89 3.65 3.46 3.30 3.17 3.05 2.96 2.87 2.80 2.73 2.62 2.53 2.45 2.39 2.34 2.25 2.39 2.34 2.23 2.15 2.09 2.09 2.09 2.09 2.09 2.15 2.09 2.09 2.15 2.09 2.09 2.15 2.09 2.09 2.15 2.09 2.15 2.15 2.09 2.15 2.15 2.15 2.15 2.15 2.15 2.15 2.15	7.14 5.91 5.12 4.57 4.17 3.86 3.62 3.43 3.27 2.84 2.76 2.69 2.58 2.49 2.58 2.49 2.58 2.30 2.49 2.42 2.35 2.30 2.11 2.05 2.01 1.94 1.89 1.85 1.80 1.69 1.63 1.61	7.11 5.88 5.09 4.54 4.14 3.83 3.59 3.40 3.24 3.10 2.89 2.81 2.67 2.55 2.46 2.39 2.32 2.27 2.16 2.08 2.02 1.90 1.85 1.82 1.76 1.66 1.60 1.58	7.09 5.86 5.07 4.52 4.12 3.81 3.57 2.87 2.78 2.77 2.64 2.53 2.44 2.36 2.30 2.25 2.14 2.06 2.00 2.25 1.88 1.83 1.79 1.74 1.63 1.57 1.54	7.06 5.82 5.03 4.48 4.08 3.78 3.54 3.34 3.34 3.05 2.93 2.83 2.75 2.67 2.61 2.50 2.40 2.23 2.267 2.61 2.50 2.40 2.23 2.267 2.21 2.20 2.21 2.02 1.961 1.84 1.78 1.58 1.52 2.53

One of the most widely used post hoc tests is Tukey's HSD (Honestly Significant Difference). Tukey's HSD is used only after a significant F has been obtained. Tukey compares the difference between any two means against the calculated HSD, and a difference between any two means is significant only if it equals or exceeds the HSD. The equation for HSD is as follows:

$$HSD = q \sqrt{\left(MS_{within} \div \overline{x}_{h}\right)} \tag{11.14}$$

where

- q = the Studentized range value from exhibit 11.3 that corresponds to the confidence level and number of groups $(k)^1$
- \overline{x}_h = harmonic mean of group sizes, which is calculated as the *k*th root of the product of the sample sizes or

$$\sqrt[k]{(n_1)(n_2)(n_3)...(n_k)}$$
 (11.15)

To illustrate HSD from the example of the public works employees,

q = 3.915 (95 percent confidence—df for $MS_{\text{within}} = 23$ – number of groups = 4).

The reader will note that df = 23 is not contained on exhibit 11.3. The *q* value of 3.915 has been interpolated between df = 20 and df = 24.

$$\overline{x}_{h} = \sqrt[4]{(7)(7)(8)(5)} = 6.654$$
$$HSD = 3.915 \left(\sqrt{(3.25 \div 6.654)}\right)$$
$$= 3.915 (.699)$$
$$= \$2.74$$

If the difference between the mean wages in any two cities equals or exceeds \$2.74, then those two cities have wage structures for public works employees that are statistically different from each other. In this example, there is one such difference—Watertown and South Bend, which differ by \$3.04 (\$15.28 – \$12.24). The other combinations of cities cannot be considered to differ from each other.

EXHIBIT 11.3. STUDENTIZED RANGE (q) DISTRIBUTION.

									Νι	ımber	of Gro	oups (l	K)									
v	α	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	α	v
1	.050	17.97	26.98	32.82	37.08	40.41	43.12	45.40	47.36	49.07	50.59	51.96	53.19	54.32	55.36	56.32	57.21	58.04	58.82	59.55 . 0	50	1
1	.010	89.98	135.0	164.3	185.6	202.2	215.7	227.1	236.9	245.5	253.1	259.9	266.1	271.8	276.9	281.7	286.2	290.3	294.2	297.9 .0	10	1
2	.050	14 03	19 02	9.798	24 72	26.63	28 20	29.53	30.68	31.69	32 59	33 39	34 13	34 80	35 42	35.91	36.53	37.03	37.50	37.94 0	10	2
3	.050	4.501	5.910	6.825	7.502	8.037	8.478	8.852	9.177	9.462	9.717	9.946	10.15	10.35	10.52	10.69	10.84	10.98	11.11	11.24 .0	50	3
3	.010	8.260	10.62	12.17	13.32	14.24	15.00	15.64	16.20	16.69	17.13	17.52	17.88	18.21	18.52	18.80	19.06	19.31	19.54	19.76 .0	10	3
4	.050	3.927	5.040	5.757	6.287	6.706	7.053	7.347	7.602	7.826	8.027	8.208	8.373	8.524	8.664	8.793	8.914	9.027	9.133	9.233.0	50	4
5	.010	3.635	4.602	5.218	5.673	6.033	6.330	6.582	6.801	6.995	7.167	7.324	7.465	7.596	7.716	7.828	7.932	8.030	8.122	8.208.0	50	5
5	.010	5.702	6.976	7.806	8.421	8.913	9.321	9.669	9.971	10.24	10.48	10.70	10.89	11.07	11.24	11.40	11.54	11.68	11.81	11.93 .0	10	5
6	.050	3.460	4.339	4.896	5.305	5.629	5.895	6.122	6.319	6.493	6.649	6.789	6.917	7.034	7.143	7.244	7.338	7.426	7.509	7.587.0	50	6
6	.010	5.243	6.331	/.033	7.556	7.974	8.318	5 815	8.869	9.097	9.300	9.485	9.653	9.808	9.951	10.08	6 0 3 0	7 020	10.43	7 169 0	10	6
7	.010	4.948	5.919	6.543	7.006	7.373	7.678	7.940	8.167	8.368	8.548	8.711	8.859	8.996	9.124	9.242	9.353	9.456	9.553	9.645.0	10	7
8	.050	3.261	4.041	4.529	4.886	5.167	5.399	5.596	5.767	5.918	6.053	6.175	6.287	6.389	6.483	6.571	6.653	6.729	6.801	6.870 .0	50	8
8	.010	4.745	5.635	6.204	6.625	6.960	7.238	7.475	7.681	7.864	8.028	8.177	8.312	8.437	8.552	8.659	8.760	8.854	8.942	9.026 .0	10	8
9	.050	3.199	5 4 2 8	4.415	4./55	5.024	5.244	5.432	5.595	5./38	5.86/	5.983	6.089	6.186	6.2/6 8 1 3 3	6.359	6.43/	6.510 8.413	6.579	6.644 .0 8 5 7 3 0	50	9
10	.010	3.151	3.877	4.327	4.654	4.912	5.124	5.304	5.460	5.598	5.722	5.833	5.935	6.028	6.114	6.194	6.269	6.339	6.405	6.467.0	5	10
10	.01	4.482	5.270	5.769	6.136	6.428	6.669	6.875	7.055	7.214	7.356	7.485	7.603	7.712	7.813	7.906	7.994	8.076	8.153	8.226 .0	1	10
11	.05	3.113	3.820	4.256	4.574	4.823	5.028	5.202	5.353	5.486	5.605	5.713	5.811	5.901	5.984	6.062	6.134	6.202	6.265	6.325 .0	5	11
11	.01	4.392	3 773	5.621 4 100	5.970	6.24/	6.4/6	6.6/I	6.842 5.262	6.992 5 305	7.127	7.250	7.362	7.465	7.560	7.649	6.023	6.080	7.883	7.952.0 6.209.0	5	11
12	.05	4.320	5.046	5.502	5.836	6.101	6.321	6.507	6.670	6.814	6.943	7.060	7.167	7.265	7.356	7.441	7.520	7.594	7.664	7.731.0	1	12
13	.05	3.055	3.734	4.151	4.453	4.690	4.884	5.049	5.192	5.318	5.431	5.533	5.625	5.711	5.789	5.862	5.930	5.994	6.055	6.112 .0	5	13
13	.01	4.261	4.964	5.404	5.727	5.981	6.192	6.372	6.528	6.666	6.791	6.903	7.006	7.100	7.188	7.269	7.345	7.417	7.484	7.548.0	1	13
$\stackrel{=}{=} 14$.05	3.033	3.701	4.111	4.407	4.639	4.829	4.990	5.130	5.253	5.363	5.463	5.554	5.63/	5./14	5./85	5.852	5.915	5.973	6.029 .0	5	14
5 15	.05	3.014	3.673	4.076	4.367	4.595	4.782	4.940	5.077	5.198	5.306	5.403	5.492	5.574	5.649	5.719	5.785	5.846	5.904	5.958.0	5	15
₩ 15	.01	4.167	4.836	5.252	5.556	5.796	5.994	6.162	6.309	6.438	6.555	6.660	6.757	6.845	6.927	7.003	7.074	7.141	7.204	7.264 .0	1	15
16	.05	2.998	3.649	4.046	4.333	4.557	4.741	4.896	5.031	5.150	5.256	5.352	5.439	5.519	5.593	5.662	5.726	5.786	5.843	5.896.0	5	16
16	.01	4.131 2.984	4./86	5.192	5.488 4 303	5./22	5.915	6.079 4.858	6.222 4 991	6.348 5.108	5 212	5 306	5 392	6.743 5.471	6.824 5.544	6.897 5.612	6.967 5.675	7.032	7.093	5 842 04	5.	16
17	.01	4.099	4.742	5.140	5.430	5.659	5.847	6.007	6.147	6.270	6.380	6.480	6.572	6.656	6.733	6.806	6.873	6.937	6.997	7.053.0	1	17
18	.05	2.971	3.609	3.997	4.276	4.494	4.673	4.824	4.955	5.071	5.173	5.266	5.351	5.429	5.501	5.567	5.629	5.688	5.743	5.794 . 0	5	18
18	.01	4.071	4.703	5.094	5.379	5.603	5.787	5.944	6.081	6.201	6.309	6.407	6.496	6.579	6.655	6.725	6.791	6.854	6.912	6.967 .0	1	18
19	.03	4.046	4.669	5.054	5.333	5.553	5.735	5.888	6.022	6.141	6.246	6.342	6.430	6.510	6.585	6.654	6.719	6.780	6.837	6.891 .0	ן 1	19
20	.05	2.950	3.578	3.958	4.232	4.445	4.620	4.768	4.895	5.008	5.108	5.199	5.282	5.357	5.427	5.492	5.553	5.610	5.663	5.714 .0	5 2	20
20	.01	4.024	4.639	5.018	5.293	5.509	5.687	5.839	5.970	6.086	6.190	6.285	6.370	6.449	6.523	6.591	6.654	6.714	6.770	6.823 . 0	1 :	20
25	.05	2.913	3.523	3.890	4.153	4.358	4.526	4.667	4.789	4.897	4.993	5.079	5.158	5.230	5.297	5.359	5.417	5.471	5.522	5.570 .05	5 2	25
30	.05	2.888	3.487	3.845	4.102	4.301	4.464	4.601	4.720	4.824	4.917	5.001	5.077	5.147	5.211	5.271	5.327	5.379	5.429	5.475 .05	5 3	30
30	.01	3.889	4.454	4.799	5.048	5.242	5.401	5.536	5.653	5.756	5.848	5.932	6.008	6.078	6.142	6.202	6.258	6.311	6.360	6.407 .01	1 3	30
40	.05	2.858	3.442	3.791	4.039	4.232	4.388	4.521	4.634	4.735	4.824	4.904	4.977	5.044	5.106	5.163	5.216	5.266	5.313	5.358 .05	5 4	40
40	.01	3.825	4.36/	4.695	4.931	5.114	5.265	5.392	5.502	5.599	5.685	5./64	5.835	5.900	5.961	5.008	6.069 5.150	6.118 5.100	6.165 5.245	5 288 04	54	10 50
50	.03	3.787	4.316	4.634	4.863	5.040	5.185	5.308	5.414	5.507	5.590	5.665	5.734	5.796	5.854	5.908	5.958	6.005	6.050	6.092 .01	1 3	50
60	.05	2.829	3.399	3.737	3.977	4.163	4.314	4.441	4.550	4.646	4.732	4.808	4.878	4.942	5.001	5.056	5.107	5.154	5.199	5.241 .05	56	60
60	.01	3.762	4.282	4.594	4.818	4.991	5.133	5.253	5.356	5.447	5.528	5.601	5.667	5.728	5.784	5.837	5.886	5.931	5.974	6.015 .01	16	50
80 80	.05 01	2.814	3.3/7 4 241	3./11	3.947	4.129	4.278	4.402	4.509	4.603	4.686	4./61	4.829	4.892	4.949 5.698	5.003	5.052	5.099	5.142	5 920 01	5 1 1	30 80
100	.05	2.806	3.365	3.695	3.929	4.109	4.256	4.379	4.484	4.577	4.659	4.733	4.800	4.862	4.918	4.971	5.020	5.066	5.108	5.149 .05	5 10	00
	.01	3.714	4.216	4.516	4.730	4.896	5.031	5.144	5.242	5.328	5.405	5.474	5.537	5.594	5.648	5.697	5.743	5.786	5.826	5.864 .01	1	
>100	.05	2.800	3.356	3.685	3.917	4.096	4.241	4.363	4.468	4.560	4.641	4.714	4.781	4.842	4.898	4.950	4.998	5.043	5.086	5.126 .05	5 12	20
x	.01	2.77	4.200	4.49/	4.709	4.072	5.005 4.17	5.118 4.29	3.214 4.39	5.299 4.47	5.575 4.55	5.445 4.62	5.505 4.69	3.301 4.75	3.014 4.81	3.002 4.86	5.708 4.93	5.750 4.98	5.02	5.06 04	ı 5 ∝	0
~	.01	3.64	4.12	4.40	4.60	4.76	5.88	5.99	5.08	5.16	5.23	5.30	5.36	5.42	5.48	5.54	5.59	5.64	5.69	5.74 .01	1	

USING SPSS FOR ANALYSIS OF VARIANCE

In statistical analysis, there are a variety of analysis of variance tests; the one presented in this chapter is more formally known as the one-way analysis of variance. SPSS can be used to make the one-way analysis of variance determination. Returning to Transportation.sav, and invoking one of the exceptions about choosing the best measure of central tendency that were discussed in chapter 6, let the researcher be interested in whether overall satisfaction with the transit system varies significantly by the educational attainment of the riders.

- 1. Access the One-Way Analysis of Variance test by selecting Analyze—Compare Means—One-Way ANOVA.
- 2. Place the variable for which means are to be used (satis_overall_q1) in the Dependent List box and the variable for which the categorical means are to be compared (education_q33) in the Factor box.

		Dependent List:	Contrasts
and the second s	4	satis_overall_q1	
💑 day			Post Hoc
💑 hour			Options
💑 superdist			Bootetran
💑 Language			Dousnap
change_service			
satis_cost_q3a		Factor	
satis_ontime_q3b	- +		7
satis fren n3r		education_q33	

- 3. In Options, select Descriptives and also make certain that cases are excluded analysis by analysis. Then press Continue.
- 4. In Post Hoc, select Tukey² (not Tukey's b) and make certain that significance is set at .05. Then press Continue.
- 5. Select OK when returned to the screen shown above.

Output for this request reveals descriptives (means) that seem to indicate greater satisfaction (scale 1–5, with 1 = very satisfied) among riders with less educational attainment—mean = 1.86 for less than high school and = 1.89 for high school graduates in contrast to mean values in excess of 2.10 for college graduates and

postgraduate degree holders. The ANOVA table indicates that this is a statistically significant finding (Sig. = .001). As such, researchers can conclude with more than 95% confidence that differences exist in overall satisfaction among the various educational categories.

One-Way

Overall Satisfaction with Bus Service

					95% Co Interval	nfidence for Mean			
	N	Mean	Std. Deviation	Std. Error	Lower Bound	Upper Bound	Mini- mum	Maxi- mum	
Some high school or less	475	1.86	1.130	.052	1.76	1.96	1	5	
High school graduate	491	1.89	.947	.043	1.80	1.97	1	5	
Some college/ tech school/assoc. degree	553	2.06	1.104	.047	1.97	2.16	1	5	
Four-year college degree	136	2.21	1.123	.096	2.02	2.40	1	5	
Postgraduate degree	54	2.11	1.110	.151	1.81	2.41	1	5	
Total	1,709	1.97	1.076	.026	1.92	2.02	1	5	

ANOVA

Overall Satisfaction with Bus Service

	Sum of Squares	df	Mean Square	F	Sig.
Between groups	22.778	4	5.694	4.967	.001
Within groups	1,953.516	1704	1.146		
Total	1,976.294	1708			

It is the post hoc test output that tells the researcher which educational categories differ significantly from which others. Significant differences among the categories are identified with an asterisk in the Mean Difference column. As such,

• Some High School or Less differs significantly from Some College/Tech School/ Associate Degree (.204 higher rating—remember that code 1 is best).

256

- Some High School or Less differs from Four-Year College Degree (.347 higher rating).
- High School Graduates differ significantly from Four-Year College Graduates (.320 higher rating).
- All other comparisons either show no asterisk or are repeats (in reverse) of these significant differences.

Post Hoc Tests

Multiple Comparison

Dependent Variable: Overall Satisfaction with Bus Service

Tukey HSD

(I) Highest Level of		Mean			95% Co Inte	nfidence erval
Education Completed	(J) Highest Level of Education Completed	Difference (I—J)	Std. Error	Sig.	Lower Bound	Upper Bound
	High school graduate	027	.069	.995	22	.16
Some high	Some college/tech school/assoc. degree	204*	.067	.020	39	02
school or less	Four-year college degree	347*	.104	.008	63	06
	Postgraduate degree	252	.154	.472	67	.17
	Some high school or less	.027	.069	.995	16	.22
High school	Some college/tech school/assoc. degree	177	.066	.059	36	.00
graduate	Four-year college degree	320*	.104	.018	60	04
	Postgraduate degree	225	.154	.584	64	.19
	Some high school or less	.204*	.067	.020	.02	.39
Some college/	High school graduate	.177	.066	.059	.00	.36
tech school/ assoc. degree	Four-year college degree	143	.102	.633	42	.14
	Postgraduate degree	048	.153	.998	46	.37
	Some high school or less	.347*	.104	.008	.06	.63
Four-year col-	High school graduate	.320*	.104	.018	.04	.60
lege degree	Some college/tech school/assoc. degree	.143	.102	.633	14	.42
	Postgraduate degree	.095	.172	.982	38	.57

(I) Highest Level of		Mean			95% Confidence Interval	
Education Completed	(J) Highest Level of Education Completed	Difference (I—J)	Std. Error	Sig.	Lower Bound	Upper Bound
	Some high school or less	.252	.154	.472	17	.67
Destand	High school graduate	.225	.154	.584	19	.64
Postgraduate Degree	Some college/tech school/assoc. degree	.048	.153	.998	37	.46
	Four-year college degree	095	.172	.982	57	.38

*The mean difference is significant at the .05 level.

Homogeneous Subsets

Tukey HSD

		Subset for all	oha = .05
Highest Level of Education Completed	Ν	1	2
Some high school or less	475	1.86	
High school graduate	491	1.89	1.89
Some college/tech school/assoc. degree	553	2.06	2.06
Postgraduate degree	54	2.11	2.11
Four-year college degree	136		2.21
Sig.		.226	.062

Note: Means for groups in homogeneous subsets are displayed. Uses Harmonic Mean Sample Size = 157.125. The group sizes are unequal. The harmonic mean of the group sizes is used. Type I error levels are not guaranteed.

The final step in completing the analysis of variance is eta-squared and eta.

- 1. Return to Analyze—Compare Means—Means and place satis_overall_q1 as the Dependent Variable and education_q33 as the Independent Variable
- 2. Select the ANOVA Table and Eta button in the Options tab; then click Continue and OK.

The resulting output shows an eta value of .107 (weak association—exhibit 10.2) and eta-squared of .012 (only 1.2% of the variation in satisfaction is explained by educational level). This is statistically significant, but much remains to be discovered about what affects overall satisfaction—education is just a small piece.

5/14/15/165.	-		Cell Statistics.
Median	1		Mean
Grouped Median			Number of Cases
Std. Error of Mean			Standard Deviation
Sum			
Minimum			
Maximum			
Range			
First		4	
Last		_	
Variance			
Kurtosis			
Std. Error of Kurtosis			
Skewness			
Std. Error of Skewness	3		
Harmonic Mean	*		
-Statistics for First Law	0.0		
Statistics for First Lay	e1		
Anova table and et	ta		
Test for linearity			

Measures of Association

	Eta	Eta Squared
Overall satisfaction with bus service* Highest level of education completed	.107	.012

Spuriousness

Regarding the significance tests shown in chapters 7, 10, and this chapter, it should be made clear that it is sometimes incorrect to draw conclusions based on the apparent relationship between only two variables. The researcher must account for the possibility that factors other than a single independent variable may also have some influence on the dependent variable and that even if these factors are very evident in the data, their effect may be obscured if they are excluded from the analysis. For example, suppose that it is found through a survey research study that football fans earn a statistically significant higher annual income than nonfans do. Prior to making a policy decision based on such a finding, the researcher should analyze other variables that might be suspected of influencing the independent variable (football fans) and the dependent variable (annual income). One such variable would be the gender of the respondent because football fans tend to be predominantly male and income levels are higher among males. It may therefore be that the preliminary finding is erroneous and that the actual independent variable is gender. If this should be the case, then the relationship between football fans and income is said to be *spurious*, or not genuine.

There are a variety of advanced statistical techniques that are designed to measure the influence of more than one independent variable on a dependent variable and thereby identify spurious relationships. Foremost among these techniques are multiple regression analysis and partial correlation, both of which are introduced briefly in chapter 12 but are otherwise beyond the scope of this book.

Notes

1. The Tukey test revolves around a measure known as the Studentized range statistic (*q*). For any particular pair of means among the k groups, let us designate the larger and smaller as ML and MS, respectively. The Studentized range statistic can then be calculated for any particular pair as

$$q = (M_L - M_S) \frac{\sqrt{MS_{\text{within}}}}{N_{p/s}} \text{ where } N_{p/s} \text{ is the number of values of } x_i.$$

2. Tukey is selected to coincide with the post hoc test used in this chapter. Tukey was chosen because its manual determination is clearer than other tests available. We note that the LSD test tends to be somewhat more generous in finding significant relationships.



CHAPTER TWELVE

REGRESSION AND CORRELATION

T his chapter considers the relationship between two interval scale variables. With interval variables as both independent and dependent, the researcher is able not only to perform tests of statistical significance and measures of association but also to quantify the exact relationship with mathematical precision through the designation of a line or curve that represents the relationship. When the researcher's interest is focused on the existence or strength of a relationship between two interval variables, tests of significance and measures of association may suffice. However, once such a relationship is found, the researcher will likely wish to predict the exact value of the dependent variable based on the value of the independent variable (s) through the application of regression analysis.

Simple Linear Regression

Interval scale variables afford the researcher significantly more informative data than do nominal and ordinal scale variables. For example, in the case of figure 12.1, the government funding agency may ask the researcher a practical question concerning the impact of specific subsidy levels and perhaps inquire how properties on a given city block can be expected to increase in value when total rehabilitation subsidies of \$118,000 are made

Block	Rehabilitation Subsidy (in thousands)	Mean Property Value Increase (in thousands)		
A	100	3		
В	85	3		
С	95	1		
D	130	5		
E	110	2		
F	75	2		
G	135	4		
Н	60	0		
Ι	120	4		
J	115	3		

FIGURE 12.1.	REHABILITATION SUBSIDY/PROPERTY VALUE
	INCREASE ANALYSIS.

available. This question can be answered by applying the technique of simple linear regression to the raw data in figure 12.1.

Simple linear regression is performed on two interval variables. Simple linear regression analysis requires that the researcher once again identify the independent and dependent variables using the principles discussed in chapter 10. In this example, the independent variable is rehabilitation subsidy (commonly designed as x), and the dependent variable is property value change (commonly designated as y). The objective of linear regression analysis is to establish the equation for the line that most closely aligns with the raw data. It is this line (the regression line) that permits the researcher to predict values of y based on values of x, thereby answering the question posed by the government funding agency. Figure 12.2 depicts the data from figure 12.1 as plotted by its values of x and y. The regression line, or *line of best fit*, is the line that minimizes the sum of the perpendicular distances from each of the plotted points to that line. One and only one line meets this criterion. The regression line is also shown in figure 12.2.

The equation for the regression line conforms to the general equation for a line and can be expressed as

$$y = a + bx \tag{12.1}$$



FIGURE 12.2. SCATTER PLOT AND REGRESSION LINE.

where

- a = the *y*-intercept (the point at which the line crosses the *y*-axis)
- b = the slope of the line (change in *y* divided by change in *x*)—also known as the regression coefficient

Linear regression provides for the determination of the *y*-intercept *a* and slope *b* of the line of best fit directly from raw data. The formulas for these calculations are as follows and are operationalized in worksheet 12.1:

$$b = \frac{\Sigma(x - \overline{x})(y - \overline{y})}{\Sigma(x - \overline{x})^2}$$
(12.2)

$$a = \overline{y} - b(\overline{x}) \tag{12.3}$$

The regression line therefore is y = -2.5275 + .051x. This means that as subsidies increase by \$1,000, value increases by approximately \$50. The question posed by the agency concerning its \$118,000 subsidy can now be answered by the researcher as follows:

$$y = -2.5275 + .051(118)$$

= 3.4905

Block	x	y	$x - \overline{x}$	$y - \overline{y}$	$(x-\overline{x})(y-\overline{y})$	$(x-\overline{x})^2$	$(y-\overline{y})^2$
A	100	3	-2.5	.3	75	6.25	.09
В	85	3	-17.5	.3	-5.25	306.25	.09
С	95	1	-7.5	-1.7	12.75	56.25	2.89
D	130	5	27.5	2.3	63.25	756.25	5.29
E	110	2	7.5	7	-5.25	56.25	.49
F	75	2	-27.5	7	19.25	756.25	.49
G	135	4	32.5	1.3	42.25	1056.25	1.69
Н	60	0	-42.5	-2.7	114.75	1,806.25	7.29
1	120	4	17.5	1.3	22.75	306.25	1.69
J	115	3	12.5	.3	3.75	156.25	.09
$\overline{x} = 102.5$	$\overline{y} = 2.7$	<i>n</i> = 1	0		$\Sigma = $ sum of the products =	$\Sigma = \text{sum of}$ squares (x) =	$\Sigma = \text{sum of}$ squares (y) =
$b = \frac{267}{5262}$	$b = \frac{267.5}{2000}$ $SP = 267.5$ $SS_x = 5,262.5$ $SS_y = 20.1$						$SS_{y} = 20.1$
5262 = .051	.5						
$a = \overline{y} - b$ $= 2.7 - b$ $= 2.7 - b$	x 0.51(102.5 5.2275	5)					
= -2.52	275						

WORKSHEET 12.1. CALCULATION OF REGRESSION LINE.

The estimated mean property value increase in the community is projected to be \$3,491.

It is important to note one of the crucial limitations on use of the line in this manner. If the subsidy were \$40,000, for instance, the line would predict that property values would decline by approximately \$500. There is no evidence that this is a logical and appropriate conclusion inasmuch as the data depict subsidies ranging from \$60,000 to \$130,000. It is not known what would happen to values when the projection is drawn for values far outside the data range. Regression lines are therefore potentially useful only within or slightly outside the range of independent variable data used to determine the line. Making predictions for independent variable values that may be far outside that range is not appropriate.

Testing the Significance of the Regression Line

The equations for the *y*-intercept and the slope can be calculated from any set of raw data. Hence, a regression equation can be established whether

or not a significant linear relationship exists between the two interval scale variables. For example, data points that are widely scattered, as in figure 12.3, will yield a regression line, just as will those that closely approximate the line, as in figure 12.4. Linear regression analysis therefore requires that all calculated regression lines be tested for statistical significance. If the line is found to be statistically significant, it may be considered for use for predictive purposes, subject to one further consideration, which we address following the test for statistical significance. If the regression line is not statistically significant, it cannot be used for predictive purposes. Generally, under similar sampling conditions and sample sizes, the closer the data conform to the line of best fit, the more likely the line is to pass the test of statistical significance.

There are several alternative tests to determine whether the regression equation is significant. The most widely used test is an ANOVA (analysis of variance) test, which has been previously discussed in the context of determining the amount of the variance that the categories of the independent variable explain in comparison to the amount that remains as residual, unexplained variance within those categories (see the "Analysis of Variance" section in chapter 11).

The ANOVA test for the significance of the regression line is similar to the analysis of variance test in that it compares the amount that the total variance is explained by using the regression line for predictive purposes



FIGURE 12.3. VIOLENT CRIMES AND POPULATION.



FIGURE 12.4. PRIOR CONVICTIONS AND SENTENCE LENGTH.

rather than using the mean. The total variance in the dependent variable using the mean for prediction is known as the *total mean square* (MS_{total}) , just as it was in the analysis of variance test, and just as with analysis of variance, MS_{total} consists of two components—*residual mean square* $(MS_{residual})$ and *regression mean square* $(MS_{regression})$. $MS_{residual}$ is the variance that remains after using the regression line, and $MS_{regression}$ is the amount that the regression line reduces MS_{rotal} to $MS_{residual}$.

where

$$MS_{regression} = SS_{regression} = df_{regression}$$
(12.4)

$$MS_{residual} = SS_{residual} = df_{residual}$$
(12.5)

$$MS_{total} = SS_{total} = df_{total}$$
(12.6)

$$df_{total} = n - 1$$

 $df_{regression} = 1$ (one for each independent variable)

$$df_{residual} = n - 2$$
Calculation of Sum of Squares

Determination of the mean squares therefore requires, as an initial step, the determination of sums of squares, as follows:

$$SS_{regression} = \frac{\left[\Sigma(x-\overline{x})(y-\overline{y})\right]^2}{\Sigma(x-\overline{x})^2}$$
(12.7)

$$SS_{regression} = \Sigma (y - \hat{y})^2$$
(12.8)

$$SS_{total} = \Sigma \left(y - \overline{y} \right)^2 \tag{12.9}$$

As was the case with analysis of variance, whenever equations contain expressions requiring the calculation of deviation, the arithmetic procedure becomes laborious. In order to minimize this problem in the case of the ANOVA calculation for the regression line, the researcher will find that such inconvenience can be minimized by calculating SS_{total} and $SS_{regression}$ and obtaining $SS_{residual}$ by subtraction ($SS_{residual} = SS_{total} - SS_{regression}$). This is the case because SS_{total} is found in the worksheet for determining the line, so it exists without additional calculations:

$$SS_{total} = SS_{y} \tag{12.10}$$

and because the equation for $SS_{regression}$ can be simplified to

$$SS_{regression} = b(SP) \tag{12.11}$$

or the slope of the line multiplied by the numerator of the slope calculation.

Were SS_{residual} to be calculated directly, it would involve another factor, \hat{y} , which is the predicted value of \hat{y} using the regression line. In the case of the subsidy example, \hat{y} is calculated by applying the regression line equation ($\hat{y} = -2.5275 + .051x$) to each value of *x* in the sample and then determining the squared differences with the actual \hat{y} values, as shown in worksheet 12.2.

In contrast to worksheet 12.2, $SS_{residual}$ can be calculated more simply by subtracting $SS_{residual}$ from SS_{total} , as follows:

$$SS_{total} = SS_y = 20.1$$
 (from worksheet 12.1)

$$SS_{regression} = b(SP) = 0.51(267.5) = 13.6425$$

 $SS_{residual} = 20.1 - 13.6425 = 6.4575$ (difference from worksheet is in rounding b = .0508 to .051).

The considerably greater ease with which $SS_{residual}$ is calculated in the subtraction method versus the calculations in worksheet 12.2 should be obvious to the reader.

Block	x	у	ŷ	$y - \hat{y}$	$(y-\hat{y})^2$
A	100	3	2.5725	.4275	.1828
В	85	3	1.8075	1.1925	1.4221
С	95	1	2.3175	‡1.3175	1.7358
D	130	5	4.1025	.8975	.8055
E	110	2	3.0825	1.0825	1.1718
F	75	2	1.2975	.7025	.4935
G	135	4	4.3575	\$.3575	.1278
Н	60	0	.5325	‡.5325	.2836
I	120	4	3.5925	.4075	.1661
J	115	3	3.3375	‡.3375	.1139
				$\Sigma = SS_{\textit{residual}} =$	6.5029

WORKSHEET 12.2. RESIDUAL SUM OF SQUARES.

Calculation of Variances and F Ratio

Once the sums of squares are calculated, an ANOVA table can be constructed (figure 12.5). As with analysis of variance, the ANOVA table compiles and simplifies the presentation of the data that are necessary to determine the significance of the regression line.

In referring to the critical values of the *F* distribution in chapter 11 (exhibit 11.1 for 95 percent confidence and exhibit 11.2 for 99 percent confidence), the researcher would find that with df numerator = 1 and df denominator = 8, the critical *F* at 95 percent is 5.32, and at 99 percent, it is 11.3. The conclusion that the researcher can draw from figure 12.5 is that there is 99 percent confidence that the regression line is a true

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F		
Regression	13.6425	1	13.6425	16.901		
Residual	6.4575	<i>n</i> – 2 = 8	.8072			
Total	20.1	<i>n</i> – 1 = 9				

FIGURE 12.5.	ANOVA TABLE FOR REGRESSION
LINE OF SU	BSIDY AND VALUE INCREASES.

improvement over the mean and potentially useful for predictive purposes subject to the discussion that follows.

Coefficient of Determination (r^2)

Some researchers stop at this point and use regression lines for predictive purposes when and if the line proves to be significant. Significance, they argue, indicates that the line is a better representation of the relationship between the independent variable and the dependent variable than is the mean of the dependent variable that bears no relationship to the independent variable.

A crucial weakness with such an approach is that significance indicates only that a relationship exists, not how strong it is. It is frequently the case that a regression line reflects only a very weak relationship that is significant in that it is an accurate reflection of the relationship, but the mere existence of such a relationship may not merit its use as a tool for making predictions and projections.

The question then becomes, "If mere significance is not sufficient to justify projection using regression analysis, what measure should be used to make that determination?"

Just as analysis of variance has a correlation ratio (E^2) that indicates the amount of the total variance in the dependent variable explained by the categories of the independent variable, regression analysis uses a coefficient of determination (r^2) to indicate the amount of the total variance that the regression line explains. The coefficient of determination is calculated as follows:

$$r^2 = SS_{regression} / SS_{total}$$
(12.12)

For the subsidy/value example,

$$r^2 = 13.6425/20.1$$

= .679

Therefore, approximately 68 percent of all the variance in market value increases, in the general population to which the sample refers, is explained by the amount of rehabilitation subsidies that flow to the properties and the blocks on which they are located.

Pearson's r Correlation

The strength of the relationship between two interval scale variables can be measured by *Pearson's* r, or what is commonly referred to as the *coefficient* of correlation.

Pearson's *r* can be calculated in accordance with the following formula:

$$r = \frac{\sum(x - \overline{x}) (y - \overline{y})}{\sqrt{(x - \overline{x})^2 ((y - \overline{y}))^2}}$$
(12.13)

The calculation of Pearson's r for the data in figure 12.1, based on equation 12.13, is demonstrated below for the original rehabilitation subsidy/property value example. It should be noted that worksheet 12.1, which was used to calculate the regression line, provides all of the information that is necessary to make this calculation, in which the numerator is the sum of the products (*SP*) and the denominator is the square root of *SS*_multiplied by *SS*_.

$$SP = 267.5$$

$$SS_x = 5,262.5$$

$$SS_y = 20.1$$

$$r = 267.5 / \sqrt{(5,262.5)(20.1)}$$

$$= 267.5 / \sqrt{105,776.25}$$

$$= 267.5 / 325.233$$

$$= .82$$

A Pearson's r of .82 can be interpreted in accordance with the guidelines presented in exhibit 10.3 (interpretation of calculated gamma except that Pearson's r also has matching negative/inverse interpretations. The strongest relationships are at +1.00 and -1.00, with 1.00 indicating a perfect relationship and the sign indicating the direction of the variables' relationship—not its strength) as a very strong positive association. In other words, rehabilitation subsidies are strongly correlated positively with property value increases: the more money that is available, the greater is the increase in property values.

It is also possible to determine Pearson's r from the coefficient of determination: r^2 . Pearson's r is, as would be supposed, the square root of r^2 with a sign (positive or negative) that matches the slope of the line. The r^2 from the simple linear regression of subsidies and value increases was determined above to be .679, the square root of which is .82, and the sign is positive to match the positive b = .051.

As with any other calculation based on a sample, statistical significance of Pearson's r must be established. If Pearson's r is derived as a by-product of regression analysis, the significance of the regression line will also indicate whether Pearson's r is significant.

If Pearson's r is determined by itself, as would be the case when the nature and strength of the relationship between two interval variables is desired and a line for predictive use is not needed, then its significance can be tested using a t test, as follows:

$$t = (r\sqrt{n-2}) + \sqrt{(1-r^2)}$$
(12.14)
$$df = n-2$$

In the subsidy/value example:

$$t = .82(\sqrt{8}) \div \sqrt{(1 - .82^2)}$$

= 2.319 ÷ $\sqrt{(1 - .6724)}$
= 2.319 ÷ $\sqrt{.3276}$
= 2.319 ÷ .5724
= 4.05
 $df = 8$

The critical *t* (from exhibit 7.2) at 95 percent and 99 percent—two tail because *t* can be positive or negative—is, respectively, 2.306 (95 percent) and 3.355 (99 percent), and significance is established by the calculated t = 4.05, as was the case when regression analysis was performed.

USING SPSS FOR REGRESSION ANALYSIS

SPSS is a particularly valuable instrument for regression analysis. Its initial value is in developing the scatter plots that are used in this chapter (figures 12.2, 12.3, and 12.4). Take, for example, the relationship between years as a customer of the Clayton Water Authority (cust in Water.sav) and final percentage of water that should come from desalination (pctnotUScomb#3), in which the hypothesized relationship is that as the years a person has been a customer increases, the less receptive he or she is to change, so that the percentage of desalinated water desired will decline as the number of years as a customer increases.

1. Click on the drop-down menu for Graphs, then Legacy Dialogs—Scatter/Dot, and then—Simple Scatter plot—Define.:

- In the Y Axis box, place the dependent variable (in this case, percentage #3—pctnotUScomb#3).
- 3. In the X axis box, place the independent variable (number of years being a customer—cust).
- 4. In the Label Cases by box, place the ID number (respnum) or however else the researcher wishes to identify each dot on the scatter plot.

Regression and Correlation

- 5. In Options, excluding cases listwise is frequently the only option available. If not, change to variable-by-variable; otherwise, remain as listwise. Then click Continue and OK.
- 6. Look at the output and double-click on the scatter plot itself.
- 7. Click on the fifth icon immediately above the chart (labeled Add Fit Line at Total), and the regression line (the line of best fit) is shown.



- 8. Make certain that the fit line is a line (as opposed to a curve) and that Linear is checked under Fit Method.
- 9. The line that is shown has an equation of y = 43.3 + .021x, indicating a positive relationship, with the percentage of desalinated water increasing by .021 for each year being a customer. The line is also shown to have a relatively low R² value (proportion of variation explained by regression line) of .004.
- 10. If the researcher wants to further examine data that might be considered to be outliers, click on the first box on the left immediately above the scatter plot itself (almost looks like a rifle scope) and then on the potential outlying dot itself—the response number will be provided adjacent to the dot itself.

All of the information provided by the scatter plot is available (without the graphics and outliers) when statistical significance is determined through regression analysis. As such, unless there is some interest in seeing the graphical depiction of the relationship and outliers, the scatter plot can be bypassed.



Testing the Statistical Significance of the Regression Line

Using the same data and hypothesis, simple linear regression analysis can be conducted in the following manner:

- 1. Analyze—Regression—Linear.
- 2. Enter the Independent(s) (y variable) and Dependent (x variable) variables in the appropriate windows.

Linear Regression	Dependent: Provious Previous Method: Enter Selection Variable: Case Labels: WLS Weight	Statistics Plots Save Options Bootstrap
	OK Paste Reset Cancel Help	

- 3. In Statistics, Model Fit and Estimates are checked as defaults. Add to those a check by Descriptives. Continue.
- 4. In Options, make certain that .05 is indicated by Entry and change listwise to pairwise for missing values. Continue.
- 5. Leave Enter as the Method and press OK.
- 6. The output can be extensive; however, there are quick items to look for first.

Descriptive Statistics

	Mean	Std. Deviation	N
q16 and q16arec combined How long customer of Clayton Water Authority	45.48 10.57	29.531 8.890	360 360

Correlations

		q16 and q16arec Combined	How Long Customer of Clayton Water Authority
Pearson	q16 and q16arec combined	1.000	.062
correlation	How long customer of Clayton Water Authority?	.062	1.000
Sig. (1-tailed)	q16 and q16arec combined		.121
	How long customer of Clayton Water Authority?	.121	
N	q16 and q16arec combined	360	360
	How long customer of Clayton Water Authority?	360	360

Variables Entered/Removed

Model	Variables Entered	Variables Removed	Method
1	How long customer of Clayton Water Authorityª		Enter

Note: Dependent variable: q16 and q16arec combined. ^aAll requested variables entered.

Model Summary

Model	R	R Square	Adjusted <i>R</i> Square	Std. Error of the Estimate
1	.062ª	.004	.001	29.516

^aPredictors: (Constant), How long customer of Clayton Water Authority.

ANOVA

Мо	del	Sum of Squares	df	Mean Square	F	Sig.
1	Regression	1,198.525	1	1,198.525	1.376	.242ª
	Residual	311,879.250	358	871.171		
	Total	313,077.775	359			

Note: Dependent variable: q16 and q16arec combined.

^aPredictors: (Constant), How long customer of Clayton Water Authority.

		Unstandardized Standardized Coefficients Coefficients				
Model		В	Std. Error	Beta	t	Sig.
1	(Constant)	43.302	2.419		17.900	.000
	How long customer of Clayton Water Authority	.206	.175	.062	1.173	.242

Coefficients

Note: Dependent variable: q16 and q16arec combined.

- 7. The ANOVA table indicates that the relationship is not significant (.242)—that any apparent association between length of being a customer and opinion about how much water should come from desalination is explained by sampling error.
- 8. That being said, nothing further is needed from this output. However, for the sake of demonstrating the parallels with the scatter plot, the reader can see that the line of best fit that was shown on the scatter plot is reflected in the Coefficients table. In the Coefficients table, the *B* value for constant is the *y*-intercept and the *B* value for customer length is the line's slope, so the equation for the line of best fit is y = 43.302 + .206x (the same as depicted on the scatter plot, except with less rounding), with y = desalination percentage and x = length

Regression and Correlation

of time as a customer. The R^2 (in Model Summary) for the line (as on the scatter plot) is .004.¹

In order to more fully explore regression analysis output, a significant relationship will be put forth—that a larger number of transfers in Transportation.sav will cause travel time to increase. Following the same steps as above—Analyze—Regression—Linear and all of the same Statistics and Options—the regression output for number of transfers (transfers#_q15) and travel time (travtime_total), with the number of transfers as the Independent Variable is:

Descriptive Statistics

	Mean	Std. Deviation	N
Travel time in hours	1.6724	1.25545	1851
Number of transfers—subject trip	1.86	1.300	1748

Correlations

		Travel Time in Hours	Number of Transfers— Subject Trip
Pearson correlation	Travel time in hours	1.000	.435
	Number of transfers— subject trip	.435	1.000
Sig. (1-tailed)	Travel time in hours		.000
	Number of transfers— subject trip	.000	
Ν	Travel time in hours	1,851	1691
	Number of transfers— subject trip	1,691	1748

Variables Entered/Removed

Model	Variables Entered	Variables Removed	Method
1	Number of transfers—subject tripª		Enter

Note: Dependent variable: Travel Time in Hours. ^aAll requested variables entered.

.188

1.13100

	Model R	R Square	Adjusted <i>R</i> Square	Std. Error of the Estimate
--	---------	----------	--------------------------	-------------------------------

.189

Model Summary

Note: Dependent variable: Travel Time in Hours.

.43.5ª

^aPredictors: (Constant), Number of Transfers—Subject Trip

ANOVA

1

	Model	Sum of Squares	df	Mean Square	F	Sig.
	Regression	503.219	1	503.219	393.400	.000ª
1	Residual	2,160.490	1,689	1.279		
	Total	2,663.709	1,690			

Note: Dependent variable: Travel Time in Hours.

^aPredictors: (Constant), Number of Transfers—Subject Trip.

Coefficients

		Unst Co	andardized efficients	Standardized Coefficients		
	Model	В	Std. Error	Beta	t	Sig.
1	(Constant)	.892	.048		18.583	.000
	Number of Transfers— Subject Trip	.420	.021	.435	19.834	.000

Note: Dependent variable: Travel Time in Hours.

The ANOVA table now shows a significant F ratio (.000) and an equation for the significant regression line of y = .892 + .420x (Coefficients Table). This indicates that travel time (variable y) increases by .42 of an hour (approximately 25 minutes) with each additional transfer (variable x). The *R*-Squared is .189 (note the adjusted R-Squared is essentially the same as R-Squared because of the absence of extraneous variables; see reference on prior page to footnote 1), which indicates that travel time variations are close to 20 percent explained by the number of transfers.

Also shown in the Correlations Table is the Pearson's r measure of association (r = .435) for the relationship between travel time and transfers and is interpreted using exhibit 10.3 to indicate a moderate positive association. Readers might also note that correlations can be obtained by themselves (without regression output) as follows: 2. Insert two variables to be correlated in Variables box (independent and dependent designations are inconsequential)—click OK.

Using SPSS to Perform Multiple Regression Analysis

To illustrate multiple regression using SPSS, return to the example used to demonstrate the computer's application for simple linear regression—number of transfers impact on travel time, which produced a significant regression equation that possessed an *R*-squared of .189. There is no specific rule about how large *R*-squared should be in order for the model to be considered a good one to use for predictive purposes. Some statisticians contend that any significant line is useable for predictive purposes in that it is the best available tool. Others argue that statistics does not typically settle for "best available" and instead seeks to be a certain degree confident (generally 95%) that the answer given is the correct one, so why not require an *R*-squared of 95%? Neither is realistic in that an *R*-squared of 95% is rarely achieved, and significance alone can still leave a great amount of variation unexplained.

The use of a regression equation must be evaluated carefully against the consequences of being wrong and against the variance that exists in the data, which may already have been reduced through data transformations such as logarithms or may be naturally low, such as time series. The less the consequences of a prediction error, the lower can be the *R*-squared and still use the regression line for predictive purposes. The tighter the variance is, the higher the *R*-squared that is expected. The researcher is invited to consult the Bibliography at the back of this book or any of a number of other references and websites that discuss the adequacy of particular R-squared values in seeking further information about *R*-squared, but as a general rule, with all things being equal, a higher *R*-squared indicates a better fit for the line and a more accurate prediction. We tend to use the regression line for predictive purposes, even for the most minor of issues, only when R-squared is at least equal to 0.50.

In that light, let it be assumed that the researcher wishes to press ahead and improve on the .189 *R*-squared from the simple linear regression model. The researcher hypothesizes that age and income will have additional effects on travel time. Income (using the recoded scale version—increc) is hypothesized to have a negative association with travel time: higher-income people can afford to live closer to their destinations (in particular, work) and older people have less tolerance for long trips, also associated negatively with travel time.

The process is as follows:

- 1. Analyze—Regression—Linear.
- 2. Travel time (travtime_total is the dependent variable and all three hypothesized independents (transfers, income, and age) are inserted in the Independent(s) box.
- 3. In Statistics, Model Fit and Estimates are checked as defaults. Add to those a check by Descriptives. Continue.

4. In Options, make certain that .05 is indicated by Entry. and change listwise to pairwise for missing values. Continue.

A ID	Dependent:
day hour superdist Language satis_overall_q1 change_service satis_cost_q3a satis_ontime_q3b satis_intre_q3c satis_trea_q3c satis_closehome satis_teleinfo_q3g satis_teleinfo_q3g satis_transferreli satis_drivecourte satis_drivecnow satis_drivecnow satis_afrevenow satis_afrevenow	Block 1 of 1 Previous Independent(s): Age_q32 increc Method: Stepwise Selection Variable: Case Labels: WLS Weight VLS Weight

5. Change Enter to Stepwise as the Method and press OK.²

The output for multiple regression is

Descriptive Statistics

	Mean	Std. Deviation	N
Travel time in hours	1.6724	1.25545	1851
Number of transfers—subject trip	1.86	1.300	1748
Age category	40.32	15.999	1823
Recoded income-midpoints	19,718.6807	25,541.97312	1804

Correlations

		Travel Time in Hours	Number of Transfers— Subject Trip	Age Category	Recoded Income- Midpoints
	Travel time in hours Number of trans-	1.000 .435	.435 1.000	.130 .061	063 173
Pearson correlation	Age category	.130	.061	1.000	004
	Recoded income-midpoints	063	173	004	1.000

Regression and Correlation

	Travel time in hours .		.000	.000	.004
Siq.	Number of trans- fers—subject trip	.000	•	.007	.000
(1-tailed)	Age category	.000	.007		.427
	Recoded income-midpoints	.004	.000	.427	
N	Travel time in hours	1,851	1,691	1,751	1,729
	Number of trans- fers—subject trip	1,691	1,748	1,651	1,639
	Age category	1,751	1,651	1,823	1,711
	Recoded income-midpoints	1,729	1,639	1,711	1,804

Variables Entered/Removed

Model	Variables Entered	Variables Removed	Method
1	Number of trans- fers—subject trip		Stepwise (Criteria: Probability-of- <i>F</i> - to-enter <= .050, Probability-of- <i>F</i> - to-remove >= .100).
2	Age category		Stepwise (Criteria: Probability-of- <i>F</i> -to-enter <= .050, Probability-of- <i>F</i> -to-remove >= .100).

Note: Dependent variable: Travel Time in Hours.

Model Summary

Model	R	R Square	Adjusted <i>R</i> Square	Std. Error of the Estimate
1	.435ª	.189	.188	1.13101
2	.447 ^b	.200	.199	1.12387

^aPredictors: (Constant), Number of Transfers—Subject Trip. ^bPredictors: (Constant), Number of Transfers—Subject Trip, Age Category.

ANOVA

Мос	del	Sum of Squares	df	Mean Square	F	Sig.
1	Regression	487.736	1	487.736	381.289	.000ª
	Residual	2094.013	1,637	1.279		
	Total	2581.749	1,638			

2	Regression	515.359	2	257.680	204.010	.000 ^b
	Residual	2,066.389	1,636	1.263		
	Total	2,581.749	1,638			

Note: Dependent variable: Travel Time in Hours.

^aPredictors: (Constant), Number of Transfers—Subject Trip.

^bPredictors: (Constant), Number of Transfers—Subject Trip, Age Category.

Coefficients

		Unstandardized Coefficients		Standardized Coefficients		
Model		В	Std. Error	Beta	t	Sig.
1	(Constant)	.892	.049		18.295	.000
	Number of transfers— subject trip	.420	.021	.435	19.527	.000
2	(Constant)	.576	.083		6.913	.000
	Number of transfers— subject trip	.413	.021	.428	19.329	.000
	Age category	.008	.002	.104	4.677	.000

Note: Dependent variable: Travel Time in Hours.

Excluded Variables

					Partial	Collinearity Statistics
	Model	Beta	t	Sig.	Correlation	Tolerance
1	Age category Recoded income-midpoints	.104ª .012ª	4.677 .551	.000 .582	.115 .014	.996 .970
2	Recoded income-midpoints	.012 ^b	.525	.599	.013	.970

Note: Dependent variable: Travel Time in Hours.

^aPredictors: (Constant), Number of Transfers—Subject Trip.

^bPredictors: (Constant), Number of Transfers—Subject Trip, Age Category.

Taken one-by-one:

1. The Descriptives output shows the means (1.68 hours of traveling, 1.86 transfers, 40.32 years of age, and \$19,719 annual income for the bus riders) and the standard deviations of the sampled population.

Regression and Correlation

- 2. Correlations shows the relationships among the variables, with the strongest being number of transfers and travel time (.435) and income and number of transfers (-.173—negative as hypothesized) and the weakest being age and income (a negligible negative association of -.004). It can be noted that the relationship between age and travel time is a weak positive, whereas the hypothesis was that it would be a negative relationship.
- 3. With the Variables Entered/Removed output, the true Multiple Regression analysis begins. Model 1 in that output represents the insertion of the most powerful of the three independent variables into the regression analysis. That variable is the number of transfers, as was noted with its .435 Pearson's *r* correlation.
- 4. In the Model Summary, one independent variable has an *R*-squared of .189, and ANOVA shows the researcher that it is significant.
- 5. The Coefficients indicate an equation for the Regression line of y = .892 + .420x—just as it was when these two variables were regressed in the above simple linear regression example.
- 6. Excluded Variables tells the researcher an interesting story: that of the two variables that were passed over for Model 1 because they were less powerful influences on travel time, one (age) is also significant. Therefore, the analysis creates a new model (Model 2) that also includes this other significant variable. It is this stepwise insertion of significant variables that lends the method its name.
- Model 2 shows age being entered into the equation in Variables Entered/ Removed
- 8. *R*-squared increases to .200 (a small but statistically significant increase of .011), and ANOVA shows that as would be expected with only significant independent variables included, the equation with both independent variables—number of transfers and age—is significant.
- 9. The new regression line equation is $y = .576 + .413x_1 + .008x_2$, where $x_1 =$ number of transfers and $x_2 =$ respondent age.³
- 10. Excluded Variables contains only income, which is not significant, so the analysis ends at this point with a final regression line equation of $y = .576 + .413x_1 + .008x_2$ and an *R*-squared of .200. The researcher can then consider continuing the process with additional hypotheses and independent variables.

Dummy Variables

Regression analysis is designed only for interval variables, and yet there are variables that are not interval and might be able to contribute significantly to a regression model. For example, if individual income data are being examined for purposes of predicting earnings, gender or ethnicity might be key predictors, but they are not interval. They can, however, be reconfigured for regression purposes as binary variables, coded as 0 or 1 only, with the 0 representing the absence of a characteristic and a 1 representing its presence.

In order to accomplish this, a variable called a *dummy variable* (sometimes referred to as an *indicator variable*) is created for k - 1 categories of the independent variable. If gender is the variable, then the two categories (k = 2)—male and female—can be accommodated by creating one new dummy variable (k - 1 = 1), where a code of 1 would indicate female and 0 would indicate male. If ethnicity is the issue and there are considered to be five ethnic categories (white, African American, Latino, Asian, other), then four dummy variables would be required, as follows:

If $x_1 = 1$, and x_2 , x_3 , and $x_4 = 0$, the individual is white. If $x_2 = 1$, and x_1 , x_3 , and $x_4 = 0$, the individual is Latino. If $x_3 = 1$, and x_1 , x_2 , and $x_4 = 0$, the individual is African-American. If $x_4 = 1$, and x_1 , x_2 , and $x_3 = 0$, the individual is Asian. If x_1 , x_2 , x_3 , and $x_4 = 0$, the individual is of another ethnicity.

Let it be assumed that multiple regression analysis produces the following line:

$$y = 30,000 + 5,000x_1 - 3,500x_3 - 7,000x_5$$
(female)
+ 1,000x_6 (years of education)

This line can be interpreted as follows. The intercept, \$30,000, is the base income, to which \$5,000 is added for predictive purposes if the individual is white or \$3,500 is deducted if the individual is African American, \$7,000 is deducted for a female, and \$1,000 is added for every year of education. Being Latino, Asian, or of another ethnic group is not significant in this example. A researcher would therefore predict that a white female with sixteen years of education would earn \$44,000 and that an African American male with fourteen years of education would earn \$40,500.

Dummy variables also serve very well as what are called *intercept variables*, where a significant event has disrupted the trend that regression relies on. For instance, if in the year 2010, Thailand is analyzing its attractiveness to tourists, the tragic tsunami at the end of 2004 will have obviously had an impact on it for some period thereafter. A dummy variable, where 0 = 2004 and before and 1 = 2005 and thereafter, would work well to account for this terrible event.

SPSS: REGRESSION AND DUMMY VARIABLES

The process of incorporating dummy variables into multiple regression is primarily a recoding function. For example, if the example of travel time is to be pursued, it may well be that variables other than interval/scale variables may need to be considered. To the two significant independent variables (age and number of transfers), let frequency of riding and gender be added, hypothesizing that men are less patient than women or that they have more employment opportunities, thereby allowing them to choose to travel shorter distances and that frequent riders would not make very long trips so regularly. Language may also have an impact on the convenience of job or other destination location.

Looking at the frequencies for frequency (freq_q6), it is noted that a huge majority of riders travels by bus four to seven days per week, which is coded as 1. A dummy variable can be made for frequent rider (freqrider), leaving the code 1 for four to seven days and coding all other responses as 0 (infrequent riders). The recoding process is shown below.

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	4–7 days per week	1,577	80.7	82.3	82.3
	1–3 days per week	288	14.7	15.0	97.3
	At least once per month	25	1.3	1.3	98.6
	A few times per year	20	1.0	1.0	99.6
	Less than a few times per year	7	.4	.4	100.0
	Total	1,917	98.1	100.0	
Missing	System	37	1.9		
Total		1,954	100.0		

Frequency Riding Bus

The language variable should be made into three dummy variables with 1 = yes, (the respondent's language) and 0 = no, it is not. This is accomplished for English by leaving the code 1 as 1 and coding 2 and 3 (all other values) as 0. For Spanish, code 2 becomes a code of 1 and codes 1 and 3 (all other values) become 0. For Vietnamese, code 3 will become 1 and 1 and 2 (all other values) become 0.

Then simply change the female 2 code to 0 for gender (genderdum) using Transform-Recode into Same Variables and these non-interval variables are set for inclusion as dummy variables after making the appropriate label changes in Variable View for each new dummy variable. Three independent variables are now included in the regression output, with Spanish language survey respondents now added in

Recode into Different Variable	es Numeric <u>V</u> ariable → Output Variable: freq_q6 → freqrider	Output Variable
Import_busbook		fregrider
simprove1_q5a		Label:
 improve3_q5c change_freq_12 length_ride_q8 reason_usebus infosource_printe infosource_ebus 	•	Ride 4-7 days or more?
infosource_trippi	Old and New Values	
♣ infosource_busst ♣ infosource_text4 ▼	(optional case selection condition)	
(OK Paste Reset Cancel Help	

Recode into Different Variables: Old and New Values	
Old Value	New Value
O <u>V</u> alue:	O Value:
	O System-missing
System-missing	Opy old value(s)
System- or user-missing	Old > Now
© Ra <u>n</u> ge:	MISSING> SYSMIS
	2 thru 5> 0
through	ELSE> Copy
Depage LOWEST through value:	Change
C Range, LOWEST through value.	Change
© Range, value through HIGHEST:	Remove
All other values	Output variables are strings Width:
	Convert numeric strings to numbers (5->5)
Continue	e Cancel Help

Value Labels Value Labels Value: Label: 0 = "less frequent bus rider" 1 = "frequent bus rider"	Spelling
Change Remove OK Cancel Help	

Regression and Correlation

Model 2, with a negative slope indicating that Spanish language respondents travel .418 hours (25 minutes) less than other language speakers. *R*-squared has increased to .220, and the new regression equation is: $y = .575 - .4445x_1 - .418x_2 + .009x_3$, where $x_1 =$ number of transfers, $x_2 =$ Spanish language rider, and $x_3 =$ age. Gender and riding frequency are not significant, although frequency, with a significance of .074, did come close.

Variables Model Variables Entered Removed Method 1 Number of Stepwise (Criteria: Probability-of-F-Transfers—subject to-enter <= .050, Probability-of-*F*-toremove >= .100). trip 2 Survey in Spanish Stepwise (Criteria: Probability-of-Fto-enter <= .050, Probability-of-*F*-toremove >= .100). 3 Stepwise (Criteria: Probability-of-F-Age category to-enter <= .050, Probability-of-F-toremove >= .100).

Variables Entered or Removed

Note: Dependent variable: Travel Time in Hours.

Model Summary

Model	R	R Square	Adjusted <i>R</i> Square	Std. Error of the Estimate
1	.435ª	.189	.188	1.13100
2	.453 ^b	.206	.205	1.11966
3	.469°	.220	.219	1.10984

^aPredictors: (Constant), Number of Transfers—Subject Trip.

^bPredictors: (Constant), Number of Transfers—Subject Trip, Survey in Spanish.

Predictors: (Constant), Number of Transfers—Subject Trip, Survey in Spanish, Age Category.

ANOVA

	Model	Sum of Squares	df	Mean Square	F	Sig.
1	Regression	491.309	1	491.309	384.084	.000ª
	Residual	2,109.354	1649	1.279		
	Total	2,600.663	1650			

Mod	lel	Sum of Squares	df	Mean Square	F	Sig.
2	Regression	534.676	2	267.338	213.251	.000 ^b
	Residual	2,065.987	1648	1.254		
	Total	2,600.663	1650			
3	Regression	571.978	3	190.659	154.788	.000 ^c
	Residual	2,028.685	1647	1.232		
	Total	2,600.663	1650			

Note: Dependent variable: Travel Time in Hours.

^aPredictors: (Constant), Number of Transfers—Subject Trip.

^bPredictors: (Constant), Number of Transfers—Subject Trip, Survey in Spanish.

^cPredictors: (Constant), Number of Transfers—Subject Trip, Survey in Spanish, Age Category.

Coefficients

		Unsta Coe	ndardized efficients	Standardized Coefficients		
	Model	В	Std. Error	Beta	t	Sig.
1	(Constant)	.892	.049		18.362	.000
	Number of transfers— subject trip	.420	.021	.435	19.598	.000
2	(Constant)	.939	.049		19.256	.000
	Number of transfers— subject trip	.449	.022	.465	20.616	.000
	Survey in Spanish	376	.064	133	-5.882	.000
3	(Constant)	.575	.082		7.015	.000
	Number of transfers— subject trip	.445	.022	.461	20.609	.000
	Survey in Spanish	418	.064	147	-6.550	.000
	Age category	.009	.002	.121	5.503	.000

Note: Dependent variable: Travel Time in Hours.

Excluded Variables

					Dortial	Collinearity Statistics
	Model	Beta In	t	Sig.	Correlation	Tolerance
1	Age category	.104ª	4.694	.000	.115	.996
	Gender dummy variable	.025ª	1.140	.255	.028	.995
	Ride 4–7 days or more?	049ª	-2.217	.027	055	.995
	Survey in English	.128ª	5.699	.000	.139	.951
	Survey in Spanish	133ª	-5.882	.000	143	.948
	Survey in Vietnamese	.012 ^{ab}	.558	.577	.014	1.000
2	Age category	.121 ^b	5.503	.000	.134	.982
	Gender dummy variable	.019 ^b	.854	.393	.021	.993
	Ride 4–7 days or more?	047 ^b	-2.135	.033	053	.994
	Survey in English	028^{b}	252	.801	006	.040
	Survey in Vietnamese	.006 ^b	.261	.794	.006	.997
3	Gender dummy variable	.011 ^c	.495	.621	.012	.988
	Ride 4–7 days or more?	039 ^c	-1.786	.074	044	.990
	Survey in English	.041°	.379	.705	.009	.040
	Survey in Vietnamese	008°	365	.715	009	.984

Note: Dependent variable: Travel Time in Hours.

^aPredictors: (Constant), Number of Transfers—Subject Trip.

^bPredictors: (Constant), Number of Transfers—Subject Trip, Survey in Spanish.

^cPredictors: (Constant), Number of Transfers—Subject Trip, Survey in Spanish, Age Category.

Choosing among Tests of Significance and Measures of Association

The significance tests and measures of association discussed in this book have been presented in the context of their application to research questions involving variables of various levels of measurement. There have been many such tests, and we believe the summary in exhibit 12.1 will be useful.

	Level of Me	asurement		
Independent Variable	Dependent Variable	Notes and Comments	Test of Significance	Measure of Association
Nominal	Nominal		Chi square	Cramér's V
Nominal	Ordinal		Chi square	Cramér's V
Nominal	Interval		Analysis of variance or independent samples/ paired samples <i>t</i> tests	Eta
Ordinal	Nominal		Chi square	Cramér's V
Ordinal	Ordinal	lf analyzing trend	Gamma	Gamma
Ordinal	Ordinal	lf not analyzing trend	Chi square	Gamma
Ordinal	Interval		Analysis of variance or independent samples/ paired samples <i>t</i> tests	Eta
Interval	Nominal		Chi square	Cramér's V
Interval	Ordinal	lf analyzing trend	Gamma	Gamma
		lf not analyzing trend	Chi square	Gamma
Interval	Interval	If prediction	Regression analysis	Pearson's r
Interval	Interval	If no prediction	Analysis of variance or independent samples/ Paired samples <i>t</i> tests	Pearson's r

EXHIBIT 12.1. CHOOSING THE APPROPRIATE STATISTICAL TEST OR MEASURE.

Notes

- 1. SPSS provides an alternative to *R*-squared, called adjusted *R*-squared. This is an adjustment of the *R*-squared that penalizes the addition of extraneous predictors to the model. Adjusted *R*-squared is computed using the formula 1 ((1 Rsq) ((N 1)/(N k 1)) where *k* is the number of predictors. Inasmuch as this example is entirely extraneous (not significant), the adjusted *R*-squared is substantially less than the *R*-squared.
- 2. The choice between Enter and Stepwise has become a hotly disputed topic within regression analysis. Enter includes all variables requested in the model, whether they are individually significant or not, as long as the overall model is significant (making

the adjusted *R*-squared especially important because it eliminates the extraneous variables from determining the model fit. Stepwise includes only variables that are individually significant. The arguments are that with the adjusted *R*-squared taking account of the variables that are extraneous, Enter more accurately reflects the way that the world works—with contributory and extraneous factors all interacting. Stepwise advocates argue that there is no reason to identify and include variables that do not contribute significantly to *R*-squared—that this produces an unwieldy model that requires more data to apply than is necessary. Enter supporters counter that the world does not operate quite so neatly—with only significant independent variables acting on the dependent variable. Once again, the reader is invited to consult the many theoretical treatises on this subject on which the authors support Stepwise for its efficiency and clarity.

3. If Enter had been used instead of Stepwise, the equation would have been: $y = .561 + .415x_1 + .008x_2 + .00000058x_3$, where $x_3 =$ income. Adjusted *R*-squared, which should be used with Enter because a nonsignificant variable has been included in the equation, equals .198.



CHAPTER THIRTEEN

PREPARING AN EFFECTIVE FINAL REPORT

The final report is the vehicle for communicating to the audience the conclusions and recommendations derived from the study. It is integral to the survey research process as a whole; therefore, the research process should not be considered complete until the final report has been prepared and disseminated. Within the report, the analysis of the data, including tables, graphs, and other statistical presentations, should be well organized and clearly explained so that the intended audience can comprehend the essential findings of the study. This chapter suggests an appropriate format and useful guidelines for preparing a formal report of survey research findings.

Report Format

Several fundamental considerations must be taken into account as the final report is prepared.

The Title

The report should have a clear and succinct title that identifies the focus of the research. Frequently a subtitle can help clarify the subject matter of the

research further. For example, a research report concerning growth control limitations in New York State was titled, "Issues Concerning Economic Growth: A Critique of Oswego County's Proposed Countywide Impact Fee Program." The title should appear on the cover of the report, along with the names and affiliations of the authors. The date of report dissemination should also be included, as should the client or sponsor for whom the study was conducted. Exhibit 13.1 is an example of a well-constructed cover page. The title should also appear within the report, either on a separate title page or at the top of the first page of the report. We recommend that the cover page of the report include color along with a tasteful design. The graphic could be an appropriate logo or simply a depiction that has a reasonable association with the sponsor or subject of the study.

EXHIBIT 13.1. EXAMPLE OF A COVER PAGE.

Issues Concerning Economic Growth: A Critique of Oswego County's Proposed Countywide Impact Fee Program

Prepared by XYZ Research Corporation for the Oswego County Taxpayers Association

March 2014

Executive Summary

A short summary of findings at the beginning of the report is helpful to readers. This summary can serve as a source of reference after the report has been read, and it can also serve as the basic source of information about the report itself for audiences that are interested in only the main findings and conclusions, not the details of the subject matter. This executive summary can be as short as one page or as long as ten pages, depending on the length and complexity of the report. We have found that three to five pages is the desirable length. The summary should briefly introduce the report and indicate the methodology employed in the survey but move ahead quickly to outline key findings that are divided into appropriate subsections and presented in a bulleted format. Graphs and charts are generally not included in the summary.

Introduction to the Study

The report should start by providing the audience with some background about the subject matter of the study and by placing the study in appropriate perspective with regard to the history and current significance of the research topic. Major social and political events of the time that the researcher feels have some bearing on the responses should be presented in the introduction. The introduction must contain a clear statement of the specific purpose of the study, including a description of the issues and an explanation of why the researchers decided to pursue the subject in this fashion and at this time.

Review of Preliminary Research

The initial processes undertaken by the researcher in identifying the research focus and helping to develop the actual research instrument should be summarized. This includes a review of existing literature consulted and a discussion of the key groups and individuals who participated in the development of the information base from which the pretest and draft questionnaire evolved. The specific questions at issue or research hypotheses to be tested should be stated and shown as having been directly derived from this preliminary research. The hypotheses should be presented in the context of the information required from the study. That is, the researcher should indicate what the research is designed to discover that was heretofore unknown.

Method of Research

It is important to include an explanation of the methodology that was employed to obtain and analyze the data. For sample survey research, there are three methodological issues: sample selection, survey procedure, and data analysis.

Sample Selection.

The report should detail the procedures employed in selecting the sample. This discussion must include an explanation of how an appropriate working population was identified to represent the general population. The determination of this sampling frame should be accompanied by a discussion of any potential systematic biases. Also to be included in this discussion are the determination of sample size (specifying the level of confidence and margin of error) and the specific sampling method employed in the selection of the final sample.

Survey Procedure.

The survey method should be discussed, including the recruitment and selection of interviewers for telephone and in-person surveys, procedures employed in the initial mailing for mail-out surveys, the web format and method of dissemination employed, follow-up procedures, response rates, and the time frame of the study. Patterns of biases, including nonresponse bias, that may have been identified in the interviewing process must be identified and explained and their potential effect on survey results indicated.

Data Analysis.

The researcher should briefly describe the statistical methods used in data analysis, including all applicable tests of significance and measures of association. Included in this section are explanations of the meaning and importance of these tests and measures. The margin of error should be mentioned once again in this section, as should the level of confidence, which is generally 95 percent.

Survey Research Findings

The major part of the report consists of the research findings. This section is composed largely of tables and graphs, with appropriate descriptive and analytical statistics accompanied by written explanations of the tabular and graphic results. A table or graph should appear in the text as close to its initial mention as possible. It has been found that this placement of tables and graphs within the text lends itself to increased convenience for readers, who can study the researcher's interpretation of the data while maintaining ready visual access to the data themselves. All charts and tables should be numbered to allow for cross-references within the text of the report. Based on our experience, we have found that clients prefer color charts and graphs to tables. We recommend that the results of each survey question be presented in a color chart.

Contingency tables should be presented at the researcher's discretion within the framework of the research issues under consideration. This discretion should be guided by the relevance of information, the statistical significance of any apparent relationships among the variables, and the relative strength of the variables' association. In most instances, contingency tables can be placed in an appendix if the researcher feels they are required at all. We find that significant relationships, detected through contingency tables, can readily be reported in the text in the form of bullet points. For example, a significant relationship between gender and opinion regarding desalination can be reported as follows:

• Men (40 percent) favor the use of desalinated water for drinking purposes more so than do women (24 percent).

This bullet point would follow the chart that reports general opinion about desalination for the entire sample.

Focus Group Findings

When focus groups are used as part of the research process, the findings from these discussions must be incorporated into the final report. This portion of the report should include a description of the number of sessions; the key characteristics of the participants; and the date, time, and place of each session. The structured question format used for the sessions should be presented, and the notes taken at each focus group session, refined by reviewing the videotape of the session, should be recorded in the form of summary minutes. These minutes constitute a listing, according to subject matter, of the key remarks offered in the session. Finally, a composite summary of the findings from all the sessions should be provided, with common themes particularly highlighted.

Conclusions

The report should conclude with a strong section that draws implications from the findings, indicates relationships and trends among the various tables and graphs, and relates the findings from the study to any relevant previous studies or literature. When appropriate, policy recommendations Preparing an Effective Final Report

should be put forth, and opportunities for further research should be discussed.

Bibliography

Important publications and documents consulted during the research process should be listed in a bibliography at the end of the report.

Appendixes

Certain information should be attached to the report in the form of an appendix. Such material always includes a copy of the survey instrument itself and a full set of frequency distributions. Other potential appendixes may be a list of verbatim open-ended responses and detailed explanations of certain statistical techniques or sampling procedures, including all applicable mathematical equations (which are much better placed in the appendix than in the body of the report itself).

There is a considerable amount of discretion involved in choosing the material to include in an appendix. The overriding principle is to provide material that the researcher feels would be beneficial to readers but would tend to compromise the readability of the report. This allows the researcher to maintain a pleasant communicative flow in the report itself while still providing all the important information to readers.

Specific Considerations for Formally Reporting Survey Results

Within the report framework, there are further issues, particularly with regard to the format and style of writing, that the researcher should address in the preparation of the final report. These issues can be organized into two general categories: (1) vocabulary, jargon, and statistical notation and (2) reporting of numerical detail.

Vocabulary, Jargon, and Statistical Notation

The audience or client must be considered in deciding the writing style and the extent of professional vocabulary to be used. If a report is prepared for a technically oriented audience (e.g., engineers, doctors, scientists, or accountants), the researcher may feel more comfortable using terms specific to their particular discipline. The more general or diversified the audience, however, the less technical the language should be, as long as meaning and substance are not sacrificed. An example of a general audience would be the people who would read a research report summarizing the results of a community public opinion survey.

The report should not rely on statistics and statistical notation as substitutes for descriptions of the relationships involved. For example, it is more informative to an audience to say that there is a moderate association between the ethnicity and income of the citizens in a community than to write only that the ethnicity/income Cramér's V = .35. In general, it is better to use descriptive words (e.g., *mean, test of significance,* and *measure of association* or *correlation*) than to rely entirely on statistical notations such as \bar{x} , χ^2 , or V. Statistical notations can be used in the written portion of the report, however, but should follow immediately after the technique's mention. For instance, when chi square is mentioned, it can be followed by the parenthetical reference (χ^2), as indicated in the following example: "The chi-square test of significance (χ^2) for the relationship between political party affiliation and sex of the respondent did not establish the existence of a statistically significant relationship between the two variables."

Word choice is very important. Be careful not to give the impression that survey findings are universal. Instead of writing "The people feel that . . . ," write, "Most people (76 percent) feel that . . . ," because it is highly unlikely that all people have the same opinion. Be careful when using words such as *strong* or *significant*; these words and certain others connote statistical relationships that may mislead the reader. It is less confusing to indicate that a particular issue has "substantial support" rather than "significant support" when reporting results. This is especially true in reporting frequency distributions. The researcher must also be careful to avoid the use of emotive or judgmental words—words that denote surprise, discomfort, or displeasure. For example, a report should not contain the phrase, "It was particularly upsetting to find that . . ."

For ease of reading, footnotes should be used sparingly. Important material should be incorporated into the body of the text as much as possible. Reference citations should always be embodied within the text in accordance with the (author, year, page) format. Each citation should correspond to a full bibliographical reference at the end of the report. Content notes (explanatory digressions that the researcher believes would tend to obstruct the flow of the text) should be used as sparingly as possible, but when they are used, they should be placed at the bottom of the page as a footnote or at the end of the chapter as an endnote for ready reference.

Reporting of Numerical Detail

The written portion of the report should indicate percentages, fractions, or ratios rather than absolute frequencies. That is, rather than reporting that 475 Democrats oppose gun control, it is more informative to say that approximately 66 percent, or nearly two-thirds, of the Democrats surveyed oppose gun control. When the overall sample is quite small or a particular subgroup within the sample is small, the researcher should report both the percentages and the corresponding frequencies. For example, rather than reporting that "40 percent of African American office workers favor a change in their union contract," it is more appropriate to report that "in a survey of twenty office workers, ten of whom are African American, four African American workers (40 percent) favor a change in their union contract." In this way, the reader will not be misled into believing that this particular finding is more substantial than it actually is.

Use whole numbers and common fractions whenever possible. Instead of reporting that "men favor an issue more than women by a ratio of 1.85 to 1," it is better to say that "nearly twice as many men as women favor this issue (1.85:1)." This approach lends itself to significantly more pleasant reading, while simultaneously providing the audience with the specific numerical detail. Similarly, it is better to report that approximately one-fourth of a population expressed a certain attitude than that seven thirty-seconds did (which may be more accurate but is much more difficult to translate into a commonly understood quantity).

Research reports should predominantly use charts and tables to report results, with charts used more frequently and tables reserved for more complex findings. Charts and graphs will help the reader recognize the findings much more easily and in a way that is more pleasing. Color and three-dimensional effects tend to promote this advantage even more. For example, figure 13.1 shows that almost three-fourths of these respondents favor, at least somewhat, the building of a seawater desalination facility. Figure 13.2 indicates ratings of police response to emergency and nonemergency calls. Figure 13.2 makes use of a five-point scale from excellent to very poor (where 1 = excellent and 5 = very poor) and also indicates mean ratings based on that scale. It can easily be seen that respondents rate emergency calls higher than nonemergency calls. This is derived from means of 2.0 and 2.6, respectively, and from 75 percent good or better ratings for emergency calls versus 54 percent for nonemergency calls.

Charts and graphs can be more complex, such as figure 13.3, which depicts mean satisfaction ratings for various characteristics of bus service.



FIGURE 13.1. FAVOR OR OPPOSE BUILDING SEAWATER DESALINATION FACILITIES.





The figure includes mean ratings on a 1 to 5 scale and percentages that indicate the higher trip ratings for each characteristic. Readers can see with relative ease that route convenience and safety are more satisfactory to riders than are waiting time and the timeliness of the buses.

Another more complex feature of graphical depictions is the *call out*, which provides more detailed information about a specific aspect of the chart or graph. Figure 13.4 shows (in the stacked column on the right) that among the 61 percent of all households that have automatically controlled

Convenience of Route2.267Safety Waiting for/Riding Buses2.265Driver Courtesy2.458Overall Bus Service2.456Buses Stop and Do Not Pass By2.555Travel Time on Bus2.554Cost of Fare Seats/Space on Bus2.648Availability of Seats/Space on Bus2.748Availability of Schedules and Route Info2.748Buses on Time Waiting for Bus2.743Time Spent Waiting for Bus2.843	Bus Features	Ratings	Percent Indicating Ratings 1 and 2
Safety Waiting for/Riding Buses2.265Driver Courtesy2.458Overall Bus Service2.456Buses Stop and Do Not Pass By2.555Travel Time on Bus2.554Cost of Fare2.648Availability of Seats/Space on Bus2.748Availability of Schedules 	Convenience of Route	2.2	67
Driver Courtesy2.458Overall Bus Service2.456Buses Stop and Do Not Pass By2.555Travel Time on Bus2.554Cost of Fare2.552Availability of Seats/Space on Bus2.648Availability of Schedules and Route Info2.748Buses on Time2.743Time Spent Waiting for Bus2.843	Safety Waiting for/Riding Buses	2.2	65
Overall Bus Service2.456Buses Stop and Do Not Pass By2.555Travel Time on Bus2.554Cost of Fare2.552Availability of Seats/Space on Bus2.648Availability of Schedules and Route Info2.748Buses on Time2.743Time Spent 	Driver Courtesy	2.4	58
Buses Stop and Do Not Pass By2.555Travel Time on Bus2.554Cost of Fare2.552Availability of Seats/Space on Bus2.648Availability of Schedules and Route Info2.748Cleanliness Inside Bus2.748Buses on Time Waiting for Bus2.843Time Spent Waiting for Bus2.843	Overall Bus Service	2.4	56
Travel Time on Bus2.554Cost of Fare2.552Availability of Seats/Space on Bus2.648Availability of Schedules and Route Info2.748Cleanliness Inside Bus2.748Buses on Time Waiting for Bus2.843Time Spent Waiting for Bus2.843	Buses Stop and Do Not Pass By	2.5	55
Cost of Fare2.552Availability of Seats/Space on Bus2.648Availability of Schedules and Route Info2.748Cleanliness Inside Bus2.748Buses on Time2.743Time Spent Waiting for Bus2.84354321	Travel Time on Bus	2.5	54
Availability of Seats/Space on Bus 2.6 48 Availability of Schedules and Route Info 2.7 48 Cleanliness Inside Bus 2.7 48 Buses on Time 2.7 43 Time Spent 2.8 43 Vaiting for Bus 2.8 43	Cost of Fare	2.5	52
Availability of Schedules and Route Info 2.7 48 Cleanliness Inside Bus 2.7 48 Buses on Time 2.7 43 Time Spent Waiting for Bus 2.8 43 5 4 3 2 1	Availability of Seats/Space on Bus	2.6	48
Cleanliness Inside Bus 2.7 48 Buses on Time 2.7 43 Time Spent 2.8 43 Vaiting for Bus 1 1 5 4 3 2 1	Availability of Schedules and Route Info	2.7	48
Buses on Time 2.7 43 Time Spent 2.8 43 Waiting for Bus 1 1 5 4 3 2 1	Cleanliness Inside Bus	2.7	48
Time Spent 2.8 43 Waiting for Bus 1 1 5 4 3 2 1	Buses on Time	2.7	43
5 4 3 2 1	Time Spent Waiting for Bus	2.8	43
Very Dissatisfied Very Satisfied	Very Dis	5 4 3 2 1 satisfied Very Sati	sfied

FIGURE 13.3. MEAN SATISFACTION RATINGS FOR VARIOUS FEATURES OF BUS SERVICE.

sprinkler systems for their lawns (as shown in the pie chart), 43 percent of the 61 percent adjust their sprinklers four or more times per year.

Sometimes the information to be conveyed is too intricate or voluminous for portrayal in a chart or graph. Such situations require tables. For instance, table 13.1 shows potential uses for recycled water, the overall mean rating for each such use, the groups that favor that particular use of recycled water more (including their mean ratings), and the groups that are more opposed (including their means). The table indicates that use





of recycled water is most favored for freeway landscaping and golf courses, with higher-income, better-educated, and white respondents more in favor than respondents of lower income, less education, and Latinos.

Once again, it is important to state that all charts and tables that are included in the report should be numbered and titled and receive some comment in the text. The researcher should avoid reporting an excessive amount of detail and should instead selectively report the few salient details that bear most directly on the focus of the study. Rather than reporting detail as follows, "Concerning mode of transportation to work, 68.2 percent use automobiles, 23.7 percent use public transit, 4.8 percent walk, 2.9 percent ride bicycles, and 0.4 percent use taxis and other dial-a-ride services," the researcher should report the information by stating that "over two-thirds of the population (68 percent) use automobiles, and nearly one-fourth (24 percent) use public transit." If the research has a particular focus regarding one of the other modes of transportation, it should be commented on, but otherwise, the written portion of the report should highlight only the critical findings.
		Statistically Significant Differences			
Potential Uses of Recycled Water	Overall Mean Index	Stronger Suppor Weaker Oppositio	t/ on	Weaker Support/ Stronger Opposition	n
Freeway/golf landscape	1.35	Whites Income \$100K+ Some college/more	1.25 1.18	Hispanics/Latinos Mixed ethnicities Income under \$25K High school/less	1.61 1.67 1.63 1.66
Toilets in new buildings	1.57	Ethnicities other than Hispanic/Latino and mixed ethnicity Some college/more	1.48 1.49	Hispanics/Latinos Mixed ethnicities High school diploma or less	1.88 1.87 1.92
Electronics manufacturing	1.66	Some college/more	1.60	High school/less	1.90
Industrial manufacturing processes	1.69	Ages 45 or more Whites Asians Some college/more	1.52 1.49 1.59 1.61	Ages 18–44 Blacks/African Americans Hispanics/Latinos High school/less	1.88 2.00 2.08 2.12
Sports fields/ parks	1.70	Asians Whites Males	1.43 1.60 1.56	Hispanics/Latinos Blacks/African Americans Females	2.04 2.27 1.84
Multifamily common areas	1.76	1–2 person households Whites Asians Own Males	1.56 1.55 1.59 1.65 1.63	13+ person households Hispanics/Latinos Blacks/African Americans Rent Females	1.97 2.49 2.35 2.01 1.90
Residential front yards	1.92	Asians Whites	1.73 1.60	Hispanics/Latinos	2.38
Agricultural irrigation	2.15	Asians Whites Males	1.96 1.78 1.95	Hispanics/Latinos Females	2.86 2.36

TABLE 13.1. STATISTICALLY SIGNIFICANT DIFFERENCES FOR VARIOUS POTENTIAL USES OF RECYCLED WATER.

(scale: 1 =strongly favor to 5 =strongly oppose)



RESOURCE A

TABLE OF AREAS OF A STANDARD NORMAL DISTRIBUTION

(B) Proportion of Area between Mean and Z	-	(A) Z	(B) Proportion of Area between Mean and Z
0.0000	-	0.16	0.0636
0.0040		0.17	0.0675
0.0080		0.18	0.0714
0.0120		0.19	0.0753
0.0160		0.20	0.0793
0.0199		0.21	0.0832
0.0239		0.22	0.0871
0.0279		0.23	0.0910
0.0319		0.24	0.0948
0.0359		0.25	0.0987
0.0398		0.26	0.1026
0.0438		0.27	0.1064
0.0478		0.28	0.1103
0.0517		0.29	0.1141
0.0557		0.30	0.1179
0.0596		0.31	0.1217
	(B) Proportion of Area between Mean and Z 0.0000 0.0040 0.0080 0.0120 0.0160 0.0199 0.0239 0.0279 0.0319 0.0359 0.0359 0.0398 0.0438 0.0438 0.0478 0.0517 0.0557 0.0596	(B) Proportion of Area between Mean and Z 0.0000 0.0040 0.0080 0.0120 0.0160 0.0199 0.0239 0.0239 0.0279 0.0319 0.0359 0.0359 0.0398 0.0438 0.0438 0.0478 0.0517 0.0557 0.0596	(B) Proportion of Area between Mean and Z (A) Z 0.0000 0.16 0.0040 0.17 0.0080 0.18 0.0120 0.19 0.0160 0.20 0.0199 0.21 0.0239 0.22 0.0279 0.23 0.0319 0.24 0.0359 0.25 0.0398 0.26 0.0478 0.28 0.0517 0.29 0.0557 0.30 0.0596 0.31

(continued)

Resource A

(A) Z	(B) Proportion of Area between Mean and Z	(A) Z	(B) Proportion of Area between Mean and Z
0.32	0.1255	0.66	0.2454
0.33	0.1293	0.67	0.2486
0.34	0.1331	0.68	0.2517
0.35	0.1368	0.69	0.2549
0.36	0.1406	0.70	0.2580
0.37	0.1443	0.71	0.2611
0.38	0.1480	0.72	0.2642
0.39	0.1517	0.73	0.2673
0.40	0.1554	0.74	0.2704
0.41	0.1591	0.75	0.2734
0.42	0.1628	0.76	0.2764
0.43	0.1664	0.77	0.2794
0.44	0.1700	0.78	0.2823
0.45	0.1736	0.79	0.2852
0.46	0.1772	0.80	0.2881
0.47	0.1808	0.81	0.2910
0.48	0.1844	0.82	0.2939
0.49	0.1879	0.83	0.2967
0.50	0.1915	0.84	0.2995
0.51	0.1950	0.85	0.3023
0.52	0.1985	0.86	0.3051
0.53	0.2019	0.87	0.3078
0.54	0.2054	0.88	0.3106

0.89

0.90

0.91

0.92

0.93

0.94

0.95

0.96

0.97

0.98

0.99

0.3133

0.3159

0.3186

0.3212

0.3238

0.3264

0.3289

0.3315

0.3340

0.3365

0.3389

0.55

0.56

0.57

0.58

0.59

0.60

0.61

0.62

0.63

0.64

0.65

0.2088

0.2123

0.2157

0.2190

0.2224

0.2257

0.2291

0.2324

0.2357

0.2389

0.2422

Resource A

(A) Z	(B) Proportion of Area between Mean and Z	(A) Z	(B) Proportion of Area between Mean and Z
1.00	0.3413	 1.34	0.4099
1.01	0.3438	1.35	0.4115
1.02	0.3461	1.36	0.4131
1.03	0.3485	1.37	0.4147
1.04	0.3508	1.38	0.4162
1.05	0.3531	1.39	0.4177
1.06	0.3554	1.40	0.4192
1.07	0.3577	1.41	0.4207
1.08	0.3599	1.42	0.4222
1.09	0.3621	1.43	0.4236
1.10	0.3643	1.44	0.4251
1.11	0.3665	1.45	0.4265
1.12	0.3686	1.46	0.4279
1.13	0.3708	1.47	0.4292
1.14	0.3729	1.48	0.4306
1.15	0.3749	1.49	0.4319
1.16	0.3770	1.50	0.4332
1.17	0.3790	1.51	0.4345
1.18	0.3810	1.52	0.4357
1.19	0.3830	1.53	0.4370
1.20	0.3849	1.54	0.4382
1.21	0.3869	1.55	0.4394
1.22	0.3888	1.56	0.4406
1.23	0.3907	1.57	0.4418
1.24	0.3925	1.58	0.4429
1.25	0.3944	1.59	0.4441
1.26	0.3962	1.60	0.4452
1.27	0.3980	1.61	0.4463
1.28	0.3997	1.62	0.4474
1.29	0.4015	1.63	0.4484
1.30	0.4032	1.64	0.4495
1.31	0.4049	1.65	0.4505
1.32	0.4066	1.66	0.4515
1.33	0.4082	1.67	0.4525
			(continued)

Resource A

3	08
-	•••

(A) Z	(B) Proportion of Area between Mean and Z	(A) Z	(B) Proportion of Area between Mean and Z
1.68	0.4535	2.02	0.4783
1.69	0.4545	2.03	0.4788
1.70	0.4554	2.04	0.4793
1.71	0.4564	2.05	0.4798
1.72	0.4573	2.06	0.4803
1.73	0.4582	2.07	0.4808
1.74	0.4591	2.08	0.4812
1.75	0.4599	2.09	0.4817
1.76	0.4608	2.10	0.4821
1.77	0.4616	2.11	0.4826
1.78	0.4625	2.12	0.4830
1.79	0.4633	2.13	0.4834
1.80	0.4641	2.14	0.4838
1.81	0.4649	2.15	0.4842
1.82	0.4656	2.16	0.4846
1.83	0.4664	2.17	0.4850
1.84	0.4671	2.18	0.4854
1.85	0.4678	2.19	0.4857
1.86	0.4686	2.20	0.4861
1.87	0.4693	2.21	0.4864
1.88	0.4699	2.22	0.4868
1.89	0.4706	2.23	0.4871
1.90	0.4713	2.24	0.4875
1.91	0.4719	2.25	0.4878
1.92	0.4726	2.26	0.4881
1.93	0.4732	2.27	0.4884
1.94	0.4738	2.28	0.4887
1.95	0.4744	2.29	0.4890
1.96	0.4750	2.30	0.4893
1.97	0.4756	2.31	0.4896
1.98	0.4761	2.32	0.4898
1.99	0.4767	2.33	0.4901
2.00	0.4772	2.34	0.4904
2.01	0.4778	2.35	0.4906

(A) Z	(B) Proportion of Area between Mean and Z	(A) Z	(B) Proportion of Area between Mean and Z
2.36	0.4909	2.70	0.4965
2.37	0.4911	2.71	0.4966
2.38	0.4913	2.72	0.4967
2.39	0.4916	2.73	0.4968
2.40	0.4918	2.74	0.4969
2.41	0.4920	2.75	0.4970
2.42	0.4922	2.76	0.4971
2.43	0.4925	2.77	0.4972
2.44	0.4927	2.78	0.4973
2.45	0.4929	2.79	0.4974
2.46	0.4931	2.80	0.4974
2.47	0.4932	2.81	0.4975
2.48	0.4934	2.82	0.4976
2.49	0.4936	2.83	0.4977
2.50	0.4938	2.84	0.4977
2.51	0.4940	2.85	0.4978
2.52	0.4941	2.86	0.4979
2.53	0.4943	2.87	0.4979
2.54	0.4945	2.88	0.4980
2.55	0.4946	2.89	0.4981
2.56	0.4948	2.90	0.4981
2.57	0.4949	2.91	0.4982
2.58	0.4951	2.92	0.4982
2.59	0.4952	2.93	0.4983
2.60	0.4953	2.94	0.4984
2.61	0.4955	2.95	0.4984
2.62	0.4956	2.96	0.4985
2.63	0.4957	2.97	0.4985
2.64	0.4959	2.98	0.4986
2.65	0.4960	2.99	0.4986
2.66	0.4961	3.00	0.4987
2.67	0.4962	3.01	0.4987
2.68	0.4963	3.02	0.4987
2.69	0.4964	3.03	0.4988
			(continued)

<i></i>	(B)
(A) 7	Proportion of Area
3.04	0.4988
3.05	0.4989
3.06	0.4989
3.07	0.4989
3.08	0.4990
3.09	0.4990
3.10	0.4990
3.11	0.4991
3.12	0.4991
3.13	0.4991
3.14	0.4992
3.15	0.4992
3.16	0.4992
3.17	0.4992
3.18	0.4993
3.19	9.4993
3.20	0.4993
3.21	0.4993
3.22	0.4994
3.23	0.4994
3.24	0.4994
3.25	0.4994
3.30	0.4995
3.35	0.4996
3.40	0.4997
3.45	0.4997
3.50	0.4998
3.60	0.4998
3.70	0.4999
3.80	0.4999
4.00	0.49997
4.01–∞	0.5000



RESOURCE B

GLOSSARY

- **AAPOR Ethical Standards in Survey Research:** A body of ethical guidelines for survey researchers and those who conduct opinion polls established by the American Association of Public Opinion Research.
- **Ambiguous words and phrases:** Confused wording that a respondent can reasonably interpret in more than one way.
- **Analysis of variance:** A test of statistical significance for use when the dependent variable is interval and there are more than two categories of the independent variable.
- **ANOVA table:** The summary presentation of the analysis of variance test. **Arithmetic mean:** See Mean.
- **Cell phone interviews**: Generally refers to a portion of a telephone sample dedicated to contacting respondents who are cell-phone-only users.
- **Chi-square test:** A statistical significance test used for variables that have been organized into categories and presented in a contingency table.
- **Closed-ended questions:** Questions that provide a fixed list of alternative responses and ask the respondent to select one or more of the alternatives as indicative of the best possible answer.
- **Cluster (multistage) sampling:** The process of randomly selecting a sample in a hierarchical series of stages represented by increasingly narrow groups from the working population.

- **Coefficient of determination** (\mathbb{R}^2): The proportion of the variance in the dependent variable that is explained by a simple linear regression line.
- **Computer-assisted telephone interviewing** (CATI): State-of-the-art technology in telephone interviewing where questions on the survey are programmed to appear on the computer screen designed to be read by the interviewer; technology enables the interviewer to follow complex skip patterns.
- **Computer software packages:** Prepackaged statistical programs that facilitate analysis of survey data.
- **Confidence interval:** A probabilistic estimate of the true population mean or proportion based on sample data.
- **Contingency table:** A tabular display presenting the relationship between two variables.
- **Control variable:** The variable that is held constant in a three-way cross-tabulation in order to display three variables by using contingency tables.
- **Convenience sampling:** Type of nonprobability sample in which interviewees are selected according to their presumed resemblance to the working population and their ready availability.
- **Correlation ratio (E2):** The proportion of the total variance that is explained by the independent variable in the analysis of variance test.
- **Cramér's V:** A measure of association used for categorical data that is calculated directly from the chi-square statistic.
- **Data entry:** The process of entering the raw data from completed questionnaires into a computer.
- **Decile deviation:** The arithmetic difference between the 10th and 90th percentiles.
- **Degrees of freedom:** Number of values in a data set that are free to vary. Once these values are known, all remaining values are also known.
- **Dependent variable:** The variable that is being explained or is dependent on another variable.
- **Difference of proportions test:** A test of statistical significance, with the dependent variable measured in percentages and with only two categories of the independent variable.
- **Direct measurement:** Information-gathering technique that involves the direct counting, measuring, or testing of data.
- **Double-barreled questions:** Questions that inappropriately elicit responses for two or more issues at the same time.
- **Draft questionnaire:** A draft of the survey instrument that is prepared at the conclusion of the preliminary information-gathering process and prior to implementation of the pretest.

Glossary

- **Dummy variable:** The use of 0 and 1 to convert a noninterval variable into one that can be used and interpreted by regression analysis.
- **Emotional words and phrases:** Words and phrases that elicit emotional responses rather than reasoned and objective answers.
- Eta: The measure of association for the analysis of variance test.
- *F* ratio: From analysis of variance, the ratio of the variance explained by the independent variable to the variance that remains unexplained.
- **Filter or screening questions:** Questions that require some respondents to be screened out of certain subsequent questions or disqualified from participating in the survey at all.
- **Finite population correction:** Adjustment of the standard error in order to account for small population sizes.
- **Fixed-alternative response categories:** A list of response choices associated with closed-ended questions.
- **Focus group:** A semistructured discussion among individuals who are deemed to have some knowledge of or interest in the issues associated with the research study.
- **Frequency distribution:** A summary presentation of the frequency of response for each category of the variable.
- **Frequency polygon (line graph):** Graphic tool for interval data in ordinal categories. A line connects points representing the midpoint of the class interval (horizontal axis) and the frequency of response (vertical axis).
- **Gamma:** A measure of association for two variables, which are either ordinal or interval.
- **General population:** The theoretical population to which the researcher wishes to generalize the study findings.
- **Harmonic mean:** A measure of central tendency that represents the *k*th root of the product of the values for each category of a variable, where k = the number of categories.
- **Inappropriate emphasis:** The use of boldface, italicized, capitalized, or underlined words or phrases within the context of a question that may serve to bias the respondent.
- **Independent samples** *t* **test:** A test of statistical significance, with the dependent variable on the interval scale and two categories of the independent variable representing independent samples.
- Independent variable: A variable that explains changes in another variable.
- **In-person interviews:** Interviews in which information is solicited directly from respondents in a face-to-face situation.

- **Intercept survey:** An in-person survey interview conducted in the course of the potential respondent's normal behavior pattern and without prearrangement of an interview time with that respondent.
- **Interquartile range:** The arithmetic difference between the 25th and 75th percentiles.
- **Interval scale data:** Data involving a level of measurement that establishes an exact value for each category of the variable in terms of specific units of measurement.
- **Interviewer instructions:** Explicit instructions to the survey administrator concerning how to properly administer and complete the questionnaire.
- **Lambda:** A measure of association for two variables, at least one of which is nominal in scale.
- **Level of confidence:** Degree of confidence associated with the accuracy of the measurements derived from sample data.
- **Level of wording:** A guideline for the development of questions that instructs the researcher to be cognizant of the population to be surveyed when choosing the words, colloquialisms, and jargon to be used in the questions.
- **Likert scale:** A scaled response continuum measured from extreme positive to extreme negative (or vice versa) in five, seven, or nine categories.
- **Mail-out survey:** Printed questionnaires disseminated through the mail to a predesignated sample of respondents. The respondents are asked to complete the questionnaire on their own and return it by mail to the researcher.
- **Manipulative information:** Explanatory information in a questionnaire that is intended to provide necessary background and perspective but serves instead to bias the respondent.
- Margin of error: The level of sampling accuracy obtained.
- **Mean (arithmetic mean):** The mathematical center of the data, taking into account not only the location of the data (above or below the center) but also the relative distance of the data from the center.
- **Measure of association:** A measure of the strength and direction of the relationship between two variables.
- **Measure of central tendency:** Statistic that provides a summarizing number to characterize what is "typical" or "average" for particular data. Mean, mode, and median are the three measures of central tendency.
- **Median:** The value of the variable that represents the midpoint of the data. Half of the data will have values on one side of the median, and half will have values on the other side.

Glossary

- **Mode:** The category or value of the data that is characterized as possessing the greatest frequency of response.
- **Multiple coefficient of determination (R²):** The proportion of the variance in the dependent variable that is explained by a regression line with two or more independent variables.
- **Multiple regression:** The statistical technique that identifies the line or curve of best fit for an interval dependent variable and two or more interval independent variables.
- **Nominal scale data:** Data that involve a level of measurement that simply identifies or labels the observations into categories.
- **Nonprobability sampling:** A method of sample selection in which the probability of any particular respondent's selection for inclusion in the sample is not known.
- **Nonresponse bias:** The departure of the sample statistics from their true population values owing to the absence of responses from some portion of the population that differs systematically from those portions of the populations that did respond.
- **Normal distribution:** Data distributed in the form of the symmetrical, bellshaped curve, where the mode, median, and arithmetic mean have the same value.
- **Null hypothesis:** The hypothesis posed in tests of statistical significance and hypothesis testing indicating that whatever relationship appears to exist in the data is merely sampling error and not indicative of a real relationship.
- **Observation:** An information-gathering technique that involves the direct study of behavior, as it occurs, by watching the subjects of the study without intruding upon them.
- **One-tail test:** See Single- and dual-direction research hypothesis testing.
- **Open-ended questions:** Questions that have no preexisting response categories and thereby permit the respondent to answer in his or her own words.
- **Ordinal scale data:** Data involving a level of measurement that seeks to rank categories in terms of the extent to which these categories represent the variable.
- **Paired samples** *t* **test:** A test of statistical significance, with the dependent variable on the interval scale and two categories of the independent variable representing samples drawn from the same population at different points in time.
- **Pearson's** *r* **correlation:** The measure of association for two interval scale variables.

- **Percentile:** Any percentile (k) is that value at which k percent of values in the data set are less than the kth percentile value and (1 k) percent of values are greater.
- **Phi:** A measure of association that is a variation on Cramér's V and used only for contingency tables where at least one of the variables contains only two categories.
- **Post hoc tests:** Multiple comparison of means tests by which the specific differences among categories of the independent variable can be identified in association with the analysis of variance test.
- **Postcoding:** The process of coding responses to open-ended questions or other questions that are not coded as part of the precoding process.
- **Precoding:** The placement of numeric codes for each category of response at the time that the questionnaire is prepared in final form for administration.
- **Pretest:** A small-scale implementation of the draft questionnaire used to assess such critical factors as questionnaire clarity, comprehensiveness, and acceptability.
- **Probability sample:** Sample with the following two characteristics: (1) probabilities of selection are equal for all members of the working population at all stages of the selection process, and (2) sampling is conducted with elements of the sample selected independent of one another. Often referred to as *random samples*.
- **Proportionate reduction in error:** The extent to which the independent variable serves to reduce the error in predicting the dependent variable.
- **Purposive sampling:** Type of nonprobability sampling in which the researcher uses judgment in selecting respondents who are considered to be knowledgeable in subject areas related to the research.
- *q* distribution: The studentized range, *q*, is a statistic used in multiple comparison methods. It is defined as the range of means divided by the estimated standard error of the mean for a set of samples being compared.
- **Questionnaire editing:** The examination of finished questionnaires for accuracy and completeness. Often referred to as the *cleanup process*. Includes postcoding.
- **Quota sampling:** Type of nonprobability sample in which the researcher deliberately selects a sample to reflect the overall population with regard to one or more specific variables that are considered to be important to the study.
- **Random-digit dialing:** Use of randomly drawn telephone numbers for the purpose of contacting potential respondents.

- **Range:** The arithmetic difference between the highest and lowest values in the data.
- **Regression analysis:** See Simple linear regression; multiple regression.
- **Research hypothesis:** The more common phrasing of a hypothesis that a relationship between variables—or a true finding—exists that the researcher can apply to the general population (generalize).
- **Respondent:** The person who replies to the questions in the survey instrument.
- **Response rate:** Percentage of the potential respondents who were initially contacted and completed the questionnaire.
- **Sample survey research:** Survey research conducted by interviewing a small portion of a large population through the application of a set of systematic, scientific, and orderly procedures for the purpose of making accurate generalizations about the large population.
- **Sampling error:** The likelihood that any scientifically drawn sample will contain certain unavoidable differences from the true population of which it is a part.
- **Sampling frame:** The process of obtaining a representative sample population from the general population.
- **Scaled responses:** Alternative responses that are presented to the respondent on a continuum.
- **Scatter plot**: Plotting corresponding values of the independent variable (*x*) and the dependent variable (*y*) when both are interval.
- **Secondary research**: A means of data collection that consists of compiling and analyzing data that already have been collected and exist in usable form.
- **Simple linear regression**: The statistical technique that identifies the line of best fit in a scatter plot of two interval variables.
- **Simple random sampling**: The random selection of members of the working population for inclusion in the eventual sample.
- **Single- and dual-direction research hypothesis testing**: The use of particular *Z* or *t* scores for determining statistical significance is dictated by how the research question or hypothesis is posed. Questions of difference in which the direction of difference is specified are *single-direction questions*; questions in which the direction of difference is not specified are *dual-direction questions*.
- **Single-sample testing (hypothesis testing):** Significance tests (both interval and percentage data) comparing the results of a sample to an established standard.

- **Skewed distribution:** A nonnormal frequency distribution with some extreme values, either high or low, that cause the three measures of central tendency to deviate from one another.
- **Snowball sampling:** A type of nonprobability sample in which the researcher identifies a few respondents and asks them to identify others who might qualify as respondents.
- **Spuriousness:** An apparent relationship between two variables that on further analysis is found to be the result of the interaction of a third variable.
- **Standard deviation:** A measure of data dispersion that depicts how close the data are to the mean of the distribution.
- **Standard error:** The standard deviation of a distribution of sample means, as opposed to the distribution of raw data of a single sample.
- Standardized Zscore: See Zscore.
- **Statistical Package for the Social Sciences (SPSS):** Extensively used statistical software package in the public and private sectors, especially in the academic arena; used as a computer-based supplement in this book.
- **Stratified random sampling:** Separation of the working population into mutually exclusive groups (strata); random samples are then taken from each stratum.
- **Student's** *t* **distribution:** An adjustment to the normal distribution to account for small sample sizes.
- **Sum of squares:** The process of summing squared differences in order to eliminate the offsetting effects of positive and negative differences.
- **Survey administration:** The implementation of the precoded final questionnaire in the survey research process.
- **Survey research:** The solicitation of verbal information from respondents through the use of various interviewing techniques.
- **Systematic random sampling:** Adaptation of the random sampling process that consists of selecting sample members from a list at fixed intervals from a randomly chosen starting point on that list.

t test: See Student's t distribution.

- **Telephone survey:** Information collection through the use of telephone interviews that involve a trained interviewer and selected respondents.
- **Tests of statistical significance:** A series of statistical tests that permit the researcher to identify whether genuine differences exist among variables.
- **Three-way cross-tabulation:** The method by which the relationship among three variables is presented in a series of contingency tables.
- **Two-tail test:** See Single- and dual-direction research hypothesis testing.

Glossary

- **Type I error:** The error associated with making a decision based on the data from the sample.
- **Type II error:** The error associated with being overly conservative and not acting on the data from the sample.
- **Tukey's HSD (Honestly Significant Difference):** A post hoc test for the analysis of variance.
- **Unit of analysis:** The element (person, household, or organization) of the population that represents the focus of the research study.
- **Variable fields:** Computer spaces allocated and corresponding to a variable in the questionnaire.
- **Venting questions:** Questions in which the respondent is asked to add any information, comments, or opinions that pertain to the subject matter of the questionnaire but have not necessarily been addressed throughout the main body of the questionnaire.
- Web-based survey: A survey administered through the Internet.
- **Web panels:** Large reservoirs or banks of potential survey respondents who are recruited to participate in various online surveys.
- **Working population:** An operational definition of the general population that is representative of that population and from which the researcher is reasonably able to identify as complete a list as possible of its members.
- **Z** score: The conversion of calculated standard deviations into standard units of distance from the mean in normal distributions.

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INDEX

Page references followed by fig indicate an illustrated figure; by t indicate a table; by w indicate a worksheet.

A

Ambiguous words and phrases, 61–62 American Association of Public Opinion Research (AAPOR) code of ethics, 34–35 American Institute of Public Opinion, 8

- ANOVA (analysis of variance) test: ANOVA table for overall satisfaction with bus service, 256–257; ANOVA table for public works employees, 249*fig*; calculation of sum of squares, 247–248; calculation of variances and *F* ratio, 248–252*e*, 268– 269*fig*; choosing the appropriate statistical test or measure, 290*e*; correlation ratio *E2* and *Eta*, 250; description and calculating the, 245–247, 249; post hoc tests, 250, 253–254*e*, 257–259; public works employee's earnings, 247*w*; for significance of the regression line, 265–266; significance of the regression line, 265–266; SPSS used for, 255*e*–259; statistical significance of, 249–250. *See also* Statistical significance
- ANOVA table: description and function of, 249; for frequency riding bus, 287–288; for overall satisfaction with bus service, 256–257; for public works employees, 249*fig*; for regression line of subsidy and value increases, 269*fig*; showing SPSS testing of statistical significance of regression line, 276–277, 278

Appendixes (final report), 297 Arithmetic mean. See Mean

B

Basic response category format, 70–71 Biases: nonresponse, 195, 196–198; systematic, 179 Bibliography, 297

С

Calculation of regression line worksheet, 264w Call out, 300–301

Cell phone only dialing samples, 20–22 Centers for Disease Control (CDC) cell phone report, 20

- Chi-square matrix (*N*= 500) worksheet, 213*w*-214 Chi-square test of significance: additional considerations for, 219–220; *Yates correction* employed for, 220; chi-square matrix (*N*= 500) worksheet, 213*w*-214; choosing the appropriate statistical test or measure, 290*e*; critical values of the chisquare distribution, 214–215*e*; description and function of, 212–213; *Fisher's exact test* employed for, 220; number of years as customer of the Richmond water district by area of residence, 205*t*, 216; opinion concerning capital punishment by political party, 212*t*, 216; SPSS used to test the, 221–226
- Closed-ended questions: advantages of using, 50–52; basic response category format, 70–71; disadvantages of using, 52–53; multiple responses format, 75–77; number of alternative responses

format, 71–72; order of response alternatives format, 74–75; scaled responses format, 77–79; structure of ordinal categories containing interval scale variables format, 72–74

Cluster (multistage) sampling, 184-185

Coefficient of correlation (or Pearson's *r*), 270–271, 290*e*

Coefficient of determination (r^2) , 269–270

- Confidence intervals: description and example of, 147–148; as determinant for large population samples, 167–168; as determinant for small population sample size, 168–171*t*; expressed as proportions, 148–150; as sampling accuracy determinant, 163–165; SPSS's Analyze–Descriptive Statistics–Explore tool for determining, 151 Confusingly phrased questions, 62–63
- Contingency tables: comparing frequency distribution table and, 204; dependent variable as variable being explained in, 205-212; description and function of, 204; independent variable as change agent in the, 205-212; number of years as customer of Richmond water district by area of residence: male respondents, 207t; number of years as customer of Richmond water district by area residence, 205t, 216; number of years as customer of Richmond water district, 205t; overall satisfaction with bus service*recoded frequency cross-tabulation, 211; overall satisfaction with bus service*recoded frequency riding bus cross-tabulation, 209; SPSS used to generate cross-tabulations, 208-209. See also Frequency distributions/tables Continuous variables, 105
- Correlation analysis: multiple regression analysis, 280–281, 282; Pearson's *r* or coefficient of correlation, 270–271, 290*e*; statistical significance of the regression line, 275, 277

Correlation ratio E2, 250

- Cramér's *V*: calculating, 218; choosing the appropriate statistical test or measure, 290*e*; description and example of, 217–218*t*; special case of phi (φ) and interpretation of, 218–219*e*
- Cross-tabulated data analysis: chi-square test of significance for, 212*t*-216, 219–226, 290*e*, Cramér's Vand phi (φ), 217–219*e*, 290*e*; cross-tabulated contingency tables for, 204–212; gamma (γ), 227–230, 290*e*, overall satisfaction with bus service satisfaction: travel time, 231
 Cumulative frequencies, 102

D

Data analysis: choosing the appropriate statistical test or measure, 290*e*, of cross-tabulated data, 203–234; description of, 33; final report's brief description on the, 295; identifying spurious relationships between variables, 259–260; measures of central tendency to ensure accuracy of, 99– 108w, 118–123, 124–130; measures of dispersion to ensure accuracy of, 108–123; regression and correlation, 261–290; scaled frequency distributions to ensure accuracy of, 130–132; testing the difference between means, 235–259. *See also* SPSS (Statistical Package for the Social Sciences) Data coding, 32–33

Data collection: interviews used for, 16–19, 32–34, 200; primary data gathered during, 5. *See also* Questionnaires

Dependent variables: choosing the appropriate statistical test or measure for, 290*e*; SPSS used to generate cross-tabulations of, 208–209; SPSS used to merge nominal and ordinal categories of, 210–212; as variable that is being explained in contingency tables, 205–212. *See also* Independent variables; Variables

Descriptive statistics: measures of central tendency, 99–108*w*, 124–130*fig*; measures of dispersion, 108–123; the normal distribution, 123*fig*–124; scaled frequency distributions, 130–132; SPSS for regression analysis, 275, 277; SPSS to perform multiple regression analysis, 280

Determination of Median Worksheet, 104w-105

- Difference of proportions test, 243-245
- Direct measurement: description of technique, 5; primary data gathered using, 5
- Discrete variables, 105
- Dispersion. See Measures of dispersion
- Double-barreled questions, 63-64
- Dual-direction (or two-tail) questions, 154–157, 159*e*–160*e*
- Dummy (or indicator) variables: regression analysis using, 283–284; SPSS used for regression and, 285–289

Е

Emotional words and phrases, 67-68

- Eta equation, 250, 290e
- Ethical issues, AAPOR code of ethics, 34-35
- Executive summary, 293-294

F

- F ratio: ANOVA table for public works employees to present, 249*fig*; ANOVA table for regression line of subsidy and value increases and, 269*fig*; calculation of variances and, 248–249; determining statistical significance of the, 249–250, 251*e*-252*e*
- Filter or screening questions, 47, 48e

- Final report format: appendixes, 297; bibliography, 297; conclusions, 296–297; example of a cover page, 293*e*, executive summary, 293–294; focus group findings, 296; introduction to the study, 294; method of research, 294–295; review of preliminary research, 294; survey research findings, 295–296; the title, 292–293
- Final report numerical details: call out used to present, 300–301; charts and graphs used to present, 299–300; when to include, 299; whole numbers and common fractions used in, 299
- Final report numerical examples: Favor or Oppose Building Seawater Desalination Facilities, 300*fig*; Frequency of Sprinkler Adjustment by Households with an Automatically Controlled Sprinkler System, 302*fig*; Mean Satisfaction Ratings for Various Features of Bus Service, 301*fig*; Rating of Policy Response to Emergency and Nonemergency Calls, 300*fig*; Statistically Significant Differences for Various Potential Uses of Recycled Water (Scale: 1=Strongly Favor To 5=Strongly Oppose), 303*t*
- Final reports: description and purpose of the, 292; format used for the, 292–297; reporting of numerical detail, 299–303*t*; vocabulary, jargon, and statistical notation used in, 297–298. *See also* Survey research *Fisher's exact test*, 220

Fixed list of telephone numbers samples, 22

- Focus group participants: identifying the critical characteristics of, 83–84; recruiting the, 87–88; sample focus group confirmation e-mail, 89*e*
- Focus group planning: choosing an appropriate facility, 84–86; determining the necessity for financial inducements, 86; establishing the number of focus groups, 84; identifying the critical characteristics of participants, 83–84; scheduling the focus groups, 86–87
- Focus group sessions: conducting the, 90–95; preparing for the, 89–90; structured format for a focus group discussion during, 91*e*–93*e*
- Focus groups: description and information gathering function of, 80–81; planning, 82–87; recruiting participants, 87–89*e*, uses of, 81–82

Focused group depth interviews, 80-81

Frequency distributions/tables: comparing contingency tables to, 204; description and function of, 204; on number of transfers during trip, 117; on satisfaction of travel time, 117–118; scaled, 130–132; SPSS used to generate, 113–118; on trip purpose, 117. See also Contingency tables

G

Gallup, George, 8 Gallup polls, 8

- Gamma (γ): choosing the appropriate statistical test or measure, 290*e*; description and formula for, 227; for determining educational level by socioeconomic status, 227–228*t*; interpreting calculated, 228–229*e*; SPSS used to test the significance of, 230; testing the statistical significance of, 229*e*–230
- General population (or universe): definition of the, 178; sample taken from, 30; sampling frame derived through use of working population and from, 178–179

Generalizing from sample, 143-145

н

Hypothesis testing: based on a sample, 152–154; calculating the number of standardized Z units, 152–154; single- and dual-direction research questions, 154–157; SPSS for one-sample, 161–162; t tests, 158–162; values of calculated Z scores to determining significance during, 157e. See also Null hypothesis

I

In-person interviews: administration of, 23–25; advantages and disadvantages of, 22–23; description of, 22; example of introduction to, 41*e*

Inappropriate emphasis phrasing, 66-67

- Independent samples *t* test: choosing the appropriate statistical test or measure, 290*e*; description of, 235; promotional examination score by department affiliation example of, 235–237; SPSS used to conduct the, 237–240
- Independent variables: as change agent in contingency tables, 205–212; choosing the appropriate statistical test or measure for, 290*e*, difference of proportions test for any two categories of the, 243–245; independent samples *t* test to compare two categories of the, 235–240; SPSS used to generate cross-tabulations of, 208–209; SPSS used to merge nominal and ordinal categories of, 210–212. *See also* Dependent variables; Variables
- Indicator (or dummy) variables: regression analysis using, 283–284; SPSS used for regression and, 285–289
- Intercept surveys: administration of the, 26–27; advantages and disadvantages of, 25–26; description of, 25; on-board bus and rail survey form of, 27
- Interval scale: calculating mean from data measured on, 105–108*w*; choosing the appropriate statistical test for, 290*e*; consultants' hourly fees paid by cities in Pennsylvania (2013) example of, 103–104*t*; description of, 70; large population sample size for variables of, 172–173, 175*fig*;

number of children per household in Mt. Pilot, South Dakota median from, 102-103t; Pearson's r to measure strength of relationship between variables of, 270-271; sample size for variables of both proportional scale and, 175-176; small population sample size for variables of, 173-174, 175fig; SPSS for calculating for ordinal variable with categories of interval/scale data, 120-122; structure of ordinal categories containing interval scale variables format, 72-74

Interview process: key respondents selected for, 200; monitoring and supervision of the, 33–34

Interviewer selection and training: description of the process of, 32; of telephone interviewers, 16–19

Introductory questions, 42-43

K

Key respondents, 200

L

Landon, Alfred, 8

- Large population sample size: determination for interval scale variables, 172–173, 175*fig*, level of confidence, level of confidence, and standard error determinants of, 167–168
- Large populations: sample size for, 167–168, 172–173, 175*fig*, sampling without replacement in case of, 178

Level of confidence: balancing risks of Type I and Type II errors with, 166; as determinant for large population sample size, 167–168; as determinant for small population sample size, 168–171*t*; as determinant of sampling accuracy, 163 Level of question wording, 60–61

Levels of measurement: applied to variables, 68–70; choosing the appropriate statistical test or, 290*e*; interval scale, 70, 72–74, 102–104*t*, 290*e*; nominal scale, 69, 210–212, 290*e*; ordinal scale, 69, 72–74, 102*t*, 103–104*t*, 210–212, 290*e*. See also Variables

Likert scale: example of question for, 130; scaled frequency distributions using, 130–132 Literary Digest poll, 8

М

Mail-out surveys: administration of, 10–11; advantages and disadvantages of, 9–10; brief return mail instructions included with, 40; description of, 8; example of introduction to, 40*e*

Manipulative question wording, 64-66

Means: analysis of variance testing difference between, 245–259; calculation of mean travel time to work in Jersey City, New Jersey worksheet, 108*w*; of children, Mt. Pilot, South Dakota, 107*w*; description of, 105; difference of proportions test, 243–245; examples of calculating, 105–108*w*; histogram of, 129*fig*–130; independent samples *t* test to calculate difference between, 235–240; normal distribution (or normal curve), 124, 128–129*fig*; paired samples *t* test to calculate difference between, 240–243; population parameters of, 136; scaled frequency distributions and calculation of, 131–132; SPSS for calculating for ordinal variable with categories of interval/scale data, 120–122; travel time to work in Jersey City, New Jersey, 107*t. See also* Sample mean

Measures of central tendency: description of, 99; mean as a, 105; median, 100–105, 124; mode, 99–100*t*, 124; negative skew distribution, 124, 125*fig*; normal distribution, 123*fig*–124, 127–130, 135–137*fig*, 138*fig*, 139*fig*–140, 144*fig*; selecting the most appropriate, 124–130; skewed distribution, 124–127*t*, 129–130; SPSS to find for individual categories of variable, 122–123; SPSS to generate measures of dispersion and, 118–120

Measures of dispersion: description of, 108–109; frequency distributions/tables, 115–118, 130– 132; police nonemergency response times, 109, 110*t*; range, 109–110; SPSS to find for individual categories of variable, 122–123; SPSS to generate measures of central tendency and, 118–120; standard deviation, 110–115*w*, 120–122, 136

Median: consultants' hourly fees paid by cities in Pennsylvania (2013) example of, 103–104*t*; description of, 100; Determination of Median Worksheet, 104*w*–105; graduate seminar student final exam scores example of, 100–101; normal distribution (or normal curve), 124; number of children per household in Mt. Pilot, South Dakota example of, 103*t*; rating of local library facilities example of, 102*t*

Mode: description of, 99; normal distribution (or normal curve), 124; political leaning, 100*t*

Montgomery city government: impression of effect of city government on business, 131*t*; scaled frequency distributions on, 130–132 Multiple responses format, 75–77

Multistage (cluster) sampling, 184-185

Ν

Negative skew distribution, 124, 125*fig*, 129 Nominal scale: choosing the appropriate statistical test for, 290*e*, description of, 69; SPSS used to merge cross-tabulations of categories of ordinal and, 210–212

Index

Nonprobability sampling, 177, 198-200

Nonresponse bias: description and issues related to, 195; efforts to compensate for, 196–198

- Normal distribution (or normal curve): area under the, 137*fig*; description of, 123*fig*; mean, median, and mode of, 124, 128–129*fig*; properties applied to hypothetical distribution of sample mean weights, 144*fig*; proportionate areas for military weights under the, 138*fig*; sampling and understanding properties of, 135–137*fig*; SPSS to help evaluate if distribution is, 127–130; standardized proportionate areas under the, 139*fig*–140. *See also* Skewed distribution
- Null hypothesis: chi-square test to identify perceived findings of, 212–213; description of, 212; example of, 153; hypothesis testing and posed, 153–154. *See also* Hypothesis testing
- Number of alternative responses format, 71-72
- Numerical details. See Final report numerical details

0

Observation: description of technique, 5; primary data gathered using, 5

One-tail (or single-direction) questions, 154–157, 159*e*–160*e*

Open-ended questions: follow-up, 53–54; using, 53; venting type of, 54

Ordinal scale: choosing the appropriate statistical test for, 290*e*; consultants' hourly fees paid by cities in Pennsylvania (2013) example of, 103–104*t*; description of, 69; rating of local library facilities median from, 102*t*; SPSS for calculating for ordinal variable with categories of interval/scale data, 120–122; SPSS used to merge cross-tabulations of categories of nominal and, 210–212; structure of ordinal categories containing interval scale variables format, 72–74

Р

Paired samples *t* test: calculating overall standard deviation of paired samples using, 240–241; choosing the appropriate statistical test or measure, 290*e*; description of, 240; SPSS used to conduct the, 241–243

Parameters (population), 136, 146-147

- Pearson's r (or coefficient of correlation), 270–271, 290e
- Phi (φ), 218-219e
- Police nonemergency response times: range of, 109–110*t*; standard deviation of, 110–115*w* Population parameters, 136, 146–147

Positive skew distribution, 124, 125 fig, 129

- Post hoc tests: description and function of, 250; SPSS used for, 257–259; Turkey's HSD (Honestly Significant Difference), 253–254*e*, 258–259
- Pretesting questionnaires, 31, 38
- Pretesting survey instruments, 31-32
- Primary data: direct measurement used to gather, 5; observation used to gather, 5; survey research used to gather, 5
- Probability sampling: characteristics of, 177–178; simple random, 182
- Proportional scale variables, sample size for variables of both interval scale and, 175–176
- Public works employees' earnings worksheet, 247w

Q

- Qualitative research, 80-95
- Quantitative research, 80
- Question formats: basic response category, 70–71; closed-ended questions, 50–53; multiple responses, 75–77; number of alternative responses, 71–72; order of response alternatives, 74–75; scaled responses, 77–79; structure of ordinal categories containing interval scale variables, 72–74
- Question phrasing guidelines: ambiguous words and phrases, 61–62; confusingly phrased questions, 62–63; double-barreled questions, 63–64; emotional words and phrases, 67–68; inappropriate emphasis, 66–67; level of wording, 60–61; manipulative information included in question, 64–66
- Questionnaire design: clarity, comprehensiveness, and acceptability of, 38; editing the completed questionnaire, 55–58; issues to consider for effective, 36–38; length of questionnaire, 54–55; process of creating the, 31; question format used in, 50–54; sequence of questions, 41–42
- Questionnaire pretesting: description of, 31; sample size used for the, 38
- Questionnaire questions: closed-ended question formatting, 50–53, 70–79; filter or screening, 47, 48¢, formatting of, 70–79; introductory, 42–43; levels of measurement used for, 68–70; logical sequence of, 46–47; open-ended, 53–54; phrasing of, 59, 60–68; related, 43–46; reliability checks, 47, 49; screening used to disqualify respondents, 48¢–49¢, sensitive, 43; series of questions demonstrating sensitizing of respondents, 45¢–46¢
- Questionnaires: coding the completed, 32–33; designing the, 31, 36–58; pretesting the, 31, 38; understood as a conversation introducing the study, 39–41. *See also* Data collection; Survey research

Questions: bias of nonresponse to, 195–198; dualdirection or two-tail, 154–157, 159*e*–160*e*; guidelines for phrasing, 60–68; questionnaire, 42–79; response formats for, 70–79; single-direction or one-tail, 154–157, 159*e*–160*e*

R

Random digit dialing (RDD), 20

- Range: description of, 109; of police nonemergency response times, 109–110*t*
- Regression analysis: calculation of the sum of squares, 267–268*w*; calculation of variances and *F* ratio, 268–269*fig*; choosing the appropriate statistical test or measure, 290*e*; coefficient of determination (*r*²), 269–270; dummy variables used in, 283–289; simple linear regression, 261–264*w*; SPSS used for, 272–279; SPSS used to perform multiple, 279–283; testing the significance of the regression line, 264–266*fig*
- Regression line: ANOVA table for regression line of subsidy and value increases, 269fig, ANOVA test for significance of the, 265-266; calculation of regression line worksheet, 264w; coefficient of determination (r^2) to indicate the total variance in the, 269-270; equation for the, 262-263; prior convictions and sentence length, 266fig; scatter plot and, 263fig; SPSS for testing the statistical significance of the, 274-279; testing the significance of the, 264-266; violent crimes and population, 265fig
- Related questions, 43-46
- Reliability checks questions, 47, 49
- Representative samples: establish a representative working population, 179–180; examples of sampling frames used to obtain, 181–187; identifying the sampling frame to select, 178–179; nonprobability sampling used to obtain, 177, 198–200; nonresponse bias issue of obtaining, 195–198; understanding probability and nonprobability sampling to select, 177–178; weighting disproportionate samples to obtain, 187–194
- Respondents: screening questions used to disqualify, 48*e*-49*e*; series of questions demonstrating sensitizing of, 45*e*-46*e*
- Responses: basic response category format, 70–71; multiple responses format, 75–77; nonresponse bias, 196–198; number of alternative responses format, 71–72; order of response alternatives format, 74–75; scaled responses format, 77–79; structure of ordinal categories containing interval scale variables format, 72–74

S

Sample mean: distribution of, 142*fig*–143; distribution of 100 hypothetical weights of, 142*t*; hypothetical normal distributions comparing to raw data, 142*fig*; properties of normal curves applied to hypothetical distribution of, 144*fig*. See also Means

Sample selection: determining procedures for, 30–31; questionnaire information on the, 39

Sample size: determination for interval scale variables, 172–175*fig*; determining the, 30–31; effect on sampling accuracy, 165–166; guidelines in the selection of, 165; for large populations, 167–168, 172–173, 175*fig*; for questionnaire pretesting, 38; for small populations, 168–171*t*, 173–175*fig*; when both proportional and interval scale variables are present, 175–176

- Sample statistics, 136
- Sample survey research: advantages of, 7–8; description of, 7
- Samples: general population or universe, 30, 179; generalizing from a single, 143–145; representative, 177–200; weighting disproportionate, 187–194; working population, 30, 179–180
- Sampling: distribution of sample means, 142*fig* 143*fig*, 144*fig*; generalizing from a single sample, 143–145; nonprobability, 177, 198–200; probability, 177–178; properties of the normal distribution implications for, 135–137*fig*, standardized *Z* scores implications for, 137–140*t*; theoretical basis of, 140–143; without replacement, 178
- Sampling accuracy: confidence interval determinant of, 163–165; effect of sample size on, 165–166; level of confidence determinant of, 163; Type I error effect on, 166; Type II error effect on, 166
- Sampling errors, 147
- Sampling frame: description and derivation of, 178–179; determining the, 30; examples of, 181–187; identification of the, 178–179; specifying the unit of analysis consideration for, 178
- Sampling frame examples: cluster (multistage) sampling, 184–185; overview of, 181; simple random sampling, 182; stratified sampling, 185–187; systematic random sampling, 183–184
- Sampling unit (working population), 182
- Sampling without replacement, 178
- Scaled frequency distributions, 130–132
- Scaled responses format, 77–79
- Screening or filter questions, 47, 48e
- Screening questions to disqualify respondents, 48e-49e

Roosevelt, Franklin D., 8

Index

- Secondary research: description of technique, 5; primary data gathered through, 5
- Sensitive questions: description and use of, 43; series of questions demonstrating sensitizing of respondents to, 45*e*–46*e*
- Simple linear regression: description of, 261–262; equation for, 262–264*w*; performed on two interval variables, 262
- Simple random sampling, 182
- Single-direction (or one-tail) questions, 154–157, 159*e*–160*e*
- Single linear regression: calculation of regression line worksheet, 264w; description and example of, 261–262*fig*; equation for the regression line, 262–263; scatter plot and regression line, 263*fig*
- Skewed distribution: negative, 124, 125*fig*, 129; positive, 124, 125*fig*, 129; salaries of employees of a small, private business, 126*e*, willingness to spend for street and highway beautification, 126–127*t*. *See also* Normal distribution (or normal curve)
- Small populations sample size: confidence interval, level of confidence, and standard error determinants of, 168–170; determination for interval scale variables, 173–174, 175*fig*; minimum sample sizes for selected, 171*t*; minimum sample sizes for variables expressed as proportions, 168–169*t*
- SPSS (Statistical Package for the Social Sciences): analysis of variance using, 255e-259; calculating a mean and standard deviation for ordinal variable with categories of interval/scale data, 120-122; chi square test of statistical significance using, 221-226; description of, 13; to find measures of central tendency and dispersion for individual categories of variable, 122-123; generating confidence intervals using, 151; generating measures of central tendency and dispersion using, 118-120; generating randomly selected cases using, 182; for hypothesis (one-sample) testing, 161-162; independent samples t test conducted using, 237-240; merging nominal and ordinal categories using, 210-211; overall satisfaction with bus service*recoded frequency cross-tabulation, 211; overall satisfaction with bus service*recoded frequency riding bus cross-tabulation, 209; paired samples t test conducted using, 241-243; performing multiple regression analysis using, 279-283; regression analysis using, 272-279; regression and dummy variables using, 285-289; testing significance of cross-tabulations using gamma, 230; testing the statistical significance of the regression line using, 274-279; weighting disproportionate samples using, 192-194. See also Data analysis

- SPSS tools: Analyze–Compare Means–Independent-Samples T Test, 238; Analyze–Compare Means– Paired-Samples T Test, 241; Analyze–Descriptive Statistics–Crosstabs, 208, 221; Analyze–Descriptive Statistics–Explore, 127–128, 151; Analyze–Regression–Linear, 279
- Spurious relationships of variables, 259-260
- Standard deviation: description of, 110–112; population parameters of, 136; SPSS for calculating for ordinal variable with categories of interval/ scale data, 120–122; worksheet for calculating for frequency distribution with single-value categories, 113*e*; worksheet for calculating for individual case data, 112*e*–113*e*; worksheet for calculating for police nonemergency response times, 114*w*–115*w*
- Standard error: as determinant for large population sample size, 167–168; as determinant for small population sample size, 168–171*t*
- Standardized Z score: description of, 137; formula for conversion of, 137–139; hypothesis testing by determining, 152–154, 157*e*; negative, 139–140
- Statistical significance: choosing among tests of measures of association and, 289–290*e*; description of, 153–154; determining if *F* ratio has, 249–250, 251*e*–252*e*; for single- and dual-direction research questions, 154–157; SPSS for testing regression line, 274–279; values of calculated *Z* scores to determine, 157*e*. See also ANOVA (analysis of variance) test
- Strata (working population), 185
- Stratified sampling: description of, 185–186; proportionate sample representation for hypothetical ethnic distribution, 186*t*–187
- Sum of squares: determining the, 267; residual sum of squares worksheet, 268w
- Survey instruments: designing the, 31; pretesting the, 31
- Survey Monkey, 13
- Survey questions. See Questionnaires
- Survey research: advantages of sample, 7–8; doubts regarding reliability of, 4; ethical considerations in, 34–35; focus groups used in, 80–95; gathering information through, 4–7; monitoring and supervision of the interview process of, 33–34; stages of process, 28–33; as technique for gathering primary data, 5; types of, 8–27; wide-spread use and acceptance of, 3–4. *See also* Final reports; Questionnaires
- Survey research process: stage 1: identifying the focus of the study and method of research, 28–29; stage 2: determining the research schedule and

budget, 29; stage 3: establishing an information base, 29–30; stage 4: determining the sampling frame, 30; stage 5: determining the sample size and sample selection procedures, 30–31; stage 6: designing the survey instrument, 31; stage 7: pretesting the survey instrument, 31; stage 8: selecting and training interviewers, 32; stage 9: implementing the survey, 32; stage 10: coding the completed questionnaires and computerizing the data, 32–33; stage 11: analyzing the data and preparing the final report, 33

- Survey research tool, 3-4
- Survey research types: in-person interviews, 22–25, 41*e*; intercept surveys, 25–27; mail-out surveys, 8–11, 40*e*; telephone survey, 14–22, 41*e*; webbased surveys, 12–14
- Systematic biases, 179
- Systematic omissions, 179-180

Systematic random sampling, 183-184

Т

- t tests: critical values of the t distribution, 159e– 160e, description of, 158; difference of proportions test, 243–245; hypothesis testing use of, 158–159; independent samples, 235–240, 290e, paired samples, 240–243, 290e, SPSS for hypothesis testing of one-sample, 161–162
- Telephone sampling: cell phone only dialing, 20–22; fixed list of telephone numbers, 22; random digit dialing (RDD), 20
- Telephone survey administration: Computer-Assisted Telephone Interviewing Program (CATI) used in, 18–19; conducting the survey, 19–20; issues of concern in, 15; monitoring and supervision of interview process, 33–34; selection of telephone interviewers, 16–17; training of telephone interviewers, 17–19; types of telephone sampling, 20–22
- Telephone surveys: administration of, 15–22; advantages and disadvantages of, 15; description of, 14; example of introduction to, 41e
- Temporary Aid to Needy Families program (TANF), 180
- Turkey's HSD (Honestly Significant Difference), 253–254*e*, 258–259
- Two-tail (or dual-direction) questions, 154–157, 159e–160e
- Type I errors, 166, 245, 250, 258 Type II error, 166

U

Unit of analysis, 178

Universe sample. See General population (or universe)

v

Variables: analyzing cross-tabulated data on, 203-234; continuous, 105; description and categories of, 68-69; discrete, 105; dummy (or indicator), 283-289; large population sample size for interval scale, 172-173, 175 fig; Pearson's r to measure strength of relationship between interval scale, 270-271; sample size for both proportional and interval scale, 175-176; small population sample size for interval scale, 173-174, 175 fig; SPSS to calculate mean and standard deviation for, 120-122; SPSS to find measures of central tendency and dispersion for individual categories of, 122-123; SPSS to merge nominal and ordinal categories of, 210-212; spurious relationships between two, 259-260. See also Dependent variables; Independent variables; Levels of measurement

W

Web-based surveys: administration of, 13; advantages and disadvantages of, 12–13; description of, 12; web panels used in, 14

Web panels, 14

- Weighting disproportionate sample (City of Davis): outdoor recreation facilities used by Davis residents, 190*t*; population of Davis planning areas, 188*t*; sample sizes and weights for City of Davis planning areas, 189*e*; sample sizes and weights for stratified sample, 191*e*; SPSS used for, 192–194; worksheet for calculating weighted citywide total for outdoor facilities, 191*w*
- Weighting disproportionate samples: example of disproportionate sampling in the City of Davis, 188*t*–194; issues to consider for, 187–188
- Working population: definition of the, 179; establishing a representative, 179–180; sample from, 30; sampling frame derived from general population through, 178–179; sampling unit in the, 182; strata groups of the, 185
- Worksheets: calculating for police nonemergency response times worksheet, 114*w*–115*w*; calculating standard deviation for frequency distribution with single-value categories, 113*w*; calculating standard deviation for individual case data, 112*e*– 113*e*; calculation of mean travel time to work in Jersey City, New Jersey worksheet, 108*w*; calculation of regression line worksheet, 264*w*; chisquare matrix (*N*= 500) worksheet, 213*w*–214; Determination of Median Worksheet, 104*w*–105; public works employees' earnings worksheet, 247*w*; residual sum of squares worksheet, 268*w*

Y

Yates correction, 220

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